



Wrocław University of Technology

Blood flow simulation for human circulatory system

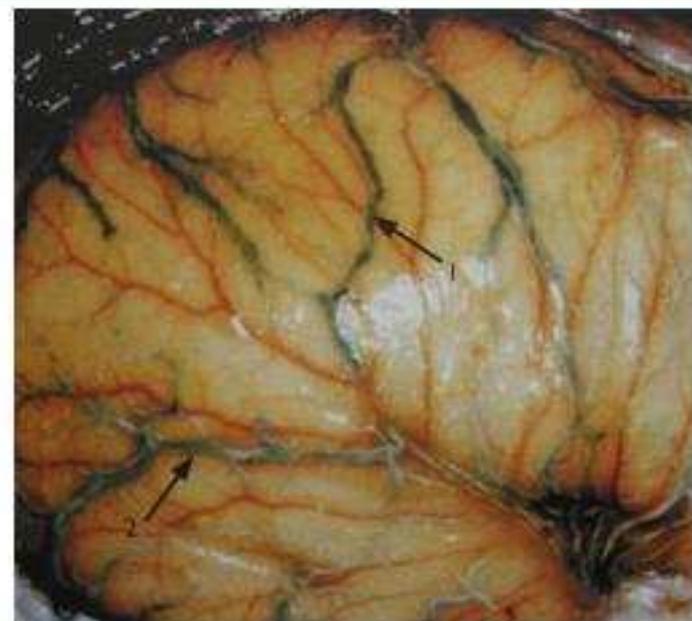
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Introduction

- Geometry of the arterial vascular system of a brain cortex has been modelled
- It is difficult to obtain model with classical method for the vessel tree
- Fractal theory permits convenient geometry representation by using simple mathematical model.
- Fractal model can be successfully applied to geometry and flow analysis of blood vessel trees
- Blood flow in circulatory system depends mostly on vascular geometry.





Research Purpose

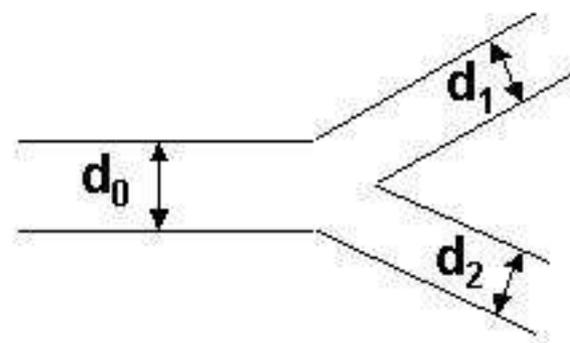
- Brain vascular system based on medical specimens with vessels diameters ranging from 20 μm to 400 μm has been creating.
- Testing the influence of model parameters variation on geometry and blood flow.
- Proposed vascular system model has been verified with physiological structures. Verification has been done with medical expert and publications.

Fractal Vascular Tree Formation

- In vascular tree vessels are bifurcating in each nodes. Bifurcation covers over 98% of all vessels divisions.
- Vessel diameter scaling according to Murray's theory.

$$d_0^x = d_1^x + d_2^x$$

$$d_1 = a * d_2$$

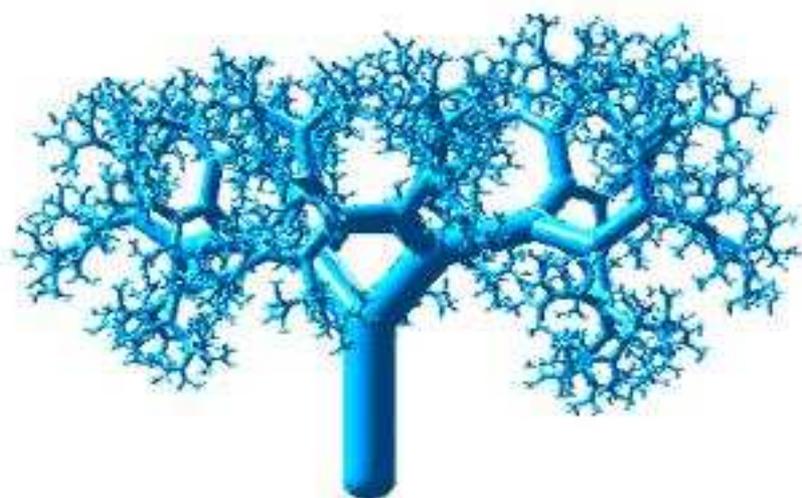


x bifurcation exponent

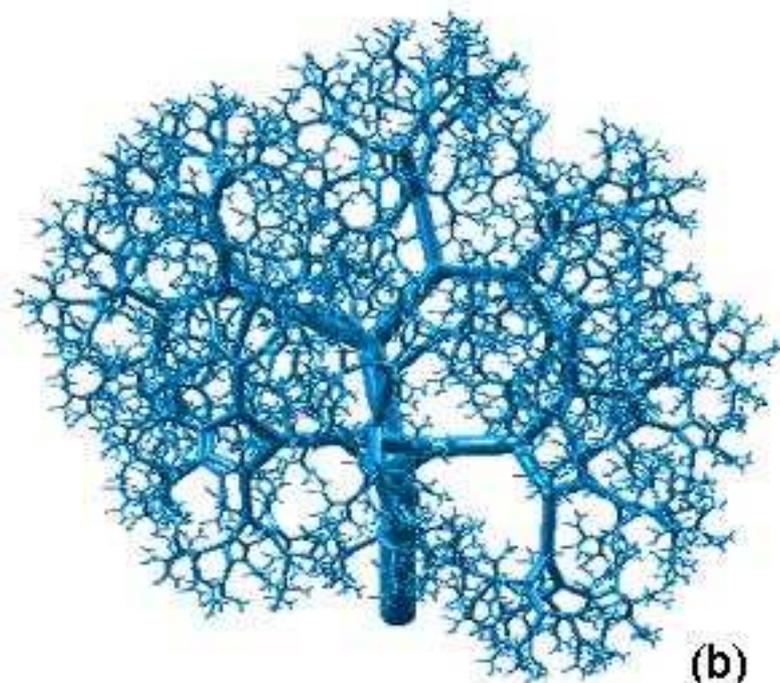
a asymmetry coefficient $a \in \langle 1:1.2 ; 1:1.7 \rangle$



Fractal Vascular Trees



(a)



(b)

Bifurcation exponent $x=2.7$, (a) asymmetrical tree 1:1.5, (b) symmetrical tree



Fractal dimension

- The main tool of fractal geometry is dimension in its many forms
- A dimension provides a description of how much space a set fills and contains much information about the geometrical properties of a set.
- In order to estimate fractal dimension fd3 programme has been used.
- Fractal dimension of terminal vessels has been estimated for different vascular trees



Fractal Dimension Estimation

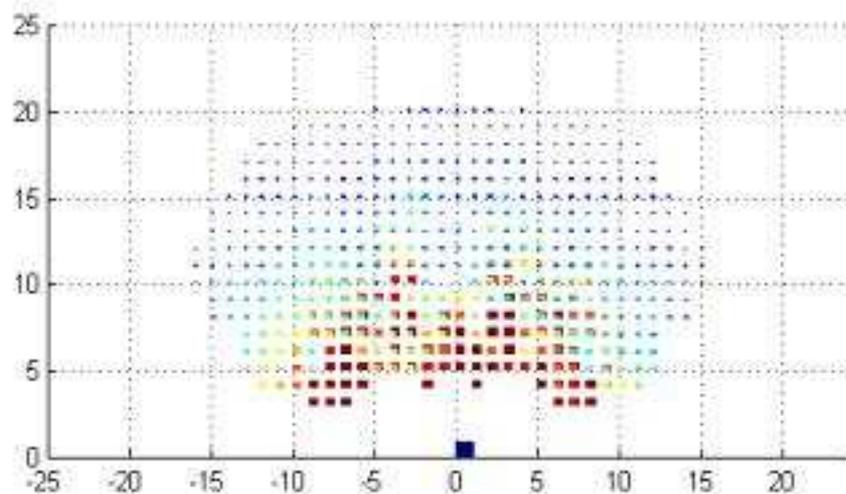
| Model | Terminal vessels number | Box-counting dimension | Correlation dimension |
|---|--------------------------------|-------------------------------|------------------------------|
| Symmetrical $x=3.0$ | 8191 | 2,22 | 2,30 |
| Symmetrical $x=2.7$ | 4095 | 2,18 | 2,21 |
| Asymmetrical 1:1.2, $x=2.7$ | 4180 | 2,18 | 2,18 |
| Asymmetrical 1:1.5, $x=2.7$ | 5536 | 2,03 | 2,07 |
| Asymmetrical 1:1.7, $x=2.7$ | 6686 | 2,01 | 2,00 |



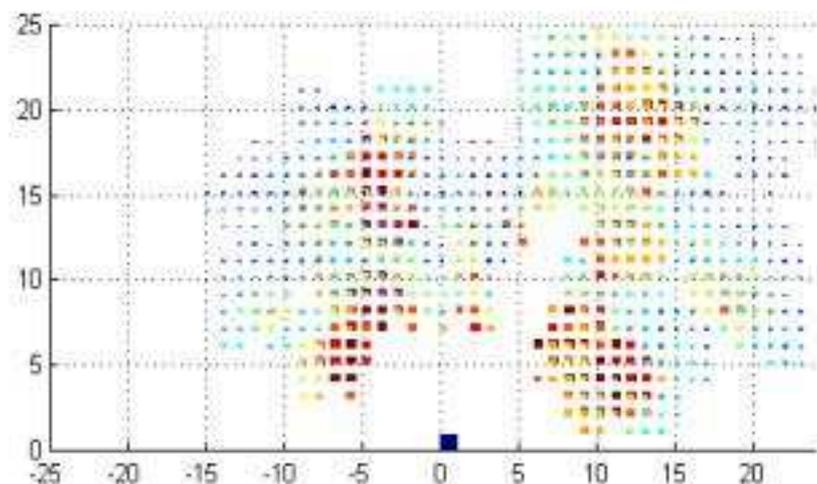
Terminal Vessel Spatial Distribution

- Terminal spatial distribution is fundamental feature of circulatory system for the sake of transport process which occur mainly in the thinnest, terminal vessel.
- Space occupied by fractal vascular tree has been divided into equal cubes $3.2 \times 3.2 \times 3.2$ mm
- In each cube number of terminal vessels have been calculated.

Terminal Distribution Example



Fractal symmetrical tree. Parallel, rectangular projection



Fractal asymmetrical tree 1:1.7. Parallel, rectangular projection



Symmetrical vessel distribution provides even space filling to secure nutrition for all surrounding tissue. Vessel terminals are uniformly distributed in space. **Asymmetrical trees** generate more scattered vessel terminals, some directions are more preferred than others.



Terminal Vessels

Spatial Distribution Deviation

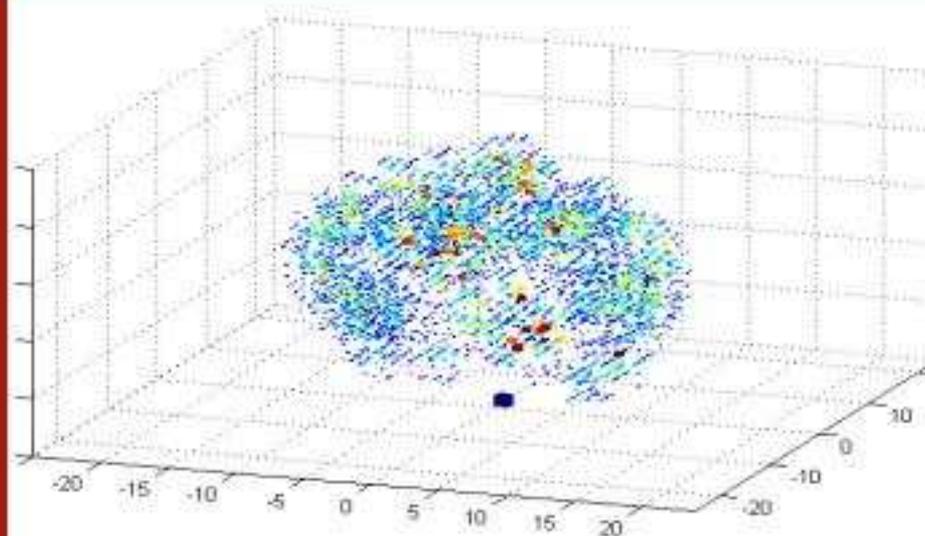
- Following analytical approach summarize observation of uniform vessel coverage.
- Propose deviation represent mean square different between terminal vessel in surrounding 26 cubes.
- For any cube with coordinates (x_0, y_0, z_0) terminal vessels deviation is defined as:

$$g_{x_0, y_0, z_0} = \sum_{x=x_0-1}^{x_0+1} \sum_{y=y_0-1}^{y_0+1} \sum_{z=z_0-1}^{z_0+1} (u_{x_0, y_0, z_0} - u_{x, y, z})^2 / 26$$

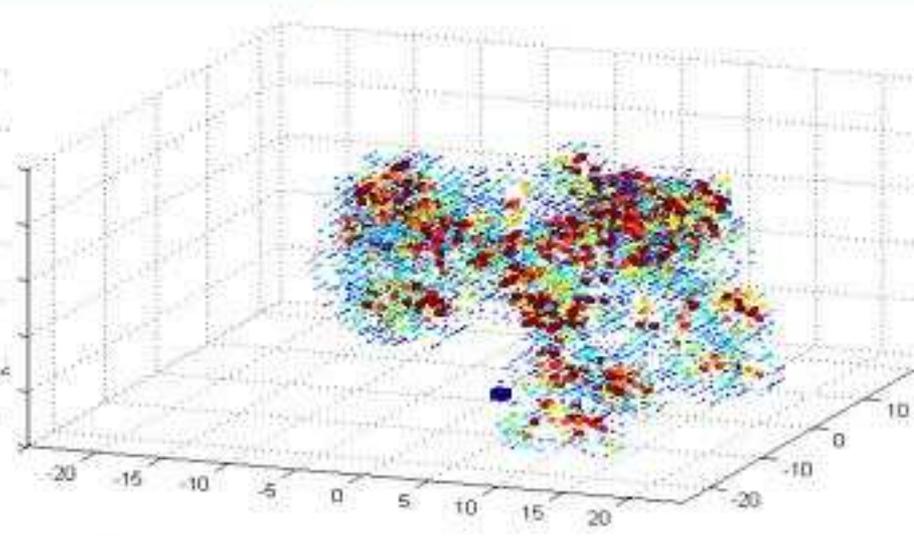
$u_{x,y,z}$ represents number of terminal vessels in cube with integer coordinates (x, y, z)



Terminal Distribution Deviation Example



Symmetrical tree.
Perspective view



Asymmetrical tree 1:1.7.
Perspective view

Symmetrical trees are dedicated for nutrition while conveying functions are more visible in asymmetrical trees



Blood flow simulation

In order to illustrate the impact of structure on functional characteristic of arterial trees blood flow has been simulated

Established assumptions:

- Rigid blood vessel wall. Vessel wall elasticity for small arteries can be omitted and do not influence essentially on obtain results.
- Blood as a Newtonian fluid. During flow in straight, health vessel, blood behave as a Newtonian fluid.
- Laminar flow. In physiological condition in circulatory system laminar flows dominate.



Hydrodynamical equations

- **Poiseuille law** gives correct result for small arteries where elasticity of vessel can be omitted

$$Q = \frac{r^4 \pi \Delta p}{8 \eta L}$$

- **Mass conservation law.** Vascular system is a closed system.

$$Q = AV_{sr}$$



Vessel Inlet Section

- Disturbances in quasi laminar flow appear in vessel nodes.
- Vessel inlet section is a part of vessel where parabolic velocity profile retrieves its shape after branching.
- Length of vessel inlet section

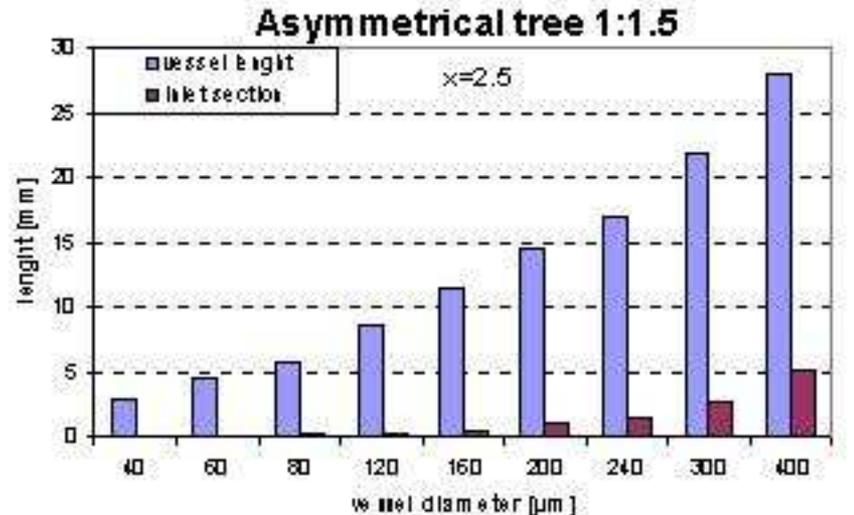
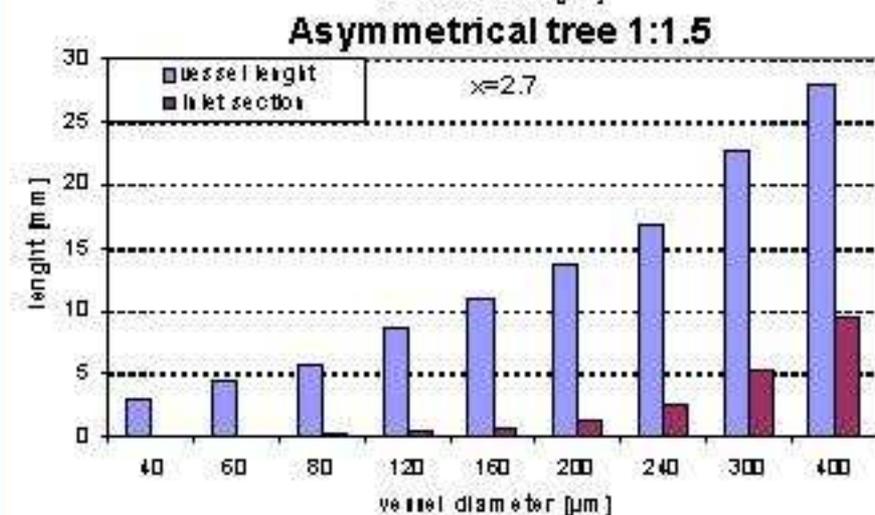
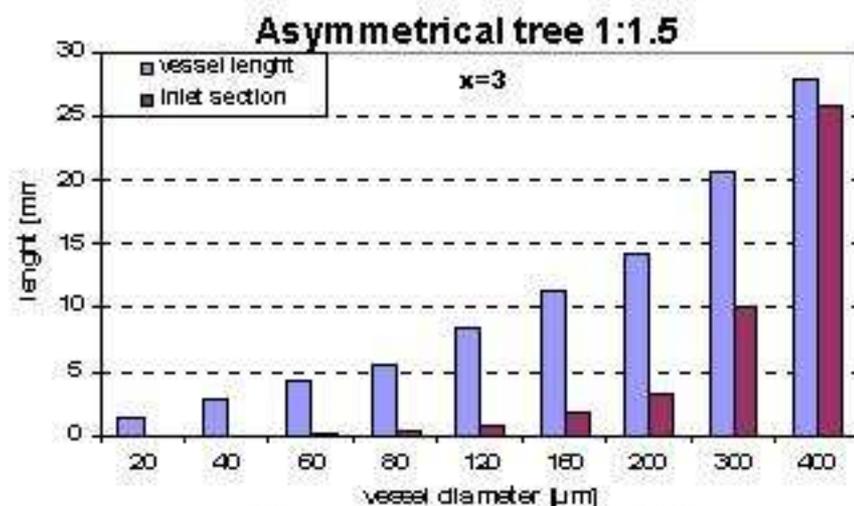
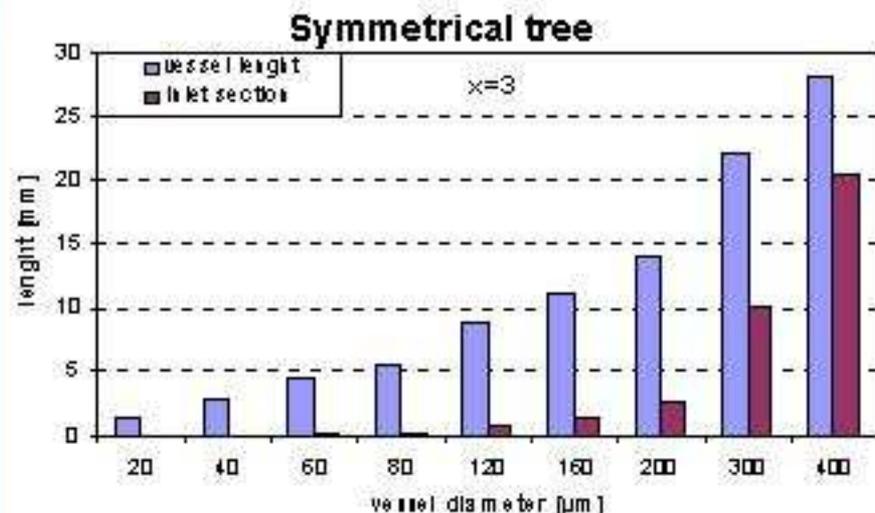
$$l_e = \lambda d R_e \qquad Re = \frac{\rho V L}{\eta}$$

R_e Reynold's number, measure of turbulence

$\lambda = 0.056$ constant value obtained from Navier-Stokes equation

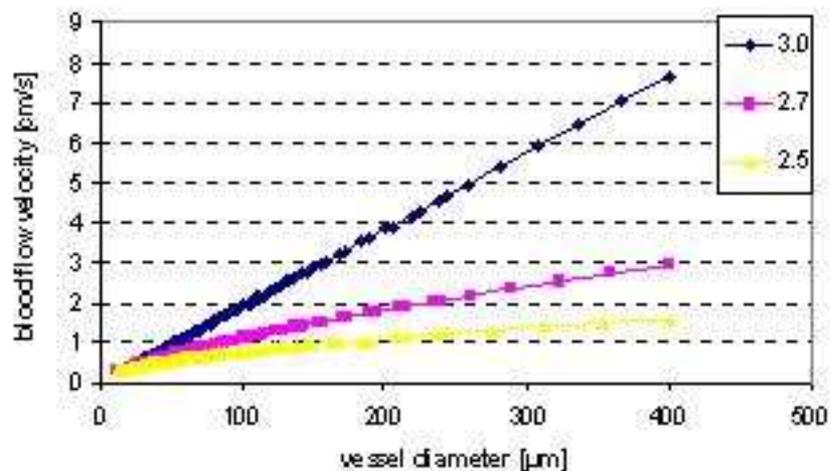
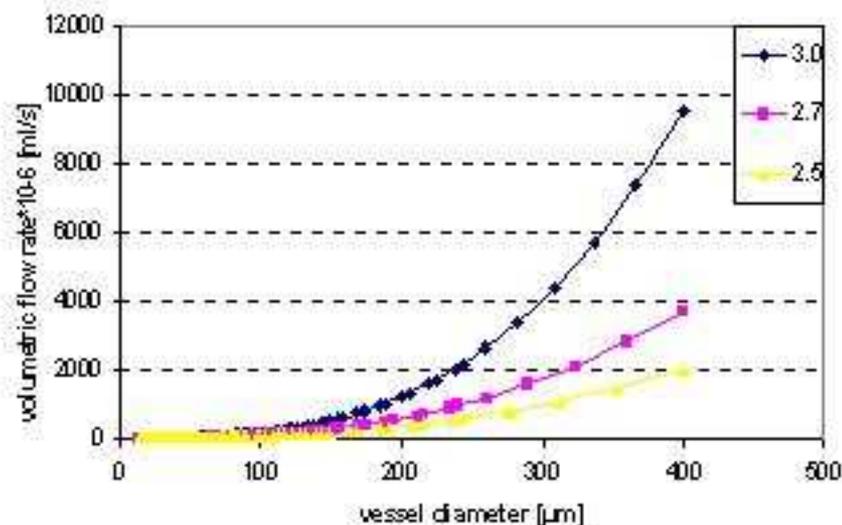
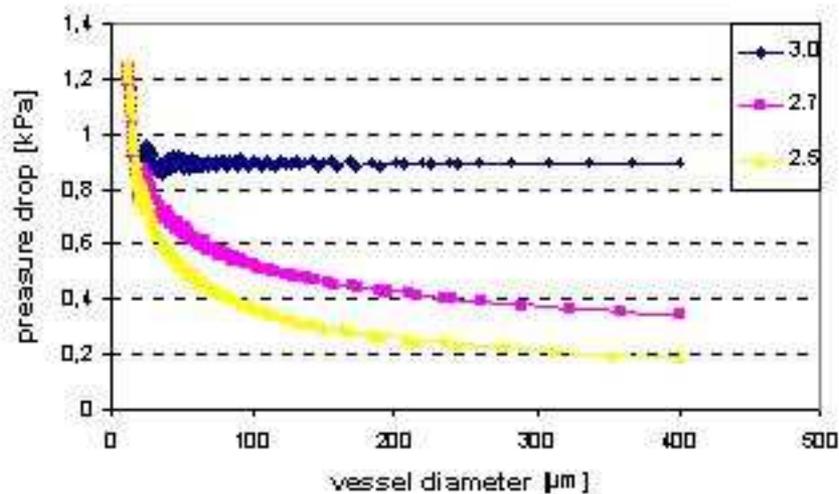


Vessel Inlet Section Example





Hydrodynamical Parameters Analysis



- Large influence of bifurcation exponent value
- Influence of asymmetry value can be neglected



Blood Flow Velocity Results In Comparison With Literature Data

| d [μm] v [cm/s] | 20 | 40-80 | 50-100 | 100-150 |
|--|-------------|--|---------------------------------|---|
| x=2.5 | 0,32 | 0,50 - 0,71 | 0,61 - 0,78 | 0,78 - 1,01 |
| x=2.7 | 0,32 | 0,58 - 0,87 | 0,78 - 1,13 | 1,13 - 1,50 |
| x=3.0 | 0,32 | 0,76 - 1,53 | 1,10 - 1,93 | 1,90 - 3,00 |
| Literature data | 0,32 | 0,30 - 1,40 1,20 - 1,59 | 2,46\pm0,34 | 4,21\pm0,47 0,94 - 2,07 |



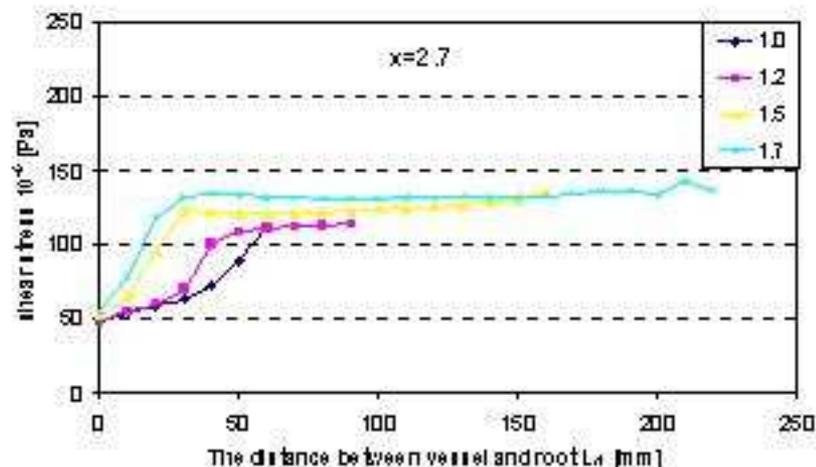
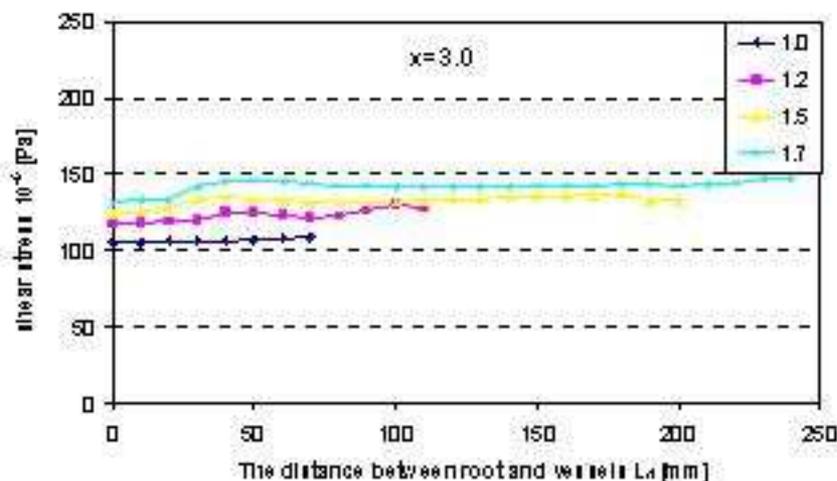
Shear Stress In Circulatory System

- Shear stress influence on vascular tree modelling, remodelling and vascular diseases.
- In physiological condition shear stress value along whole vessel system should be equal
- The biggest shear stress appear near vessel wall, but in the place blood plasma is flowing

$$\tau_i = \frac{4\eta * Q_i}{\pi * r^3} = \eta \gamma_i$$



Shear Stress Along Vessel Tree



Shear stress for different asymmetry value

- Equal shear stress value along vessel tree have been observed for symmetrical trees with bifurcation exponent 3
- Elasticity is a factor which compensate shear stress decrease



Conclusions (1)

- Proposed fractal model correctly describes geometry of vascular system.
- Calculations based on fractal model give results consistent with real circulatory system.
- Vessel symmetry and bifurcation exponent value visibly affects geometry and flow parameters.
- Differences in structure of blood vessel trees, emphasised also by estimated fractal dimension, are a result of biological function.
- Main function of asymmetrical tree is conveying blood to symmetrical tree where the blood is delivering to the tissue.



Conclusions (2)

- Fractal model of blood vessel system is a certain geometrical simplification but it suffices for acceptable blood flow simulation.
- Flow analysis permits understanding influence of hemodynamic forces and their role in the development of vascular diseases.
- Proposed approach enables straightforward analytical evaluation of flow parameters, which usually involves very sophisticated numerical computational methods like FEM.
- Taking advantage of physiological verification, pathological conditions and their influence on flow in degenerated fractal vascular trees can be simulated.

Thank you for yours attention

