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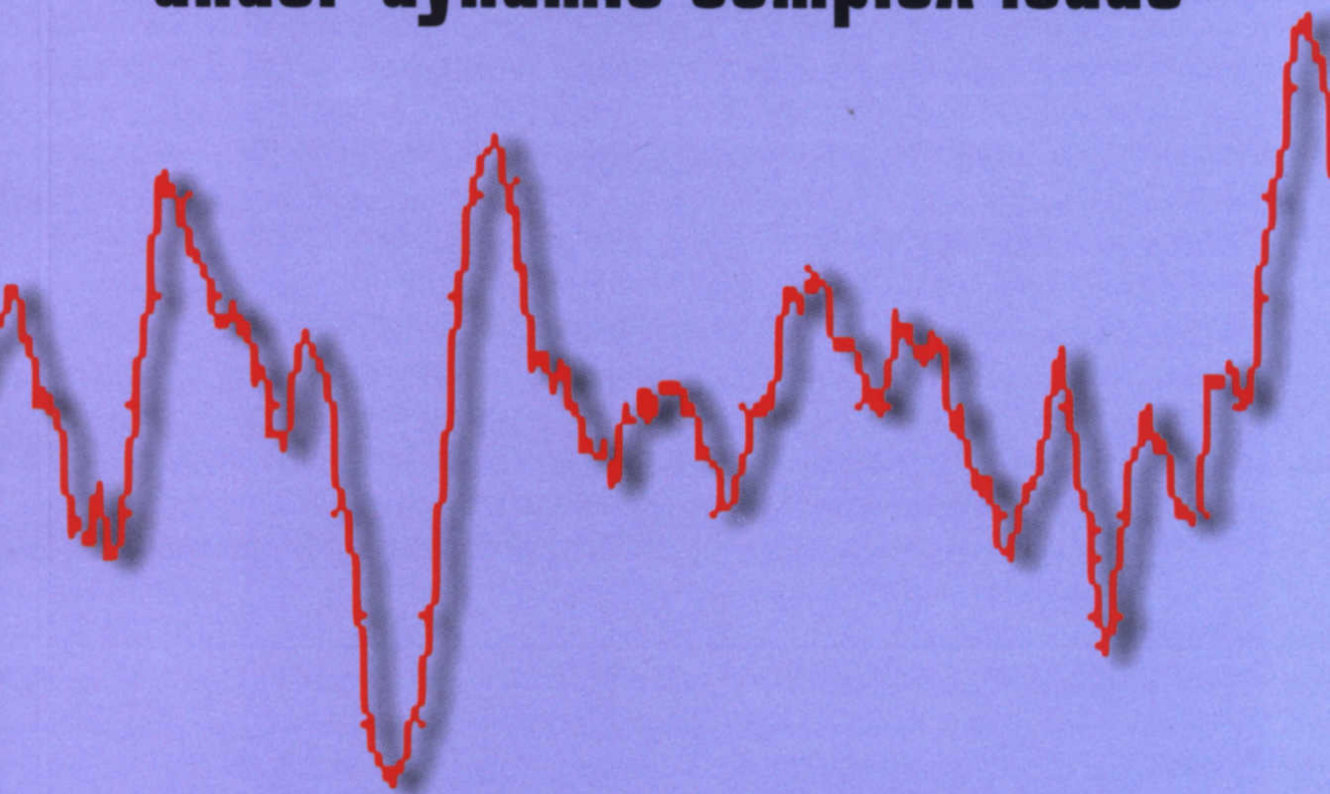
Modeling and identification of nonlinear mechanical systems under dynamic complex loads



Oficyna Wydawnicza Politechniki Wrocławskiej

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MODELING AND IDENTIFICATION OF NONLINEAR MECHANICAL SYSTEMS UNDER DYNAMIC COMPLEX LOADS

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Contents

Introduction

The measurement and proper modeling of the vibration properties of mechanical systems subjected to complex dynamical loads (e.g. random loads) are indispensable for a wide range of engineering applications. The process of constructing a model of a material system belongs to one of the most demanding branches of mechanics. Still its importance for technological applications has grown rapidly in recent years. This is due to the high requirements which a constructed model must meet with regard to the precision of the description and analysis of the behavior of the whole system under investigation. This applies particularly to automobile and aerospace applications in which the elasto-plastic properties of individual components have a significant effect on the vibrations of the whole system and the reliability in cyclic load conditions must be defined precisely.

The prevalent methods of investigating mechanical systems are the application methods related to experimental modal analysis. The method of experimental modal analysis allows one to determine the so-called "modal model" and more precisely, to ascertain its parameters such as natural frequency and modal damping, and coefficients of a modal matrix whose columns correspond to the particular forms of vibration. One should notice that a modal model determined in this way allows one to establish a system's response to any deterministic or random dynamic excitation. In this context, a question arises: How accurately does a model constructed in this way reflect the properties of the real system? If we notice that the shape of the modal model has been derived from the theory of linear systems, then the use of modal analysis in the case of nonlinear systems reduces itself to searching for an optimal linear model in which nonlinear effects, nonetheless, do not occur. It should be emphasized, however, that though the shape of the modal model is known, its concrete form does not stem from the knowledge of the structure of a real system (whose structure allows one to obtain a model in the form of differential equations by using, for example, the Lagrange equations), but is an outcome of the applied experimental method in which the system is the so-called "black box".

Therefore the method of experimental modal analysis plays rather a role of the functional identification of a real system, which does not detract, however, from its universality in the sphere of applications.

From the point of view of technological applications, the identification or modeling of a real system (so-called material system) is effective if the model reflects accurately enough the behavior of the whole construction investigated in a specified interval of its operation (e.g. a machine's service life). It can be assumed that the majority of engineering constructions fulfill both the linearity condition (the loads superposition condition) and the time invariant condition in the case of weak vibrations and in a specified (finite) time of their observation.

However, the situation is slightly different when the changes occurring in a machine's component under continuous cyclic loads are the object of our interest. As is well-known, the basic measure of such changes can be the energy loss. However, to estimate it correctly, a precise model of the vibration damping, taking into account not only the viscous friction but also the resistance to motion associated with dry friction, must be introduced. Therefore a nonlinear-type model, which would describe much more accurately the dynamic properties of the tested element of a machine than the commonly used linear model, is adopted. It is especially interesting from the point of view applications how the nonlinear damping characteristic would change during the operation of the component until, for example, its failure. It seems that the changes in the shape of this characteristic could oftentimes provide important clues as to the life of the tested machine element and thus could be used in its diagnosis (similarly as the *Frequency Response Function*). A significant breakthrough in this area would be brought about if we managed to develop some identification methods of designing material systems on the basis of a suitable gamut of nonlinear models.

The present work provides a corpus of numerous solutions in this domain, worked out mainly by the author and his team. These solutions (their concepts, including the theoretical basis and experimental verification) have been presented separately many times at various conferences, and some of them have also been published. Nevertheless, a comprehensive study in this area, covering general methods and procedures, seems to be advisable. Only this kind of treatment can bring out the totality of the presented solutions and contribute to their universal use in technological applications. However, to understand fully the methods considered here, they must be presented against the background of the general problems of the identification and modeling of material physical models, which has been done in the first chapter of this work.

All the methods discussed have been divided into two parts. In the first part (chapter 2), procedures whose use requires the selection of proper excitations (e.g. sinusoidal resonance frequency excitations) are presented, whereas methods suitable for any type of dynamic excitation, both determined and random, are described in the second part (chapters: 3, 4, 5). It should be added that some of them have been presented in Polish by Stanisław Piesiak – co-author of papers [3.1, 3.2, 4.5, 4.6] (see [3.9]). The last (chapter 6) provides general way for an arbitrary real-life dynamic system.

This book is addressed to engineering offices and groups that need people with a knowledge of the testing of mechanical systems under complex dynamical loads. The methods presented can also be useful in material laboratory testing being specialized especially in an evaluation of dynamical properties of composite materials. The readers who are interested only in practical applications can reduce the study to chapters 1 and 6 of the book.

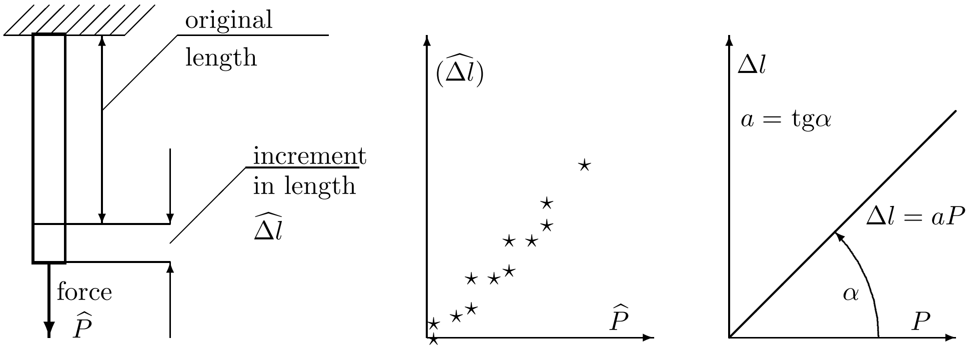
Chapter 1

Depicting of the general problem

When analyzing complex vibrations of real-life material systems (e.g. buildings, bridges, vehicles, machines, etc.) we usually apply the well-known mathematical methods related to spectral analysis, the theory of linear systems, the Laplace operator technique or correlation analysis. However, the automatic application of these methods can often lead to an erroneous interpretation of the results. This can happen especially in the area of the testing of mechanical systems. Therefore, let us begin with a sketch of a certain way of perceiving obvious things whose nature one does not fully comprehend.

Those who deal mainly with real mechanical systems (so-called physical systems) stand at the boundary of two worlds. On the one hand, having abstract notions (such as a square, a circle, a function, an operator, density, acceleration) at their disposal, they believe in the reliability of the methods based on these notions. On the other hand, often frustrating experiences from their practice usually connected with discrepancies between the results of observation and theory, teach them to treat calculated results with some reserve. They often find the latter baffling or sometimes even shocking. We shall call this other world which can be explored through our senses (sight, touch, smell, etc.) the **world of material facts**. Let us also notice that the world of material facts is independent of man (e.g. he/she cannot change the sense of the gravitational force). Whereas man governs the **world of abstractions**, which he/she creates according to his/her wishes and needs (similarly as in the case of rules and methods of playing chess).

For instance, while observing the relationship between the increment in a bar's length and the value of the tensile force (Fig. 1.1a), the results of the observation itself can be presented schematically as in Fig. 1.1b. At this stage, by selecting from the arsenal of abstract notions such concepts as a variable and a linear function, we are constructing something which we shall call a model of our system. Let us then assume that the cause (i.e. the force) will be described by real variable P, whereas the increment in length (the effect), by real variable Δl (see Fig. 1.1c). Because of certain "similarity" between observation results



(a) diagram of the real system

(b) representation of the results of observation

(c) a proposed model in the form of a linear function

Figure 1.1. A sample description of a material system

(b) and linear function (c) we decide to use linear function $\Delta l = aP$ to describe our system, where the fitting of value a constitutes a separate problem (lying in the sphere of approximation and optimization). Naturally, here we can try to adopt, for example, random variables instead of real variables P and Δl, or assume a nonlinear function $\Delta l = a_1 P + a_2 P^3$. Therefore, we come to the conclusion that the choice of a model for the system depends on the individual who deals with the problem. In our example, the real bar (Fig. 1.1a) will be called a material system or an object of an inquiry, whereas its linear function $\Delta l = aP$ will be labeled a model. Variable Δl (the effect) will be referred to as a model of the object's output quantity (a model of length increment), whereas variable P (the cause) will be called a model of the object's input quantity (a model of force).

In other words, we can say that in the world of material facts we find a relationship between empirical variable $\widehat{\Delta l}$ (length increment) and empirical variable $\widehat{P}$ (force), which we model by a linear function that assigns real variable P to real variable Δl (see Fig. 1.2).

Another example of a physical system can be a motor mounted on a shop floor (Fig. 1.3). If we are interested in the movement of the floor produced by the motor's rotation at a specified speed, then we usually reduce the cause (the input quantity) to certain force whose model can be certain sinusoidal function $p(t)$.

Whereas the effects (the movement of the floor) can be described by a certain number of time-dependent functions $x_i(t)$, put in the form of certain

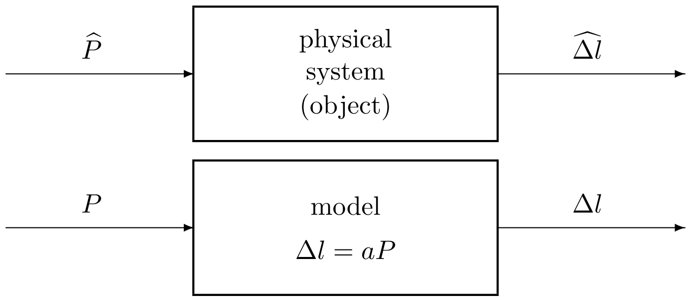


Figure 1.2. An object and model as a relation between input and output quantities

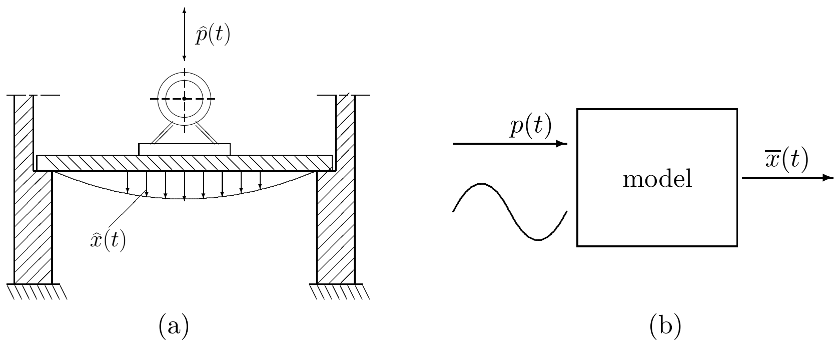


Figure 1.3. A diagram of a physical system (a) and its model (b)

vector $\overline{x}(t) = (x_1, x_2, \ldots, x_n)^T$. The definition of the relationship between $p_i(t)$ and $\overline{x}$ is more complicated.

Let us now consider a more complex system, such as a bridge, whose movements (usually vibrations) are produced by the vehicles going over it and by weather conditions (jumps in temperature, winds, etc.). In this case, the definition of "input quantities" becomes much more complex. If by the "input quantities" of a material system (an object) we understand all the causes producing specific changes (movements) of this system, then we should specify the number of vehicles, as well as their speeds, accelerations and positions, directions and velocities of winds, jumps in temperature, etc. The same applies to the notion of an object's "output quantity". We can observe the movements of a specific point on the bridge in three different directions (the vertical one and two horizontal ones), and there is an infinite number of such points. A general scheme of an object can have the form shown in Fig. 1.4. By an object (a physical system) we shall understand a certain observed relationship between the set of output quantities (the effects) and the set of input quantities (causes). In the case of dynamic systems (the causes and the effects are time-dependent),

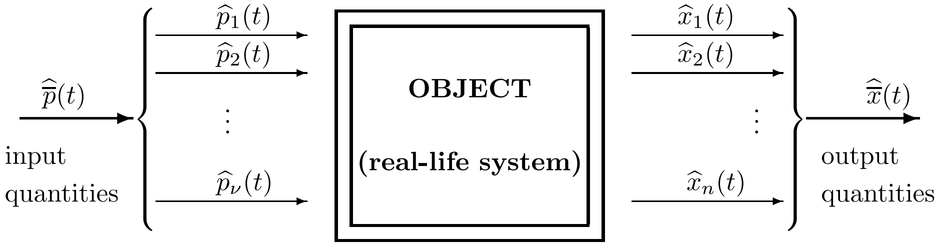


Figure 1.4. A general scheme of a complex dynamic object

input quantities and output quantities are modeled by means of deterministic or random time functions (random processes). The use of scientific methods in the investigation of the real world of physical facts consists in the adoption of a specific model fitting the analyzed phenomenon (an object of the inquiry). Such a model constructed on the basis of exact abstract notions (e.g. concentrated force, random function, operator, vector, etc.) is then subjected to close scrutiny. The model should be constructed on the basis of a carefully planned identification procedure which (when carried out properly) would eliminate if possible the often deceptive engineering intuition.

For example, if we consider a typical vibrating system consisting of mass supported horizontally by an element having elasto-damping properties (e.g. a rigidly fixed beam made of a specific material whose dynamic properties are unknown (see Fig. 1.5)), then the equation of dynamic equilibrium for mass shall be written as

$$m\ddot{x} + F(?) = p(t) \tag{1.1}$$

where the particular symbols denote: m – real variable describing the value of mass $\widehat{m}$, $\ddot{x}$ – function of a real variable of time t describing acceleration $\widehat{a}$, $F(?)$ – certain function with unknown arguments (usually assumed as a function of speed $\widehat{\dot{x}}$ and displacement $\widehat{x}$ of mass $\widehat{m}$)) which describes the effects of the element (a beam) on mass $\widehat{m}$, $p(t)$ – certain real function describing an exciting force.

It is then assumed (most often in this case) that the dynamic properties of the elasto-damping element are described by the elastic coefficient (c), and damping coefficient (k). This approach follows from the adoption of a linear model whose configuration is parallel (see Fig. 1.5 model (a)). However, we could just as well assume for this case a model in other form (see Fig. 1.5). A parallel-configuration model (model (a)) is commonly assumed for complex systems with many degrees of freedom because the adoption of a model in this form for each elasto-damping component of a machine leads to a system of

Physical system $\Longrightarrow$ physical model $\Longrightarrow$ mathematical model

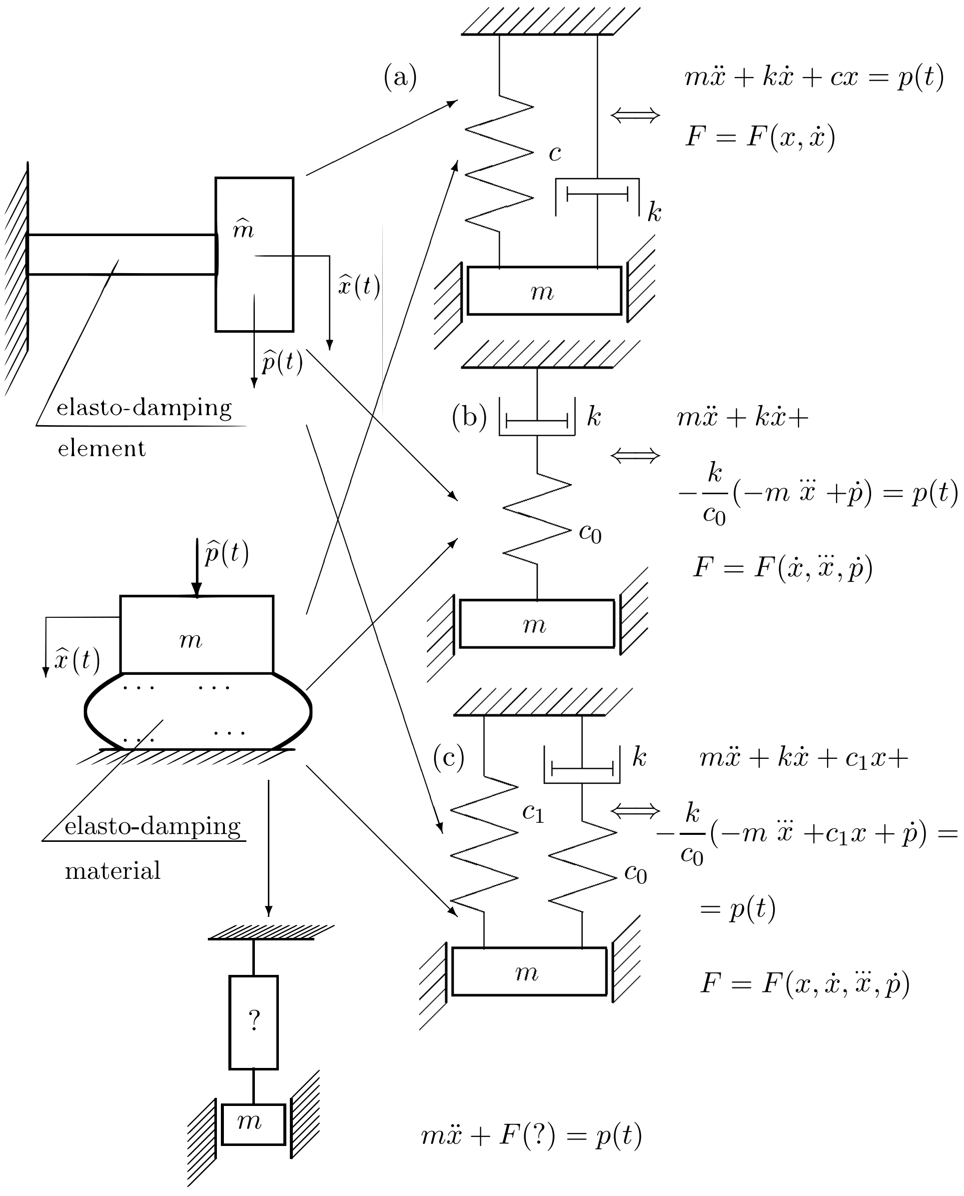


Figure 1.5. It is the art to assume "a priori"
a specific proper class of model of the function $F(?)$

linear differential equations with constant parameters, the theory of which is well known and which are easy to apply now (professional computer software).

It is worth mentioning that a professional method of the experimental analysis of dynamic systems called **Experimental Modal Analysis**, has been developed [1.1, 1.2, 1.5, 1.6, 1.19–1.22]. The method is described from the theoretical viewpoint, for example, in [1.19], while interesting practical applications can be found in papers [1.15, 1.18]. To apply the experimental modal analysis method in practice, one needs special equipment (multi-channel analyzers, impact testing machines, accelerometers). There are, however, situations when linear physical models can simplify a real system too much.

We shall exemplify this using one of the real systems shown in Fig. 1.5. Let us assume that this system functions exactly in accordance with the following equation

$$m\ddot{x} + h\,\mathrm{Sgn}\ v + k_1 v + k_3 v^3 + k_5 v^5 + cx = p(t) \tag{1.2}$$

($v = \dot{x}$) and that under continuous cyclic loads, the dynamic properties of this system change gradually (aging of the material, creep, etc.). The damping properties of the dynamic system are defined by the parameters of certain (nonlinear!) function $F_d(v)$ which describes a damping interaction force and which is dependent usually on velocity. Thus when the dynamic properties of a real system change (fatigue), it follows that parameters of the function $F_d(v)$ also can change gradually in time (see Fig. 1.6). Thus, the form of a real system damping function can change gradually. One should note that the damping function can provide valid indications as to the necessity of replacing the elasto-damping element of a machine.

However, to ascertain this, one should know how to **determine the form of the damping function of a given elasto-damping element of a machine at any time in its service life**. Therefore it becomes necessary to develop new research methods (identification procedures) based on complex nonlinear dynamic models.

The construction of a mathematical model of an object on the basis of measured outputs $\widehat{\overline{x}}$ and inputs $\widehat{\overline{p}}$ can be divided into two stages:

1. **Modeling** which consists in determining the model's "class", i.e. operator $\overline{x} = \varphi(\overline{p}, t, D)$, where $\overline{p}, \overline{x}$ are called the model's input and output values, respectively.

2. **Parametric identification** whose aim is to determine such constants of matrix D so that operator φ will describe the investigated object best (according to a specific criterion) (see Fig. 1.7).

A typical example of this process in the dynamics of material systems is the determination of the values of mass m_{ij}, damping k_{ij} and rigidity c_{ij}, i.e.

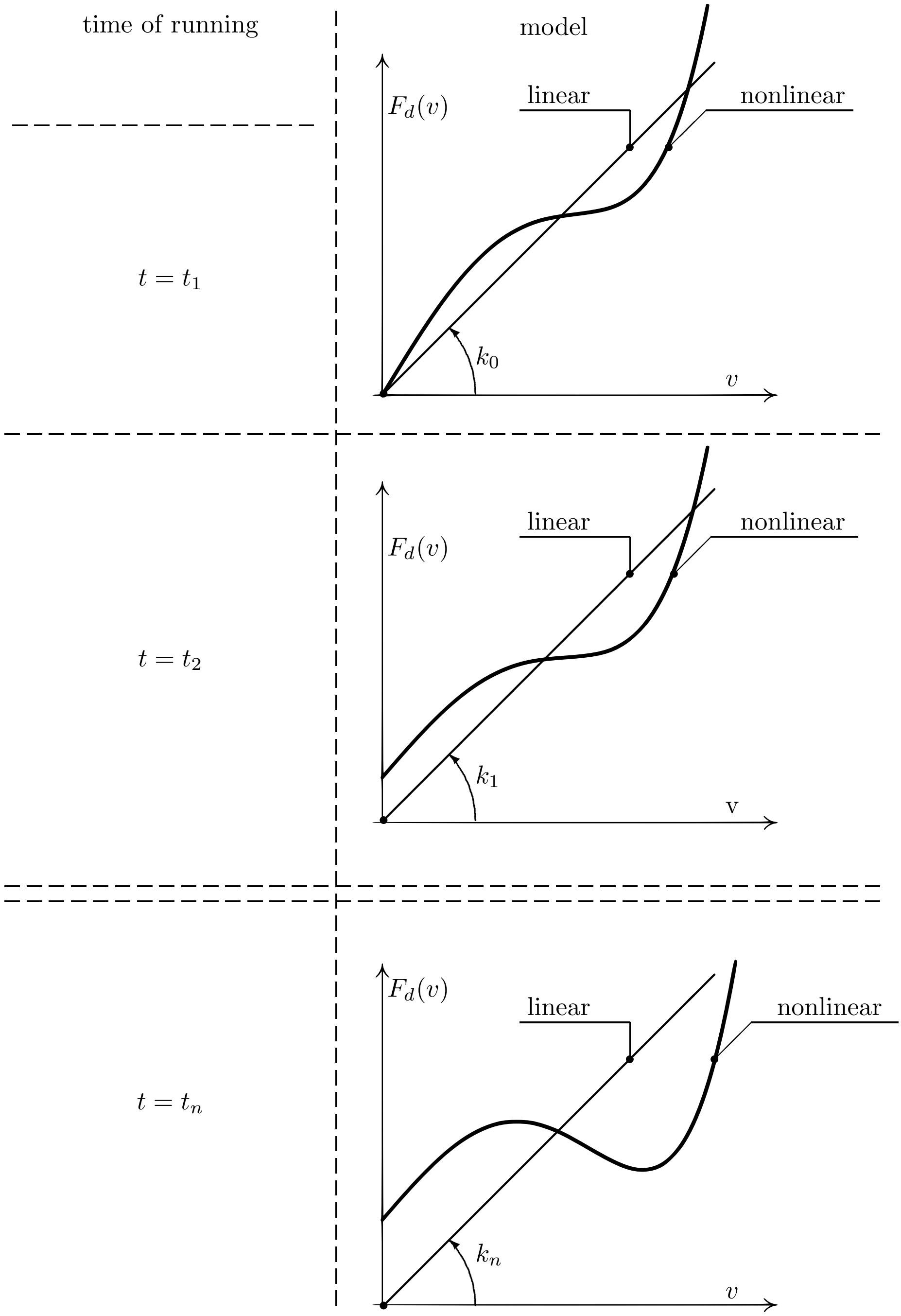


Figure 1.6. Some changes of damping functions during the service life can be noticed on the basis of nonlinear models ($k_0 = k_1 = \ldots = k_n$)

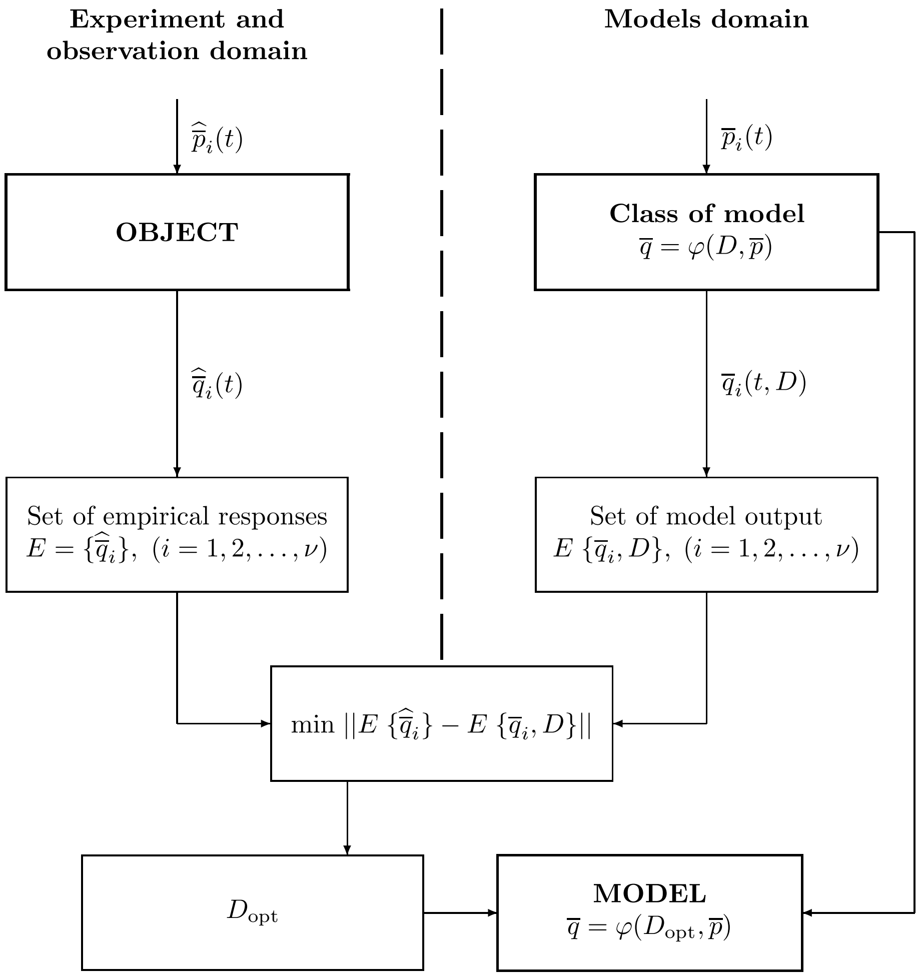


Figure 1.7. A general scheme of the typical parametric identification process

the terms of matrix $D = [\mathbf{M}, \mathbf{K}, \mathbf{C}]$ of an a priori assumed model in the form

$$\mathbf{M}\ddot{\overline{x}} + \mathbf{K}\dot{\overline{x}} + \mathbf{C}\overline{x} = \overline{p} \tag{1.3}$$

where $\overline{p} = (p_1, p_2, \ldots, p_N)^T$, $\overline{x} = (x_1, x_2, \ldots, x_N)^T$ [1.2, 1.7–1.9]. Depending on what identification criterion is assumed (this consists in the adoption of a specified measure of "distance" δ between the model and the object: $\delta = ||\widehat{\overline{x}} - \overline{x}||$, if $\widehat{\overline{p}} = \overline{p}$), there is a great variety of methods of determining matrix coefficients $\mathbf{M}$, $\mathbf{K}$, $\mathbf{C}$, difficulties with the identification are encountered only in the case of very high values of number N. For linear systems with a large number of degrees of freedom there are modal analysis methods available that have been developed in recent years.

A model in the form of the system of equations (1.3) is usually based on the prior construction of so-called physical model [1.4, 1.16] which is a discrete mechanical system with N degrees of freedom with generalized displacements and exciting forces.

A typical way of constructing a model of a real object, consists of the two activities:

– **the idealization of the object** aimed at obtaining a material model (e.g. by assuming that certain components of the object are ideally rigid, the foundation is motionless, the linear elastic stresses are without damping or with viscous damping, etc.)

– **the close analysis of the physical model** by means of laws of mechanics (d'Alambert's principle, the Lagrange equations) aimed at obtaining the so-called mathematical form of the model.

A schematic diagram of this method of searching for a reliable model of an object is shown in Fig. 1.8. Since the model's class can be determined in this case through analytical considerations, we are dealing here, from the point of view of the theory of identification, with the so-called "gray box". Such a model, however, can describe the object inaccurately if its idealization is not accurate enough. Therefore, the "construction" of a physical model is regarded to be of fundamental importance for the process of identification in the dynamics of mechanical structures.

If one assumes that the physical model of the dynamic object under investigation is a certain material system, then the preliminary activities consist in:

a) the determination of the number of particles, and the number and quality of constraints (i.e. determination of the degree of freedom),

b) the assumption of a specific form of the internal forces reacting between the particular particles,

c) the adoption of a specific form of the external forces acting on the particles of a discrete mechanical system, i.e. a model of the object's input signals (a model of exciting forces $\overline{p}$).

The assumptions about the character of the internal forces (mentioned in point b) always raise most doubts. If the aim of the activities mentioned in point a) is to perceive the similarities in the internal organizational structure between the construction investigated and a specified discrete dynamic system and those in point c) to approximate measured time run by specific time function $p(t)$ the assumptions in point b) are usually made without any preliminary investigation and that is why they determine the so-called distance

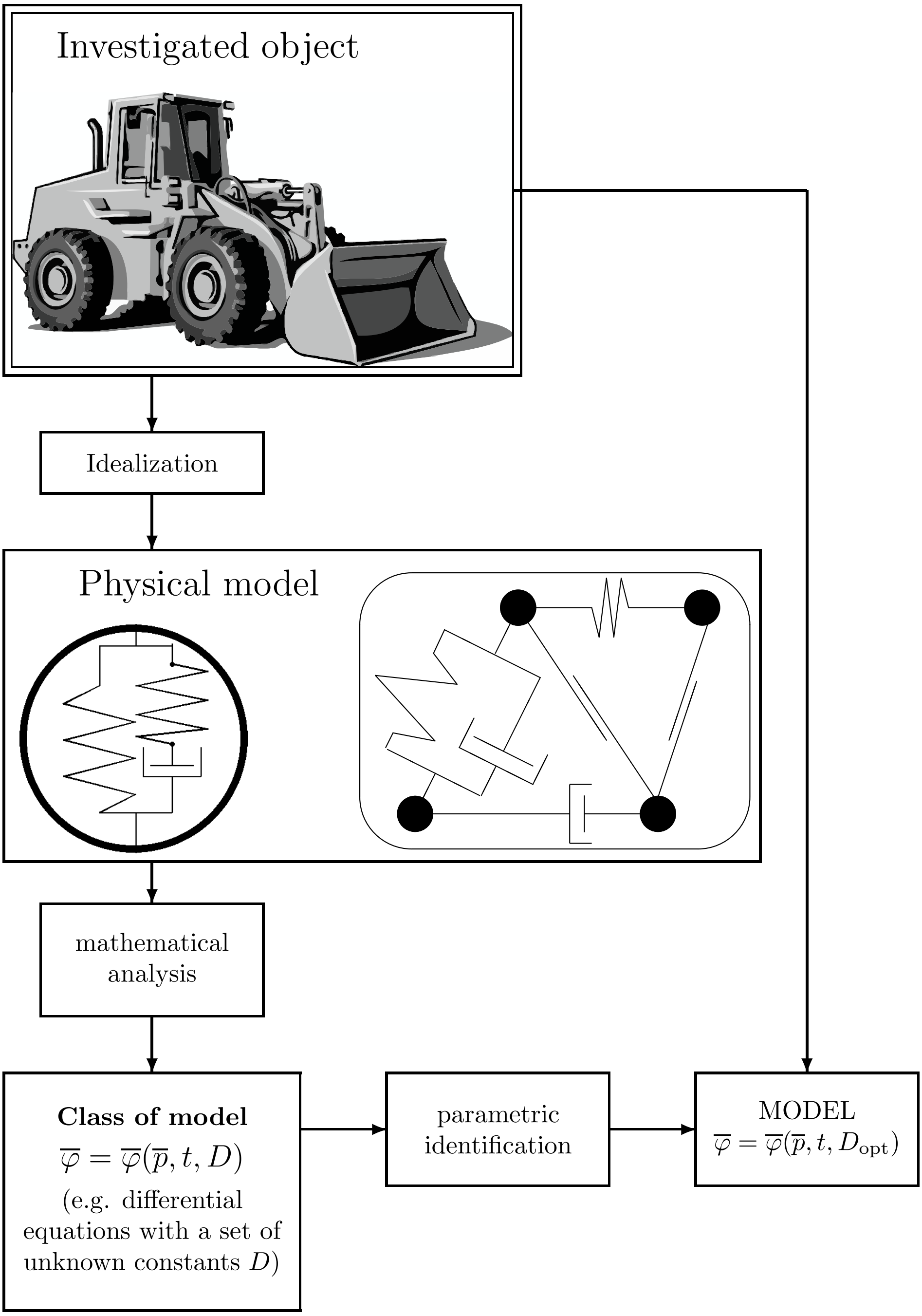


Figure 1.8. A typical approach to the modeling of dynamic objects in mechanics

of the model's ultimate form from the analyzed object. According to a generally advanced hypothesis, "elastic force" $F_s(x)$ and "damping force" $F_d(\dot{x})$ act parallelly – usually as linear functions of arguments $x, \dot{x}$, where x denotes the relative displacement of two selected particles of a discrete mechanical system (see Fig. 1.9). Consequently, a model in form (1.3) is obtained. This model describes precisely enough most of the mechanical structures, especially in the case of weak vibrations, and being relatively simple, it is easy to use. Nevertheless, the requirements which modern mechanical constructions must meet, especially the ones stemming from the need to save materials and energy, force persons dealing with the dynamics of machines and structures to use models more precise than model (1.3). Such models will be, obviously, more complex and thus more difficult to handle in engineering practice, but this does not constitute an insurmountable obstacle because of the development of computer technology.

One of the procedures which will be considered in this book is created without assumptions b), i.e. no concrete form of the internal forces is assumed a priori. The primary task of the methods is to determine the internal forces treated as certain unknown functions of velocity and displacement (Fig. 1.10).

To determine precisely function $F(x, v)$ which describes the dynamic properties of a selected element of a given mechanical structure, this element must be isolated maximally during testing. Ideally, the test stand should be built in such a way that the element could be treated as a system with one degree of freedom. This is associated with costs of the experiment (the disassembly of

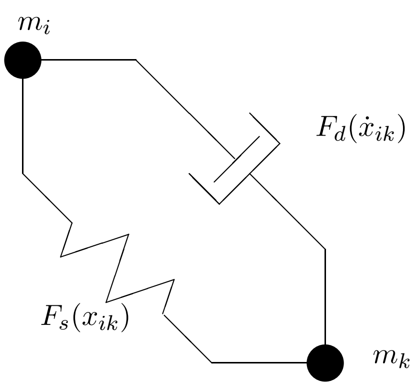


Figure 1.9. A schematic diagram of the commonly assumed parallel configuration of idealized components in a discrete material system

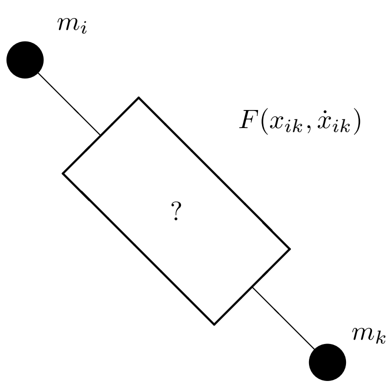


Figure 1.10. A linking element with an indeterminate configuration

the construction, the disabling of some of its segments, etc.). Hence a recent trend in methods of the identification of complex material systems consists in, for example, such controlling of the internal forces (excitation) that they generate such vibration of a selected component of the construction, which will be in a specific way "similar" to the vibration of a system with one degree of freedom. This approach has been used commonly in the domain of linear systems, but the same concept has been applied successfully also to some nonlinear systems (e.g. [1.12]). That is why the identification of a system with one degree of freedom, in which the whole problem of identification reduces itself to the determination of function $F(x, \dot{x})$ in the following equation

$$m\ddot{x} + F(x, \dot{x}) = p(t) \tag{1.4}$$

describing the motion of a particle with constant mass m, is treated in this work as the most important case.

The problem of determining the proper form of function F, treated as certain relation $F(x, \dot{x})$, has been considered many times in papers dealing with the identification of nonlinear systems. It is worth mentioning here works by Kononienko and Plachtenko in which methods that allow one to obtain function $F(x, \dot{x})$ in a linearized form (additional dependence on the amplitude of vibration) under certain added preliminary assumptions as to its shape are described [1.11].

In 1979, Masri and Caughey pointing to the drawbacks of the then existing methods of nonparametric identification, presented [1.13, 1.14] an identification procedure whose idea consists in the direct measurement at the same instants of all the variables of equation (1.4) (i.e. values $\ddot{x}_k = \ddot{x}(t_k), \dot{x}_k = \dot{x}(t_k), x_k = x(t_k), p_k = p(t_k)$ for $k = 1, 2, \ldots, K$) and the approximation by Czebyshev polynomials of empirical relation $\widehat{F}_k = \widehat{F}_k(x_k, \dot{x}_k)$, obtained as follows

$$F_k(x_k, \dot{x}_k) = p_k - m\ddot{x}_k.$$

This book presents, among other things, two methods (see chapter 2) that allow one to verify experimentally the assumption which neglects component $\Delta F(x, \dot{x})$ of function $F(x, \dot{x})$ written as

$$F(x, \dot{x}) = F_s(x) + F_d(\dot{x}) + \Delta F(x, \dot{x}). \tag{1.5}$$

The first of them requires periodic excitation producing a harmonic response in the system under investigation, while the second one consists in the passive observation of free vibration. One should notice that the absence of component

$\Delta F(x, \dot{x})$ corresponds to the so-called parallel configuration of a material model (see Fig. 1.5a) whose motion is described by the following equation

$$m\ddot{x} + F_d(\dot{x}) + F_s(x) = p(t). \tag{1.6}$$

Then a method of determining damping function $F_d(\dot{x})$ for systems described by (1.6) was devised, assuming that the elastic force is described by any nonlinear function $F_s(x)$ (section 2.4). The method was extended (section 2.5) to systems with many degrees of freedom, but having a chain-like structure (see Fig. 2.47). The method presented in section 2.2 is, however, somewhat different since it is based on the so-called universal material model (see Fig. 2.3), for which the equation of motion of mass m assumes the following form

$$m\ddot{x} + F(x, \dot{x}, \dddot{x}, \dot{p}) = p(t).^1 \tag{1.7}$$

The primary task of this method is to describe accurately such phenomena occurring in mechanical constructions, which in order to be incorporated require that the spring and the damper are connected in series (see Fig. 2.1b), or maybe also that the so-called Reid spring is introduced into the material model. Because both the methods described in section 2.3 and the method presented in section 2.2 can be used to determine the proper configuration for the material model of an investigated dynamic system (especially, to verify hypothesis about parallel configuration constituting a basic assumption for the method described in section 2.4), they can be treated as preliminary to the procedures described in sections 2.4 and 2.5. For this reason, the general procedure of using these methods is explicated in the last section of chapter two (section 2.6). This procedure allows one to determine a form for the mathematical model of a real system. However, to carry out tests according to this procedure, specially controlled dynamic excitations must usually be used. Therefore a separate group of methods suitable for any excitations is presented in the further chapter. And so the methods described in section 3 were developed to be used in arbitrary operating conditions. Therefore they can be used for any periodic excitations, both continuous and impulse ones. Similarly the methods explicated in chapter 4 can be applied in the cases when random input functions with an arbitrary but stationary probability distribution act on the object. However, the use of these methods requires the prior knowledge of the model's form and thus they belong rather to methods of the parametric identification of dynamic systems. For this reason, the recommended methodology of applying all the methods described in this work to a concrete physical system is presented in the last chapter.

[1] It is a system with 1.5 degree of freedom [1.10] and hence the relationship between force F and variables $\dddot{x}$ and $\dot{p}$.

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Chapter 6

Conclusion. Application guidelines

In this chapter a review of the methods described in the previous chapters, focusing on their application is presented. Against the background of the general problems (chapter 1), we shall show possible applications of these methods to the modeling and identification of elasto-damping elements (EDE) of machines.

As has been established (see chapter 1), the following two stages can be distinguished in a typical process of the identification and modeling of real systems:

a) determining the class of a model,

b) determining the values of the parameters of a model of this class (parametric identification).

When such a model is constructed for static load conditions, the relationship between the effect (e.g. deformation) and the cause (the loading of the structure) is usually defined by a function whose form (class) is determined on the basis of certain "observation" of the results of the experiment. In the case of one input value and one output value (e.g. tension, compression), this "observation" is limited to the visual determination of the empirical relation between these variables systematized in a specified way (e.g. points on plane xy). The situation gets slightly more complex when there is a larger number (more than two) of independent variables. Then it becomes impossible to determine the type of relation $y(x_1, x_2, ..., x_n)$ on the grounds of the overall "visual shape" of empirical relation $\widehat{y}(\widehat{x}_1, \widehat{x}_2, ..., \widehat{x}_n)$. A typical approach in such a case is to a priori assume the following linear dependence

$$y = c_0 + c_1x_1 + c_2x_2 + \cdots + c_nx_n \tag{6.1}$$

as a certain "output" form of a model of empirical relation $\widehat{y}(\widehat{x}_1, \widehat{x}_2, ..., \widehat{x}_n)$ and to determine the optimal values of parameters $c_0, c_1, c_2, \ldots, c_n$ using the regression analysis [2.22, 3.6]. The specific parameters of this analysis (the standard deviation, the confidence intervals and the correlation coefficients) determine the degree of accuracy of the model obtained but only for the range of variables $x_1, x_2, ..., x_n$ in which the function was tested.

We have to deal with a similar but much more difficult situation when modeling phenomena produced by variable-in-time input quantities. For example, to analyze fast-changing cyclic loads acting on the supporting structure of a machine a rather accurate dynamical model is required. If one assumes that input quantities in such a case are defined by certain excitation column vector $\overline{p}(t)$ and output quantities – by certain generalized displacement vector $\overline{q}(t)$, then relation $\overline{q}(\overline{p})$ must have the character of an operator since variables $\overline{q}$, $\overline{p}$ are time functions.

A linear model is a priori assumed (similarly as in statics) for typical vibrating engineering systems. It usually has the form

$$\mathbf{M}\ddot{\overline{q}} + \mathbf{K}\dot{\overline{q}} + \mathbf{C}\overline{q} = \overline{p} \tag{6.2}$$

and is determined on the basis of either the knowledge of the structure of a real system or an analysis of measured response $\widehat{\overline{q}}(t)$ to specified set of excitations $\widehat{\overline{p}}(t)$ (e.g. by means of experimental modal analysis methods [1.1, 1.17–1.20]). In both cases, however, a form of the model depends on the test vibration range (the range of excitation frequency and amplitude). If this range is changed, not only the values of the model parameters within the same class need to be changed but also nonlinear elements of specific form must be added to this model. Therefore to determine the ultimate form of the model, one must specify the range and the kind of dynamic excitations which act on the real system. Since this is closely connected with the goal of modeling, we shall discuss these aspects in the context of the modeling of elasto-damping elements of machines.

Taking into account the fact that each element of a real system in mechanics has its mass and is deformable under the action of forces, it is difficult to define precisely an element like this. Nevertheless such elements are distinguished in real systems by reason of their relatively high flexibility and small mass (e.g. springs, rubber suspensions, pneumatic suspensions, etc.). The flexibility characteristics of such elements play a special role in a system and are usually selected specifically for this purpose (e.g. a windscreen pillar in a bus, a bar in a plane's or helicopter's load-bearing structure, etc.).

In a real system, an elasto-damping element carries over dynamic loads from part $\mathcal{A}$ to part $\mathcal{B}$ of the system, being subjected itself to variable-in-time deformations. It is extremely difficult to model it in such conditions and that is why two parameters associated with a linear parallel-configuration model, i.e. coefficient of elasticity (c) and viscous damping coefficient (k), are usually assumed and determined in such cases (see Fig. 6.1). Since values of these parameters influence strongly the so-called modal parameters of the system, an experimental modal analysis is generally used to determine them. However,

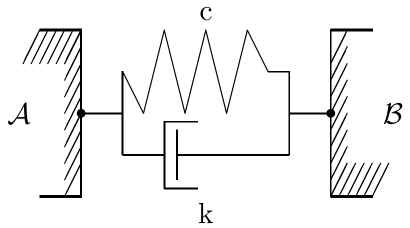

Figure 6.1. Commonly used model of EDE of a real system

even a simple static loading and unloading test carried out on the element reveals a certain shortcoming of this model because the linear reaction function

$$F(x, v) = kv + cx \tag{6.3}$$

does not describe the phenomenon of hysteresis.

As is well known, the simplest description of this system requires, apart from a model of linear viscous damping, a model of dry friction. Therefore it becomes necessary to use nonlinear modeling. Then the motion in the case of single mass m would be described by the following equation

$$m\ddot{x} + F(?) = p(t). \tag{6.4}$$

The force of reaction of an elasto-damping element (EDE), acting on mass $\widehat{m}$ of the system investigated is described by unknown function $F(?)$ (see Fig. 6.2).

Therefore we shall now consider the problem of determining the class of function $F(?)$, which resolves itself to the determination of:

– the independent variables (i.e. what to put in place of question mark "?" in function $F(?)$),

– the class (form) of function $F(?)$,

– the values of the constant parameters of function $F(?)$ (so-called parametric identification).

Let us notice that the assumption that function $F(?)$ depends only on velocity v and displacement x, i.e. $F(?) \equiv F(x, v)$ can indeed be erroneous. It is obvious that this assumption is erroneous for a system in which the EDE consists of real springs connected as in Fig. 6.3 with a real damper. This can be demonstrated through a careful analysis of the differential equations for such a system (by reducing two equations to one having form (6.4)). However when the EDE is made of an elasto-plastic material, the formulation of an equation which would correspond exactly to (6.4) and the definition of a set of independent variables in terms of function $F(?)$ become problematic. In the latter case, i.e. the determination of a form of function F for a specified set of independent variables, the procedure generally consists in the a priori assumption of a specific class for this function. Most frequently the function is assumed as separated into purely elastic part $F_s(x)$ and purely dissipative part $F_d(v)$, i.e.

$$F(?) = F(x, v) = F_s(x) + F_d(v) \tag{6.5}$$

where $F_s(x) = cx$, $F_d(v) = kv$ are linear functions. This approach leads further to considerable difficulties in the determination of constant parameters of these functions if assumption (6.5) does not occur in the real system. This is so because different methods (a free vibration damping decrement, a harmonic-excitation frequency characteristic, static tests, etc.) used to determine these parameters yield different evaluations of these parameters.

Therefore it becomes essential to develop various methods (methods based on different criteria) of determining the constants of the models because the scatter of results indicates to what degree our assumption concerning the class of function $F(?)$ is correct. Thus methods of identifying dynamical systems for various types of dynamic excitations (e.g. random pulse excitation) should be developed.

Now a brief review of solutions in this field, suitable for the modeling of dynamical properties of elasto-damping elements of machines follows. The aim is to provide a certain methodology for the identification and modeling of machine elements subjected to cyclic dynamic excitations. If model of the

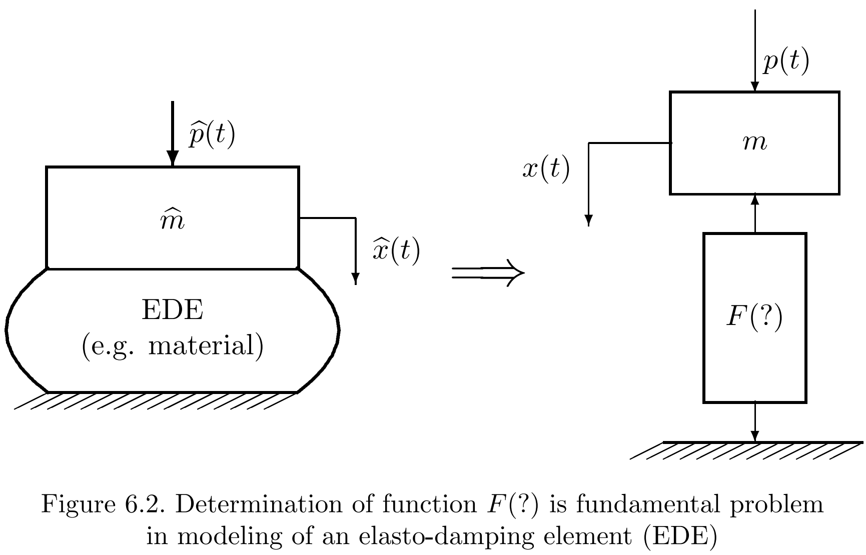

Figure 6.2. Determination of function $F(?)$ is fundamental problem in modeling of an elasto-damping element (EDE)

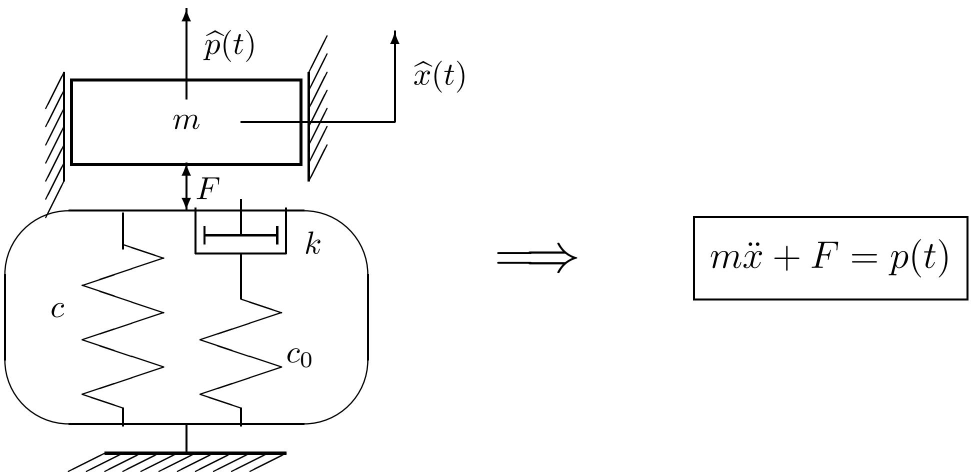

Figure 6.3. An example of EDE for which $F(?) \neq F(x, v)$

function F is to be accurate both for dynamic excitations and static loads, the identification criterion must include the information associated with the two kinds of loads.

As is shown in [6.9], one of the ways of searching for the class of function $F(?)$ consists in the application of an identification algorithm to an output model with an a priori assumed, possibly complicated configuration and strongly nonlinear dissipative-elastic elements followed by the verification of a hypothesis that certain constant parameters within set D of this model are equal to zero (see Fig. 6.4). This procedure, however, requires identification algorithms for a wide range of nonlinear dynamical systems. A typical example here is the method developed for a universal-configuration system with so-called Reid spring (cf section 2.2). In this method, it is a priori assumed that the EDE investigated works as a system whose complex configuration is shown in Fig. 6.5. This system covers set $\mathcal{D}$ which contains six constant parameters, i.e. h_0, h_1, k, c_0, m, c. It is assumed that the system includes two nonlinear elements described by the following functions

$$R_1(v) = h_0 \operatorname{Sgn} v, \quad R_2(x, v) = cx + h_1|x| \operatorname{Sgn} v \tag{6.6}$$

where function $R_1(v)$ describes the so-called dry friction and function $R_2(x, v)$ is the so-called Reid spring (cf chapter 2).

Let us notice that identification carried out for this type of a model may result in the following radical extreme cases (see [6.10]):

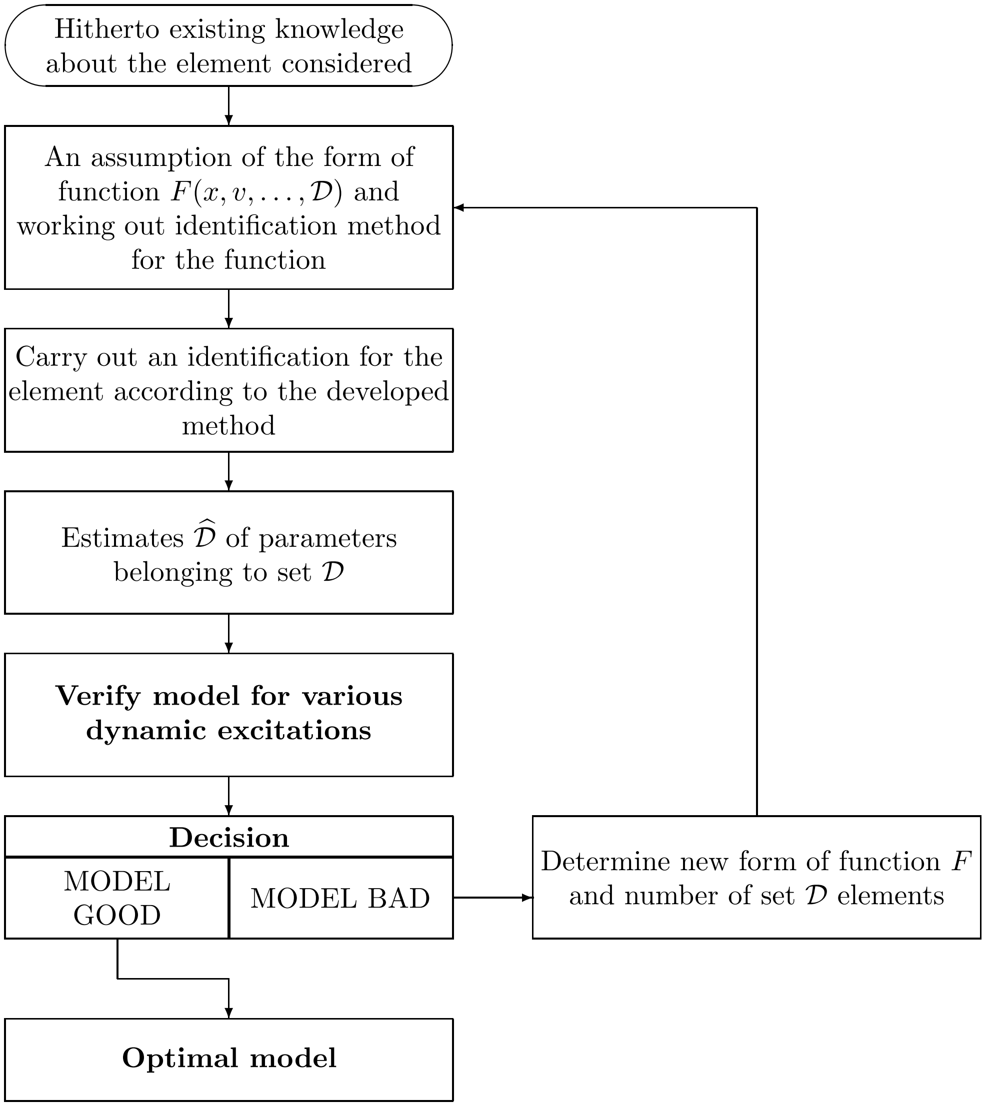

Figure 6.4. The way of searching for an optimal function F (acc. to [6.9])

1) parameters h_0, h_1 equal to zero and parameter $c_0 = \infty$ – the EDE works as a parallel-configuration linear system of the Kelvin type (Fig. 6.6a);

2) parameters h_0, c, h_1 equal to zero – the EDE works as a Maxwell-configuration linear system (Fig. 6.6b);

3) parameters h_0, h_1 equal to zero – the EDE works as a universal-configuration linear system of the standard type (Fig. 6.6c).

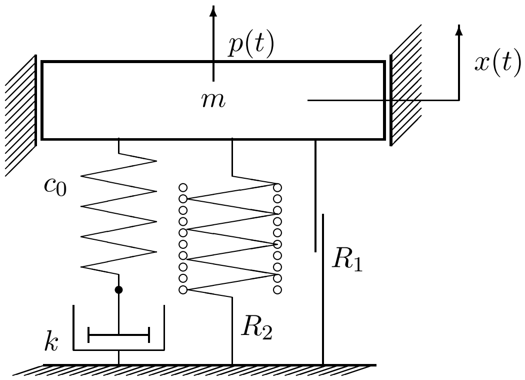

Figure 6.5. A diagram of the output model

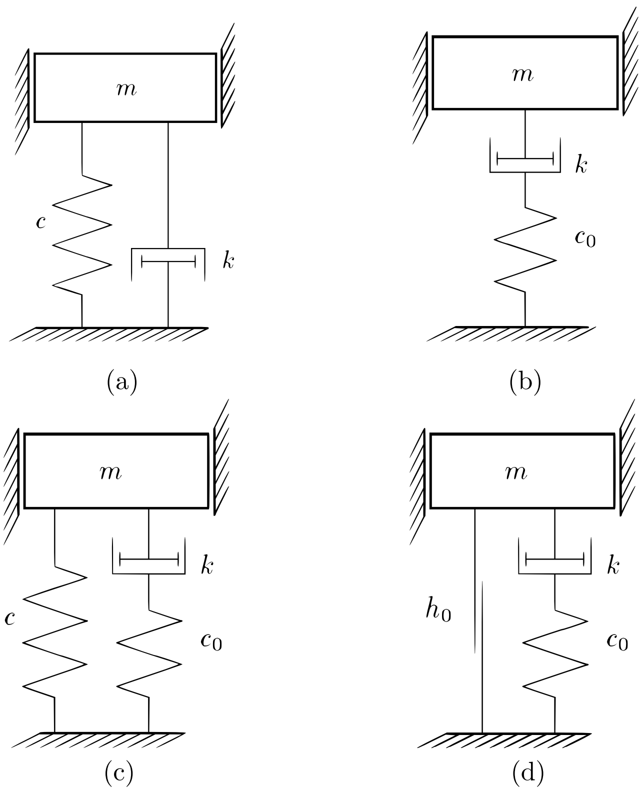

Figure 6.6. Four basic types of simple models that the system in Fig. 6.5 covers: (a) – parallel configuration, (b) – series configuration, (c) – universal configuration, (d) – series with dry friction

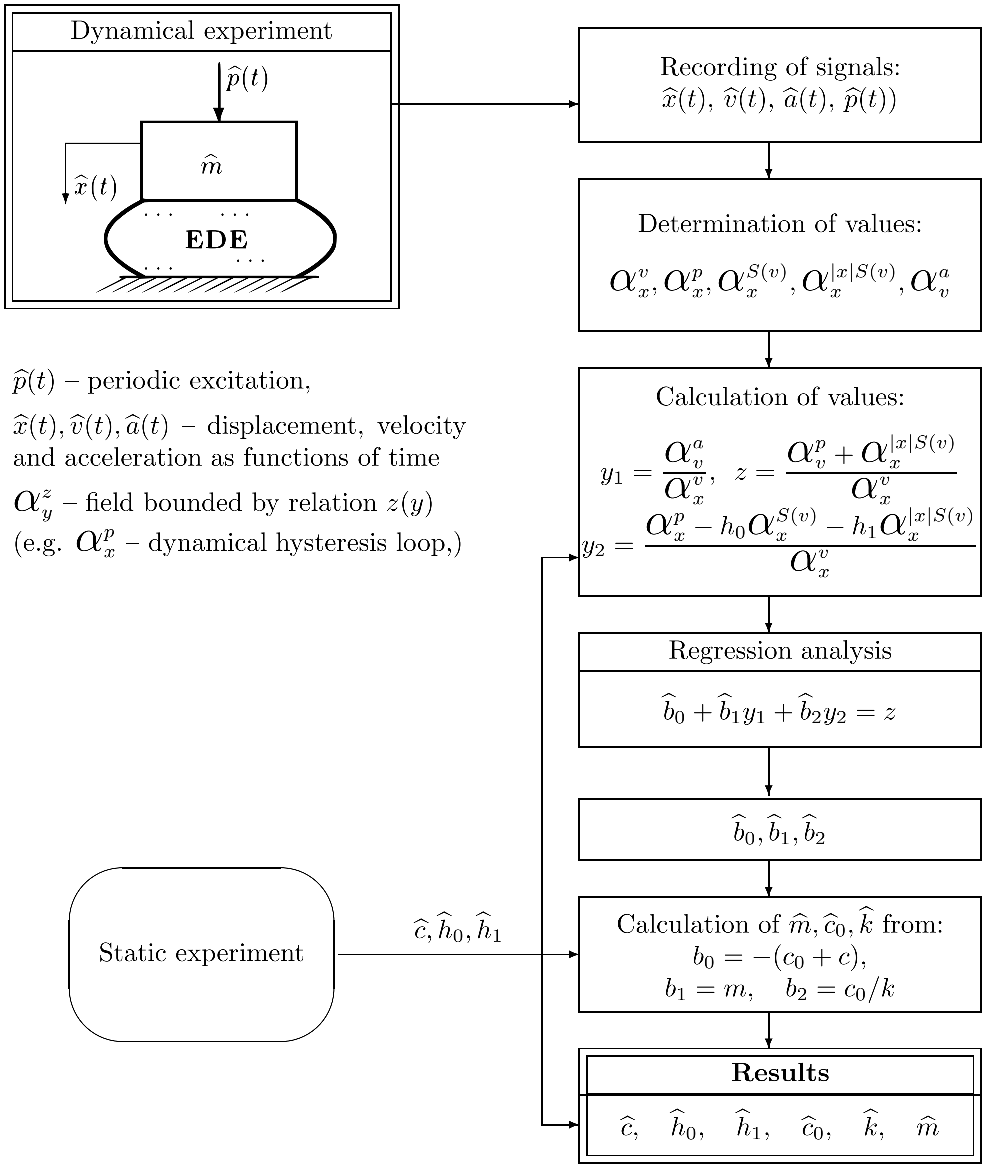

Figure 6.7. The identification algorithm for the model in Fig. 6.5 (acc. to [6.10])

Other cases are also possible, such as $c = 0$, $h_1 = 0$ for which the model in Fig. 6.5 reduces itself to the system in Fig. 6.6d. In each of the above cases, the identification process should be repeated for a new set of parameters $\mathcal{D}$, using the algorithm described in section 3.3. This algorithm assumes that

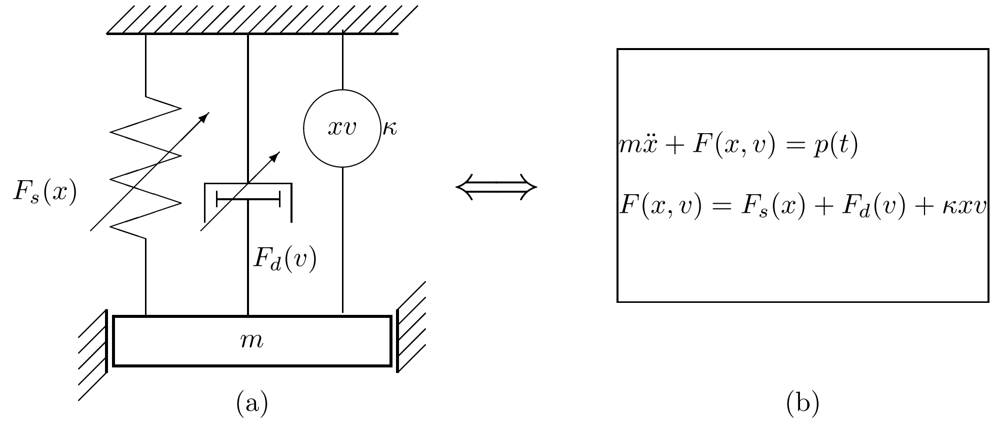

Figure 6.8. A diagram of a model with the mixed element (a) and its mathematical form (b)

constants h_0, c, h_1 are determined through the suitable approximation of the static hysteresis loop (cf Fig. 2.5) and the constants k, c_0, m – by subjecting the system to arbitrary periodic excitations.

Figure 6.7 shows a diagram of the identification algorithm for the EDE, assuming that the model corresponds to the one shown in Fig. 6.5. It is worth mentioning that this algorithm works also for any periodic excitations for which the response of the system is any periodic function. Therefore when a model obtained is to be verified, this solution can be applied to a wide range of excitations.

Another method used for estimating reaction function $F(?)$ is the one described in section 3.4. It can be used, similarly as the previous method, in a way described in Fig. 6.4. The main difference is that the output model here is a system in which, besides purely elastic interactions $F_s(x)$ and purely damping ones $F_d(v)$ (functions $F_s(x)$ and $F_d(v)$ can be of any shape), there is mixed element xv whose significance for the considered system is specified by coefficient κ (see Fig. 6.8). Let us notice that when the value of coefficient κ is determinable, one can test the hypothesis that in the system elastic interactions can be separated from purely dissipative ones, i.e. that function $F(x, v)$ can be written as follows

$$F(x, v) = F_s(x) + F_d(v). \tag{6.7}$$

This method assumes that functions $F_s(x)$ and $F_d(v)$ are arbitrarily nonlinear (the symmetry of dissipative function $F_d(v)$ is odd, i.e. $F_d(-v) = F_d(v)$) and they can be written as follows

$$F_s(x) = \sum_{\mu=1}^{q} c_\mu v^\mu, \quad F_d(v) = h \text{ Sgn } v + \sum_{\nu=1}^{n} k_\nu v^\nu \tag{6.8}$$

(index ν takes only odd values!).

This method was derived on the basis of the following three identification equations (cf section 3.4):

– the energy balance equation

$$h\,\alpha_x^{S(v)} + \sum_{\nu=1}^{n} k_\nu \alpha_x^{v^\nu} + \kappa \alpha_x^{xv} = \alpha_x^p, \tag{6.9}$$

– the power balance equation

$$m\alpha_v^a + \sum_{\mu=1}^{q} c_\mu \alpha_v^{x^\mu} + \kappa \alpha_x^{ax} = \alpha_v^p, \tag{6.10}$$

– and the so-called auxiliary equation

$$m\alpha_x^{ax} + h\,\alpha_x^{S(v)x} + \sum_{\nu=1}^{n} k_\nu \alpha_x^{v^\nu x} + \kappa \alpha_x^{vx^2} = \alpha_x^{px}, \tag{6.11}$$

where α_y^z $(z(t), y(t)$ – signals of relevant responses and their functions) denote a field bounded by loops of relation $z(y)$ for periodic excitations.

Since these equations are satisfied for any periodic excitations which generate a periodic response (impulse excitations of any form), they are essential for the process of identification (the verification of a model in different operating conditions of a machine's EDE). A diagram of an identification algorithm for this method is shown in Fig. 6.9. Let us notice that the static hysteresis loop in this case can be used to verify the results obtained. From the point of view of the modeling of real systems, however, the fact that it is possible to determine estimate $\widehat{\kappa}$ of coefficient κ which defines the share of mixed element xv in the system is most important.

It is this value that decides whether this element can be neglected (in the range of vibration) or not. Let us now discuss the problem of determining the class of the dissipation function in parallel-configuration systems. If an elasto-damping element can be described by a parallel-configuration system ($\kappa \approx 0$ or $c_0 = \infty$), then the reaction force can be written by (6.7). In such a case, the identification task reduces itself to the determination of dissipation function $F_d(v)$. The relevant solutions are described in chapter 2. They were constructed in a way similar to the one shown in Fig. 6.4. But the

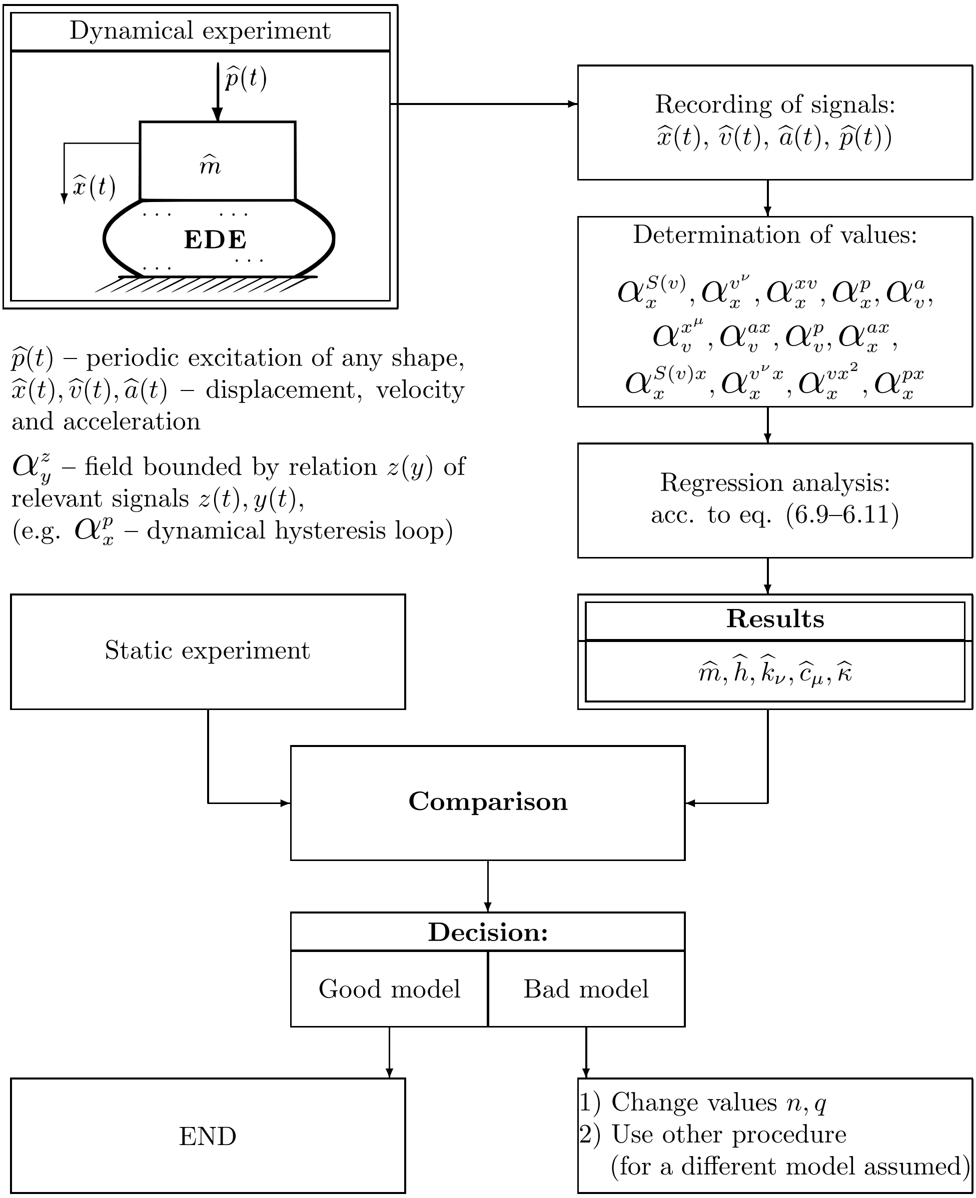

Figure 6.9. An identification algorithm for the model in Fig. 6.8 (acc. to [6.10])

choice of an output set of parameters $\mathcal{D}$ is much easier here since it exploits a relationship between two variables. The shape of this relationship can be estimated visually in a special plot.

For example, assuming that the dissipative function is described by the expression

$$F_d(v) = h \text{ Sgn } v + \sum_{\nu=1}^{n} k_\nu v^\nu \tag{6.12}$$

where h, k_ν – constant unknown parameters and ν, n – any odd numbers, the value of number n is not assumed a priori but on the basis of a relation amplitude P of harmonic excitation as a function of the amplitude of velocity in resonance (V_r). This relationship can be visualized in a finite interval of variables P and V_r as a set of points on a plane (see chapter 2, Figs. 2.32, 2.34, 2.39). This shape makes it much easier to determine number n which describes the degree of polynomial (6.12). The method is applied by calculating constants h, k_ν, on the basis of an optimal approximation of experimentally determined relation $P(V_r)$ (see section 2.4).

Having a rough form of reaction force $F(?)$, now the main task is to determine its constant parameters, i.e. their optimal values for a specified class of dynamic excitations. This task is called parametric identification. It should be performed for different dynamic excitations since only such a procedure can verify fully the model class being constructed. However, methods which are good for any dynamic excitation conditions can often be used in the natural operating conditions of an investigated EDE. A sample solution here is a method in which the energy balance equation and the power equation are applied to systems for which it has been established preliminarily that reaction function $F(?)$ can be written in form (6.7).

Let us assume that as a result of static tests and the use of the harmonic resonance excitation method it has been established that the form of elasticity function $F_s(x)$ and dissipation function $F_d(v)$ is described by (6.8), in which numbers q, n are already known. Now the task is to determine the values of parameters k_ν, c_μ, h again but for non-harmonic and non-resonant dynamic excitations. **By comparing the new values of these parameters with the old ones it will be possible to state whether the model can be used to describe the operation of the investigated EDE in different excitation conditions or ought to be rejected because the differences are unacceptable (so-called verification)** (see [6.10]).

As was shown in chapter 4, the energy and power equations for parallel-configuration systems and functions $F_s(x), F_d(v)$ of form (6.8) can be written as follows

$$hE[|v|] + \sum_{\nu=1}^{n} k_\nu E[v^{\nu+1}] = E[pv], \tag{6.13}$$

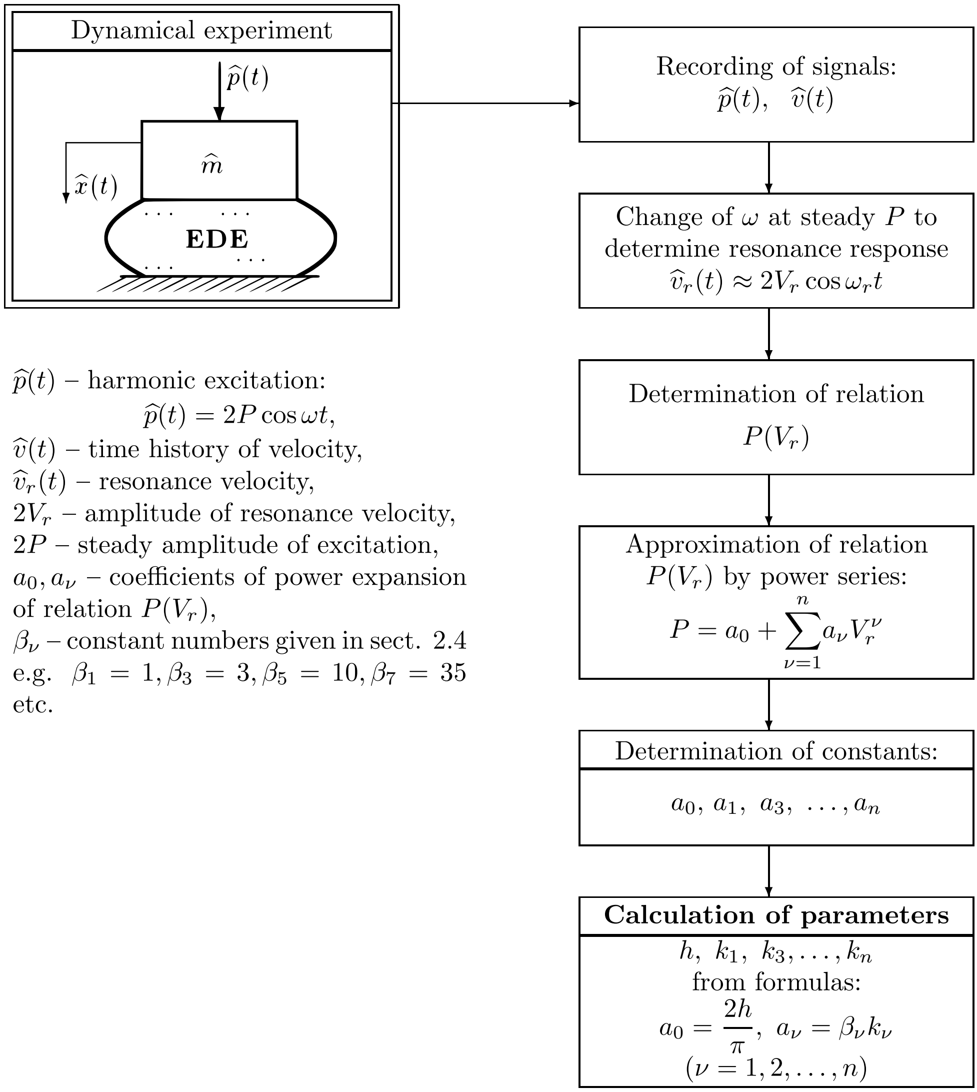

Figure 6.10. The algorithm for determining the class of the dissipative function in the case of odd symmetry

$$mE[a^2] + \sum_{\mu=1}^{q} c_\mu E[x^\mu a] = E[pa], \tag{6.14}$$

where symbols $E[\ldots]$ represent means of relevant signals (e.g. $E[a^2]$ – the mean of an acceleration signal raised to the 2-nd power). It has been demonstrated that these equations are satisfied for such dynamic excitations for

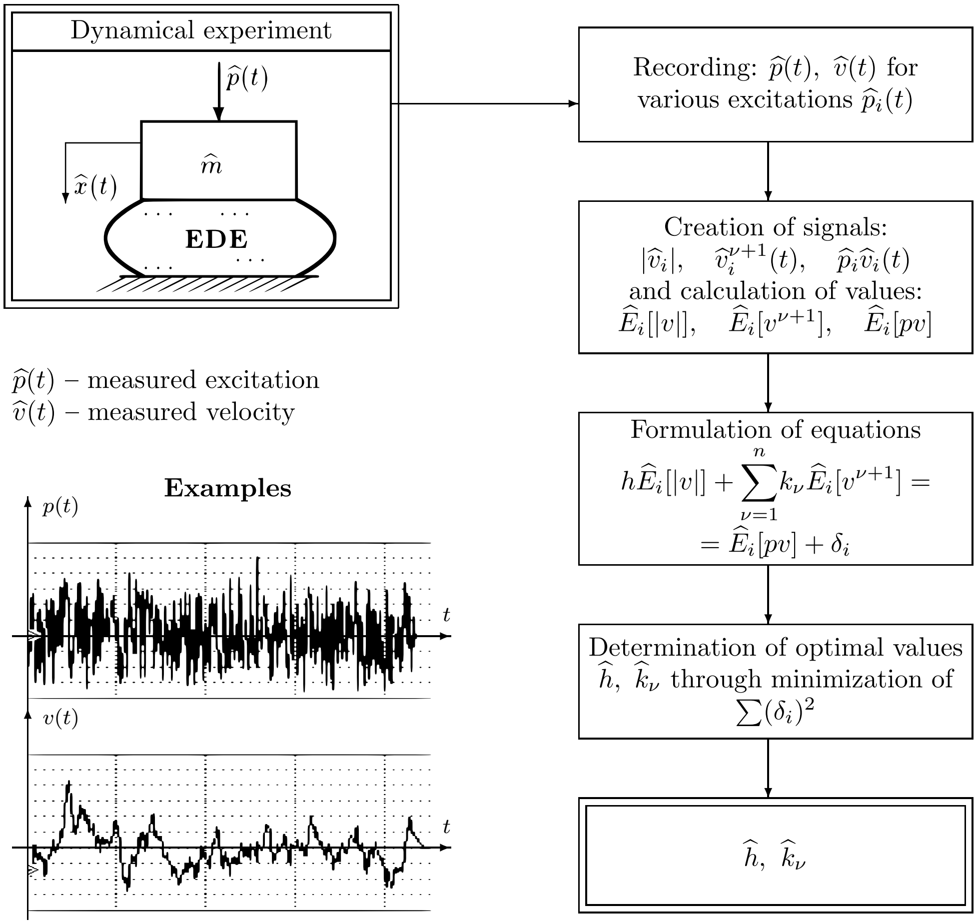

Figure 6.11. The way for determining the parameters of arbitrarily nonlinear dissipation function in the case of any complex dynamical load (acc. to [6.10])

which the response of the system is steady. Let us notice that in the case of real systems, this assumption is true practically for any periodic excitations and also for random excitations with steady probability distribution. It is quite easy to determine mean values $E[\ldots]$ of relevant signals experimentally using multi-channel spectrum analyzers. It should be noticed that the identification testing of the dissipation function can be carried out independent of the investigation of the restitutive function (see Figs. 6.11 and 6.12).

All procedures described in this book have been developed with an appropriate software and tested on several practical applications. For example these procedures have been used to identify influence of the persistent peri-

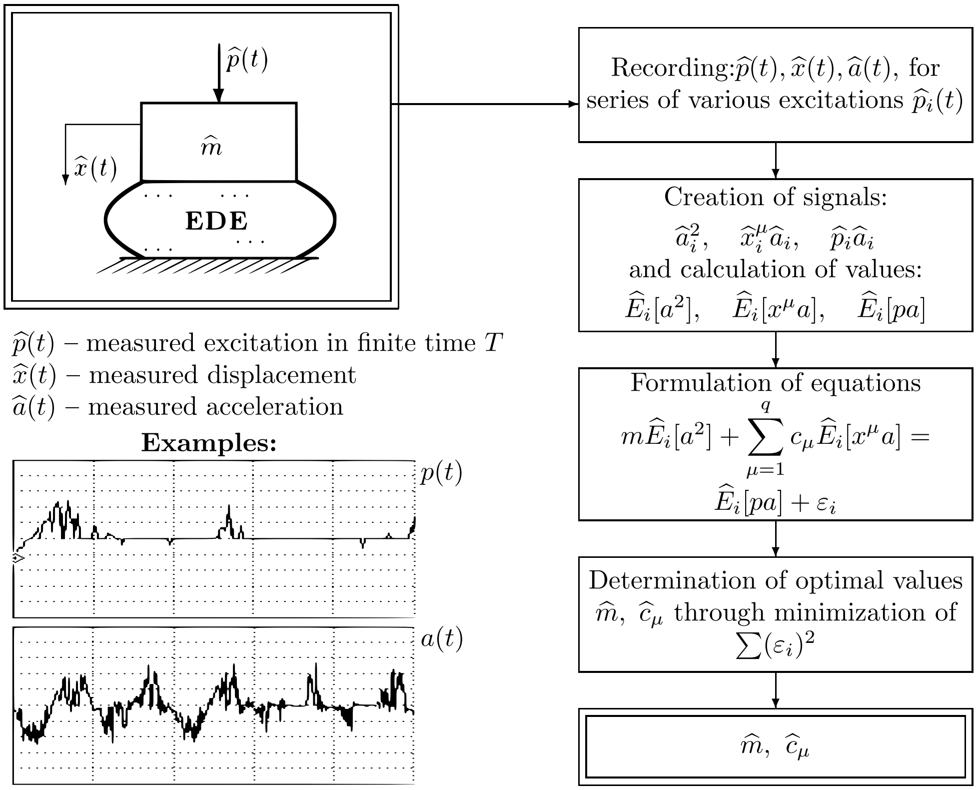

Figure 6.12. The way for determining the parameters of arbitrarily nonlinear restitutive function in the case of any dynamical complex load (acc. to [1.10])

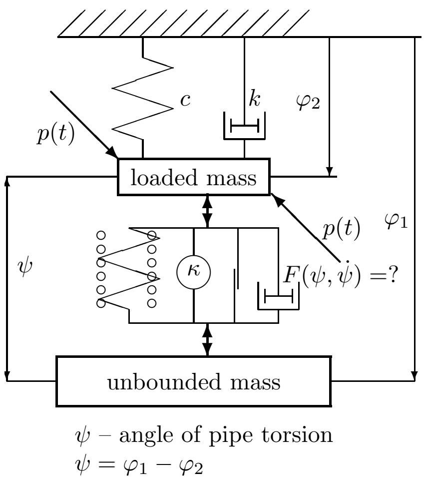

Figure 6.13. Scheme of a model of torsional vibration measurement

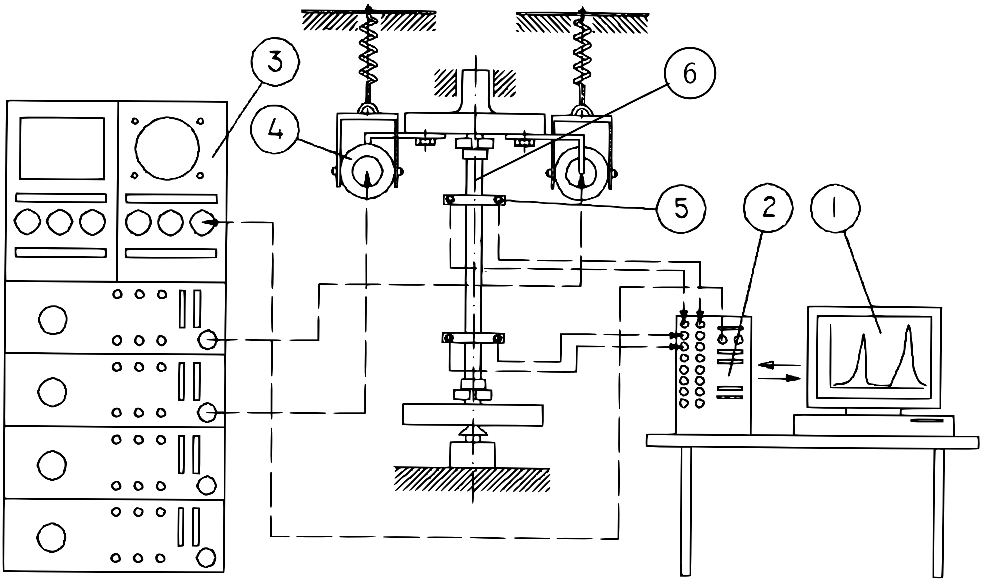

Figure 6.14. The measuring stand for pipe torsional properties identification: 1 – computer with HPVEE software, 2 – HP-E 1473A spectral analyzer, 3 – PRODERA power amplifier, 4 – PRODERA shaker, 5 – PCB accelerometers, 6 – pipe tested

odic excitations on fluctuation of damping properties of some brass, copper and aluminium pipes under torsional loads [6.9]. To achieve this aim the proper measuring stand was created (see Fig. 6.14) in which the tested pipe works as an EDE in a dwo-degrees-of-freedom system (Fig. 6.13).

Summing up, let us notice that by using suitable methods **one can determine the set of arguments of function F and its class (the degree of nonlinearity). This function can be determined accurate to the value of constant parameters for any dynamic excitation. In this process, one can use the original procedures developed for strongly nonlinear models**. The presented methods allow one to verify models fully also by using random loads.

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