Wrocław University of Technology

HORIZONTAL PARAMETRIC RESONANT VIBRATION OF SUSPENSION FOOTBRIDGES

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PARAMETRIC VIBRATON IN FOOTBRIDGES

- The possibility of exciting horizontal parametric resonance vibration in a cable stayed footbridge as a result of a premeditated action of a group of people, or of crowd movement, is analysed.
- Large horizontal transverse vibration are caused by the vertical periodic synchronised movement of people, which is analogical to the movements of a person on a swing.
- The problem studied is how large a group of people is able to excite dangerous parametric transversal vibration in the system.

SCHEME OF PRESENTATION

1. Aim of the paper
2. Introduction
3. Physical interpretation of parametric resonance
4. Problem formulation
5. Simplified analysis – energy balance
6. Stability analysis of pendulum with variable length
7. Results and conclusions
8. Summary

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AIM OF THE PAPER

To establish if it is possible for resonance parametric vibrations to occur in footbridges.

To estimate how many people are needed to cause horizontal vibration in a footbridge by synchronized squatting and standing up.

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NEWLY DESIGNED CABLE-STAYED FOOTBRIDGE OVER DUNAJEC RIVER IS THE MATTER OF THE ANALYSIS

- Border crossing – Sromowce Niżne (Poland) and Červeny Kláštor (Slovakia)
- The pylon is on the Polish river bank
- The main span is 90m long and two riparian spans are 10m each
- The bridge girder is made of glued pinewood
- The total length of the object is 150m
- The total length of the girders is 112m

FOOTBRIDGE OVER DUNAJEC RIVER

- Vertical projection of the formation line is a straight line
- The formation line is inscribed in vertical circle with radius 1,4 km
- The bridge deck is suspended to the pylon with a radial system of ten ion members (wire rope)
- The pylon is made of steel pipes

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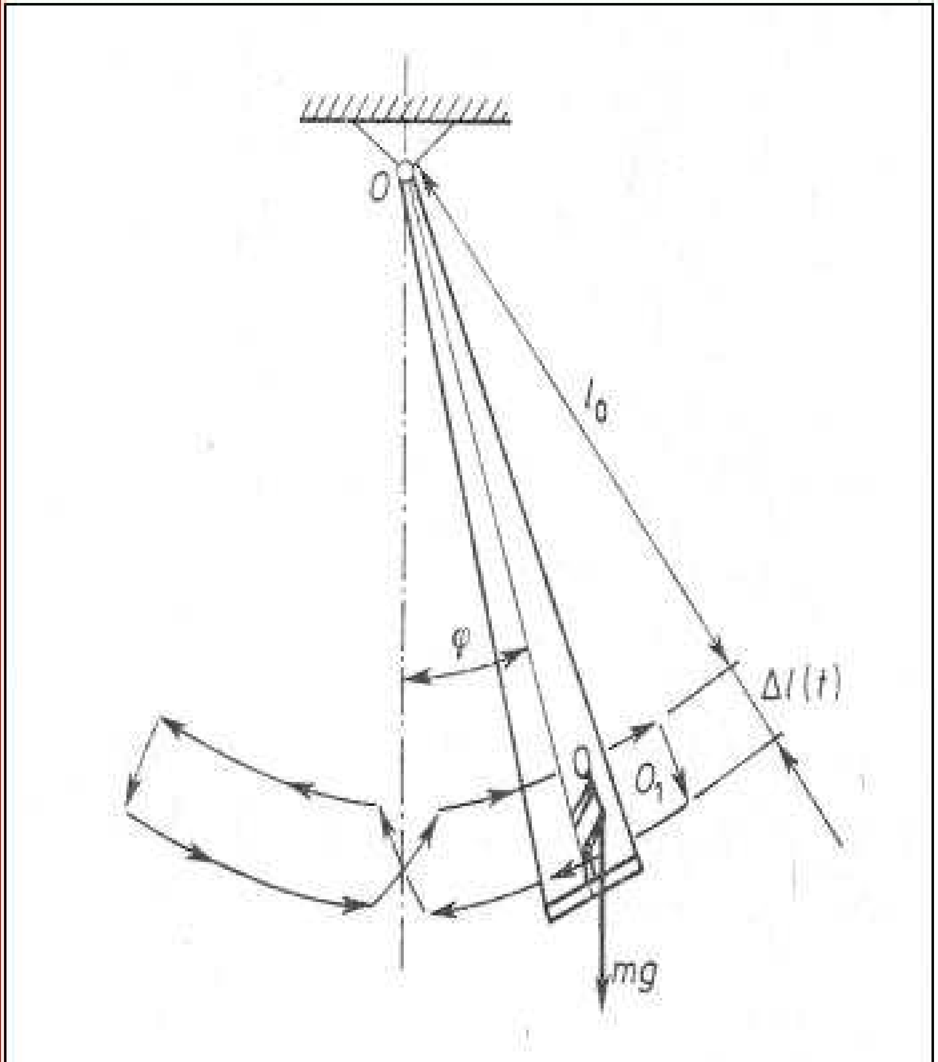
THE SWING



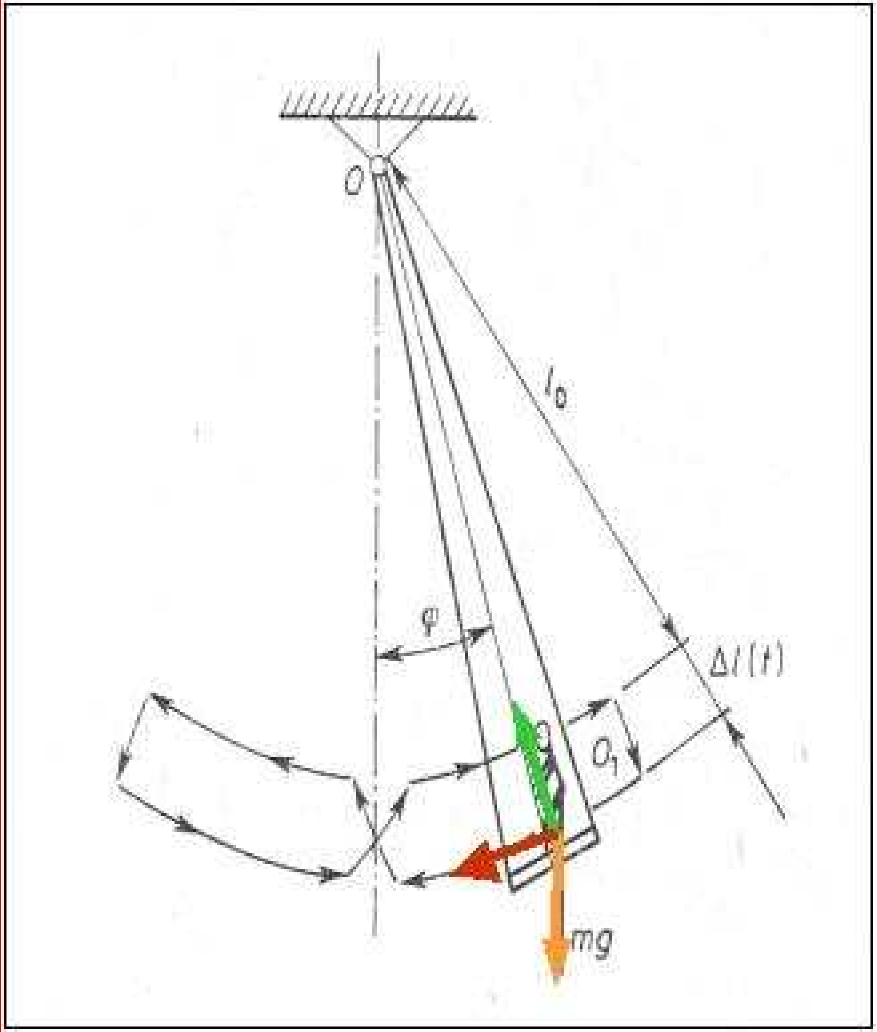
- What should one do to swing the swing?
- When the swing is in the equilibrium point one should stand up.
- When the swing is maximal deflected from equilibrium point one should squat down.

QUESTIONS

How is it possible that a man's motion can cause large vibration amplitude, despite the fact that there are no external forces acting on the system ?

How is man's motion, perpendicular to the motion of the system, transferred into energy of increasing system's vibrations ?

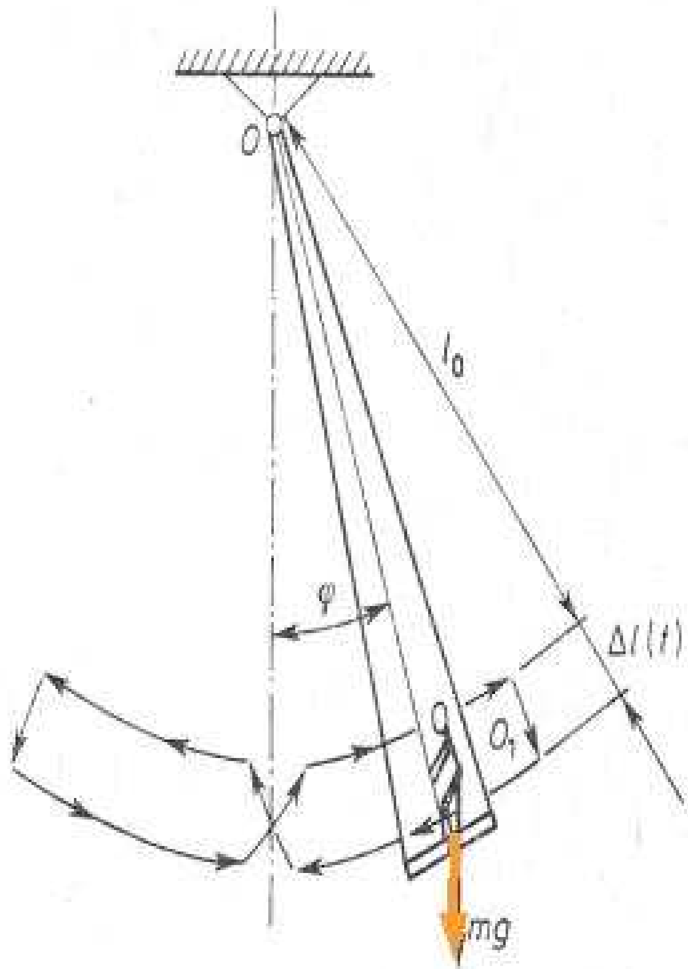
PHYSICAL INTERPRETATION OF THE PHENOMENON



During one's movement on the swing three forces may act:

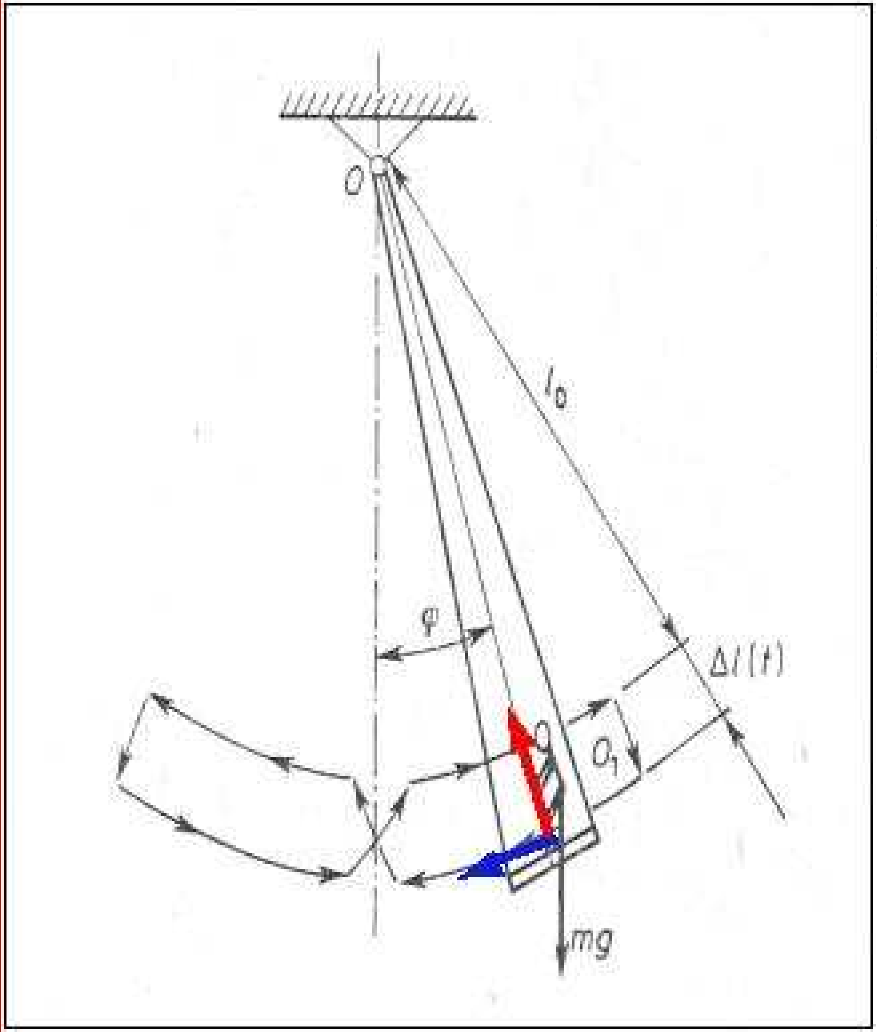
- gravity force
- elastic reaction
- Coriolis force

PHYSICAL INTERPRETATION OF THE PHENOMENON



- the gravity force, as a potential force, doesn't do work because the trajectory of a swinging man's centroid is a closed curve

PHYSICAL INTERPRETATION OF THE PHENOMENON

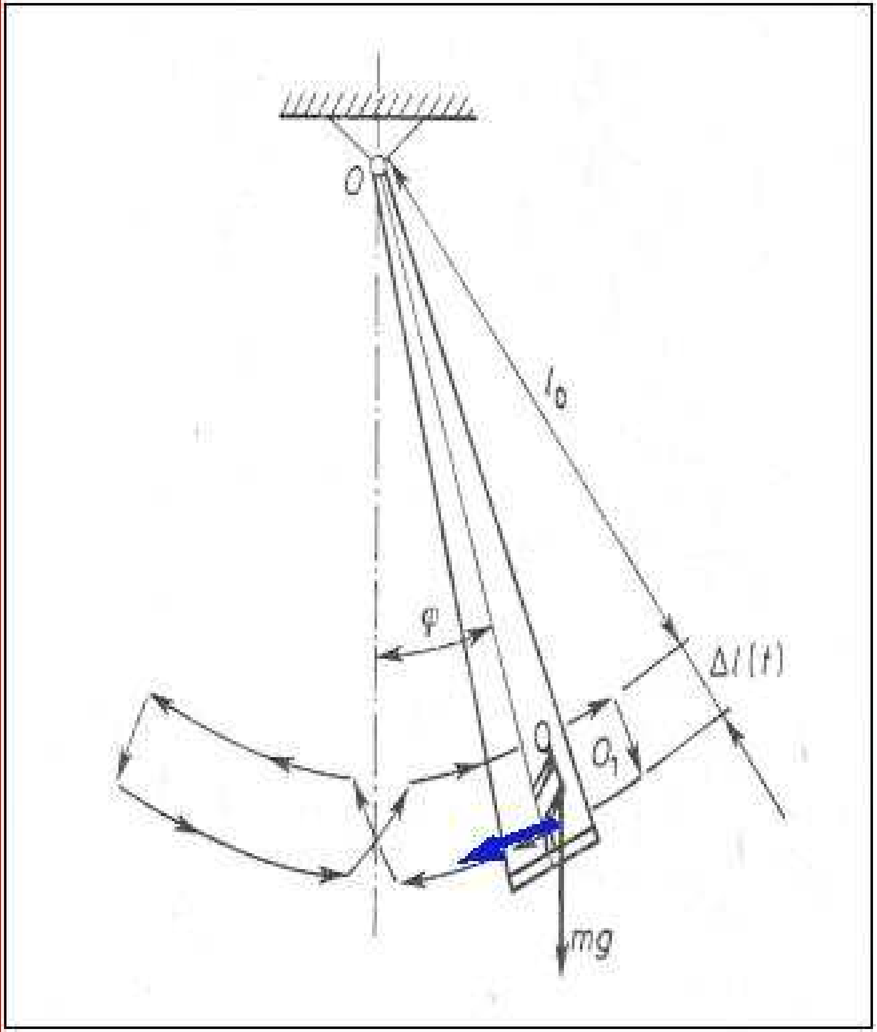


The three forces are equal to the resultant force with two components:

– tangent

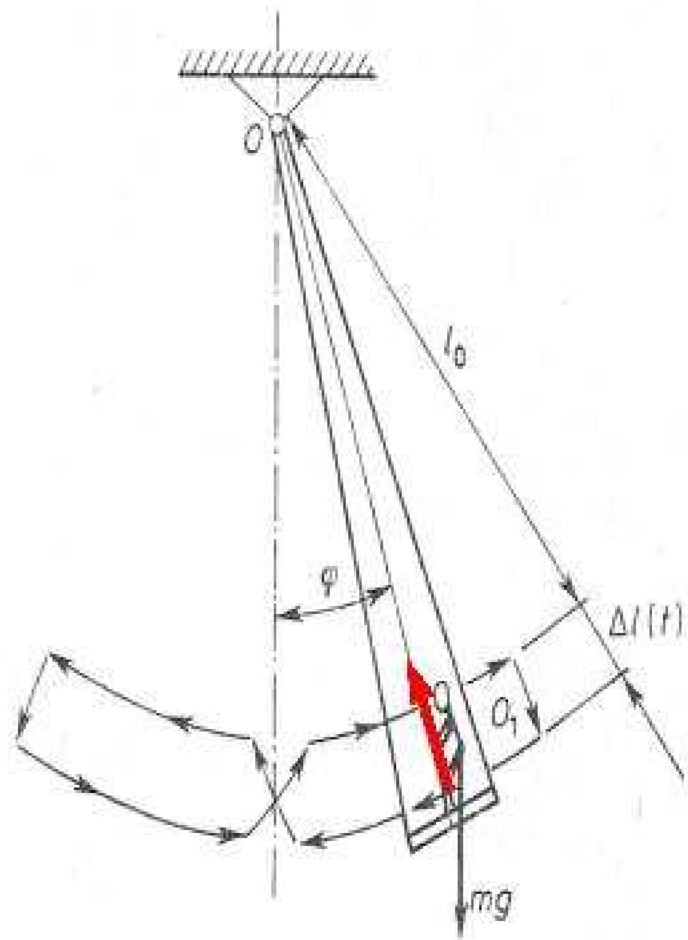
– centripetal

PHYSICAL INTERPRETATION OF THE PHENOMENON



- **the tangent force** also doesn't do work because it is perpendicular to the displacement of man's centroid during squatting and standing up.

PHYSICAL INTERPRETATION OF THE PHENOMENON



The centripetal force

- in the extreme position the centripetal force doesn't do work because it's value is zero
- in the equilibrium point the centripetal force has the maximal value and in this moment one stands up
- **the force does positive work which transforms itself into vibration energy of the system !**

PARAMETRIC RESONANCE

- the swing is a pendulum with variable length
- it is a so-called parametric system
- the parametric resonance occurs when there is specified ratio between the eigenfrequency and the frequency of parametric excitation
- the most dangerous resonance is the subharmonic one i.e. frequency of parametric excitation is approximately two times bigger then eigenfrequency
- parametric resonance can also occur when the frequency of parametric excitation is equal to the eigenfrequency of the system
- parametric resonance and force resonance can occur simultaneously

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FEM MODEL

FEM SYSTEM- COSMOS/M ver 2.5

17176 degrees of freedom

2875 nodes

3143 finite elements

1960 shell elements (4/6):

- bridge girder made of glued pinewood
- bridge deck

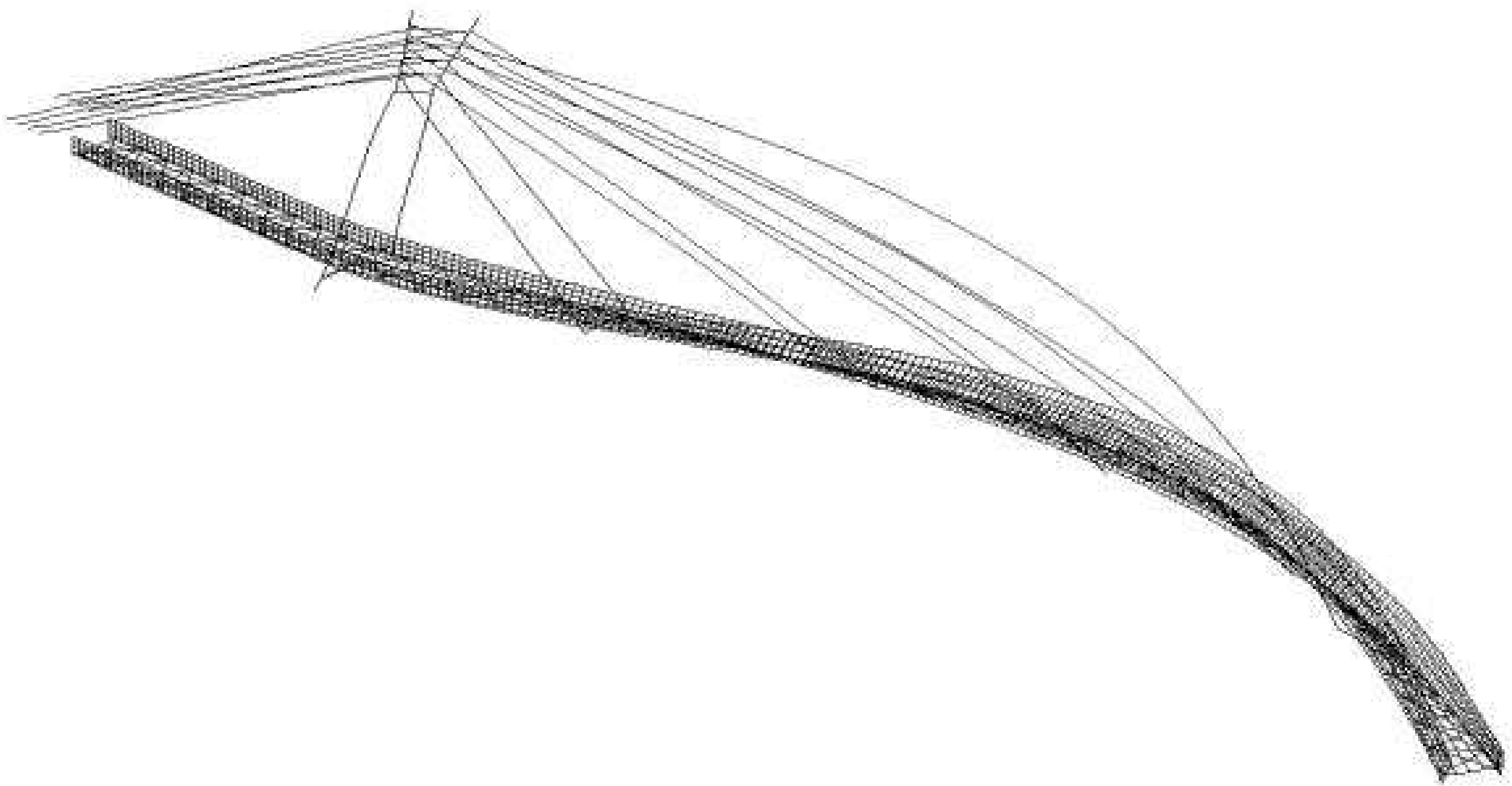
1045 spatial beam elements (2/6):

- half-frame connecting main girders
- pylon
- beams connecting tension members and girders

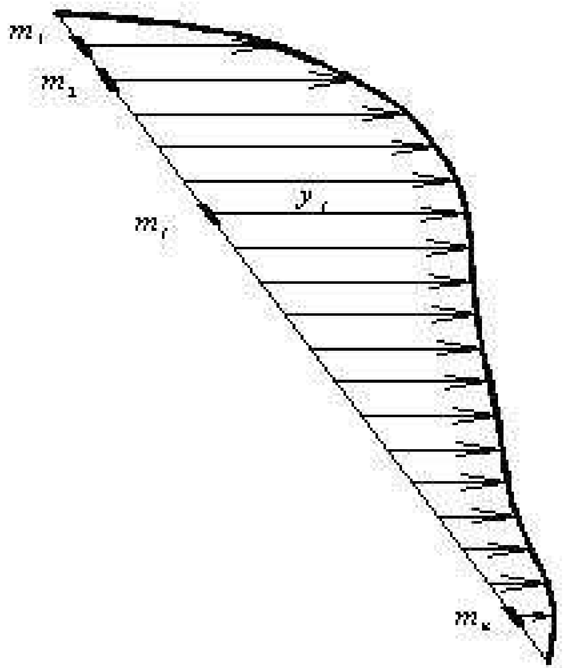
138 spatial truss elements (2/3):

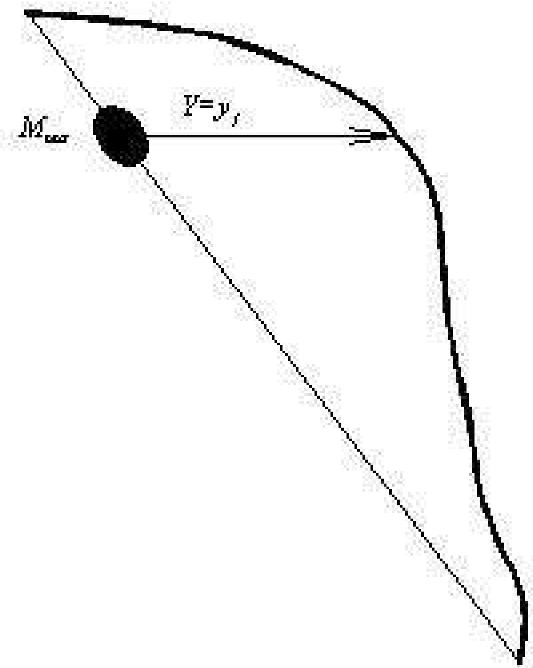
- horizontal truss between main girders
- bearings elements

3rd EIGENVALUE - 0,898Hz



EQUIVALENT MODAL MASS ENERGY BALANCE



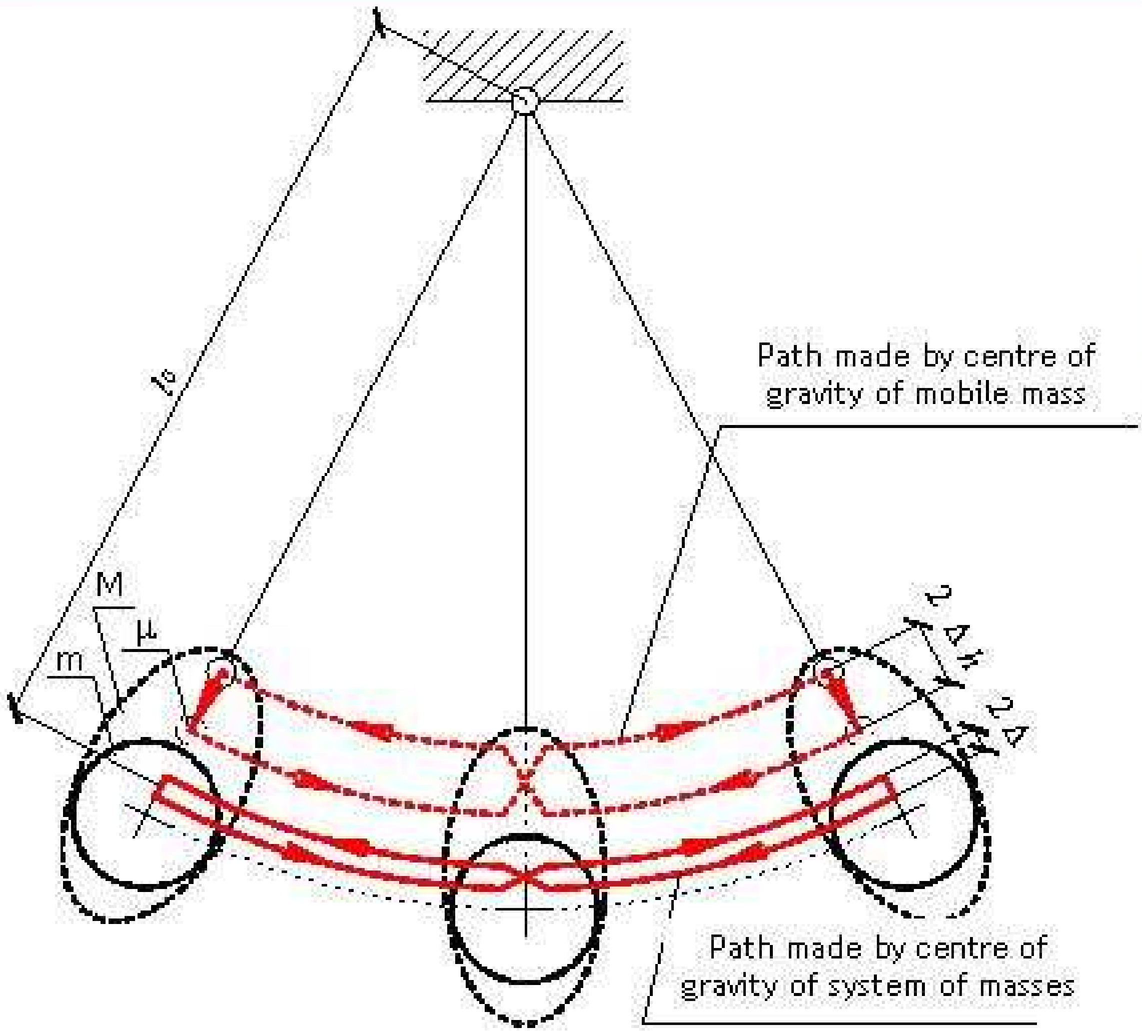


$$m = \sum_{i=1}^{n} m_i \, \eta_i^2$$

$$m = 54164 \ \text{kg}$$

$$E_k = \frac{1}{2}\sum_{i=1}^{n} m_i \, \dot{y}_i^2 = \frac{1}{2}\left(\sum_{i=1}^{n} m_i \, \omega^2 \, y_i^2\right) = \frac{1}{2}\omega^2 Y^2 \left(\sum_{i=1}^{n} m_i \, \eta_i^2\right) = \frac{1}{2} \, m\dot{Y}^2$$

VARIABLE LENGTH PENDULUM



$$l(t) = l_o + \Delta l(t)$$

$$\Delta = \frac{\Delta S}{M} = \frac{\mu \Delta h}{M}$$

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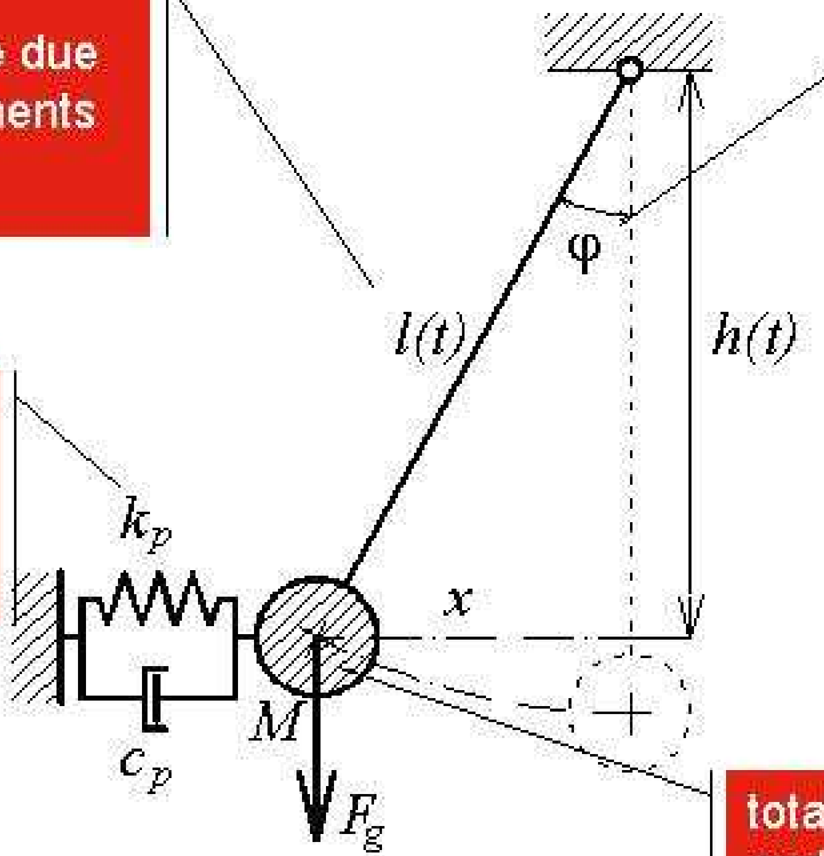
CALCULATION DIAGRAM
parametric system

pendulum length changeable in time due to people's movements $l(t)=l_o+\Delta l(t)$

deflection angle of pendulum

equivalent stiffness and viscosity of girders

total mass of girders and people , $M=m+\mu$



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ENERGY INTRODUCED PARAMETRICALLY

In case of small vibration of pendulum, displacement from equilibrium point and velocity of vibration are described by harmonic functions

$$x(t) = x_{\max} \sin \omega t$$
$$\dot{x}(t) = \dot{x}_{\max} \cos \omega t$$

Value of centripetal force is given by the well-known formula

$$F_d = \frac{\mu \dot{x}^2}{l_o} = \frac{\mu \dot{x}^2_{\max}}{l_o} \cos^2 \omega t = F_{\max} \cos^2 \omega t$$

ENERGY INTRODUCED PARAMETRICALLY

With the assumption that vertical motion of mass μ is described by harmonic function with frequency $2f$

$$\boxed{\Delta h(t) = \Delta h \sin 2\omega t}$$

Work done by centripetal force is equal to

$$\begin{aligned} W &= \int_{h_1}^{h_2} F_d(\Delta h)\, d\Delta h = \int_0^T F_d(\Delta h(t))\Delta h'(t)\, dt = \\ &= 2\Delta h \omega F_{\max} \int_0^{2\pi/\omega} \cos^2 \omega t \cos 2\omega t\, dt = \\ &= 2\Delta h \omega F_{\max} \frac{\pi}{2\omega} = \pi F_{\max} \Delta h \end{aligned}$$

WORK DONE BY CENTRIPETAL FORCE DURING ONE PERIOD OF PENDULUM'S VIBRATION

Finallny

$$\boxed{W = \pi F_{\max} \Delta h = \pi \frac{\mu \Delta h}{l_o} \dot{x}_{\max}^2}$$

ENERGY DISSIPATED DURING ONE PERIOD OF PENDULUM'S VIBRATION

mean damping power during one period of pendulum's vibration

$$\overline{\Phi} = \frac{1}{2}c\overline{\dot{x}}^2 = \vartheta\alpha\omega m\overline{\dot{x}}^2 = \pi\alpha m f\,\dot{x}_{\max}^2$$

where

$$\overline{\dot{x}}^2 = \dot{x}_{\max}^2/2 \qquad c = 2\alpha\omega m$$

dash - mean value

dot - time derivative

ENERGY DISSIPATED DURING ONE PERIOD OF PENDULUM'S VIBRATION

Finallny

$$\boxed{E = \overline{\Phi} T = \pi \omega n \dot{x}_{\max}^2}$$

CONDITION OF VIBRATION'S GROWTH

Vibration will grow if energy introduced to the system parametrically will be bigger than the energy dissipated

$$W > E$$

$$\pi \frac{\mu \Delta h}{l_o} \dot{x}_{\max}^2 > \pi c \omega m \dot{x}_{\max}^2$$

CRITICAL MASS ESTIMATION

Resonant parametric vibration will occur in the system if the mobile mass is bigger than

$$\mu_{\min} = \frac{\alpha\, l_o}{\Delta h} m$$

and greater than

$$\mu_{\min} = \frac{\alpha\, l_o}{\Delta h - \alpha l_o} m$$

when taking into account the influence of people's mass on damping ($M=m+\mu$, instead of m)

$\alpha, \Delta h, m, l_o$ should be known

PENDULUM LENGTH ESTIMATION

It is possible to write down two equations

$$k_w = mg / l_o$$

$$k_w = \omega^2 m - k_p$$

From these equations one can find the value of equivalent pendulum length (preliminary value)

$$l_o = mg / (m\omega^2 - k_p)$$

INPUT DATA

- $m = 54164$ kg
- $l_o = 1{,}93$ m
- $\alpha = 0{,}005 - 0{,}0125$
- $\Delta h = 0{,}15$ m
- $k_p = 1{,}45 \cdot 10^6$ N/m

RESULTS

$$\mu_{\min} = \frac{\alpha l_o}{\Delta h - \alpha l_o} m$$

$\gamma = 2\alpha = 0{,}010$

$\mu_{\min} = 3724$ kg

$\gamma = 2\alpha = 0{,}025$

$\mu_{\min} = 10381$ kg

CRITICAL MASS CALCULATION

According to the formula, pendulum length depends on frequency, frequency depends on mass, mass is changing - so pendulum length is changing too, therefore the iteration procedure is necessary.

$$l_o = mg/(m\omega^2 - k_p)$$

Results received from iteration

$\gamma = 2\alpha = 0{,}010$

$$l_o = 2{,}00\mathrm{m}$$

$$\Delta = 0{,}010m$$

$\mu_{min} = 3869$ kg

$\mu_{min} = 3724$ kg

$\gamma = 2\alpha = 0{,}025$

$$l_o = 2{,}20\mathrm{m}$$

$$\Delta = 0{,}025m$$

$\mu_{min} = 12159$ kg

$\mu_{min} = 10381$ kg

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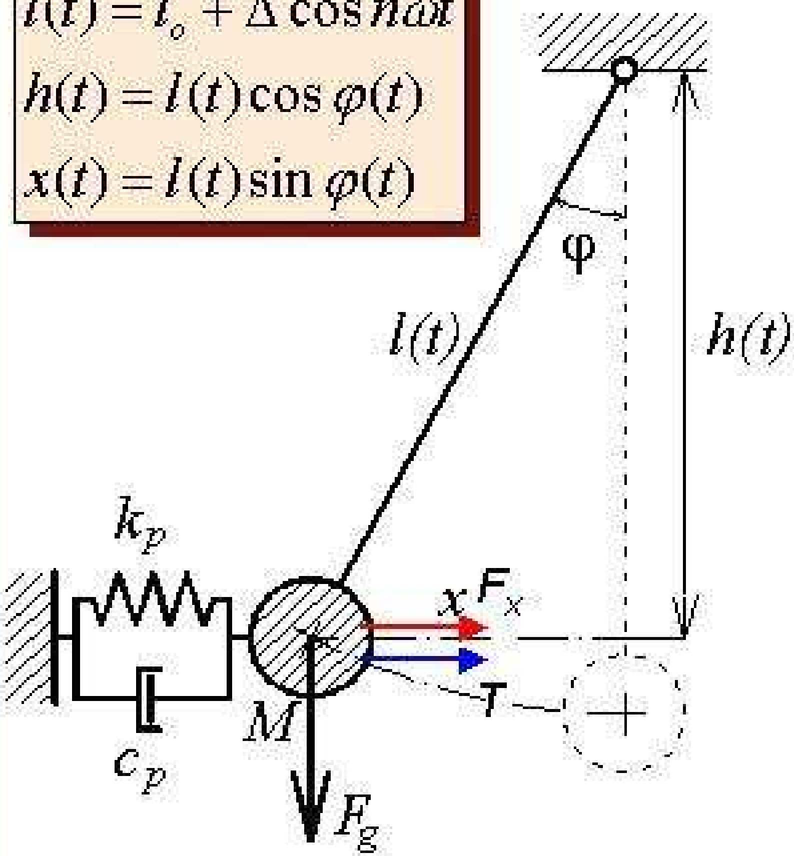
VARIABLE LENGTH PENDULUM

$$l(t) = l_o + \Delta \cos n\omega t$$
$$h(t) = l(t) \cos \varphi(t)$$
$$x(t) = l(t) \sin \varphi(t)$$



d'Alembert principle

Moment of momentum change rate is equal to moment of force

$$\frac{dL(t)}{dt} + T\,h(t) + F_x h(t) + F_g x(t) = 0$$

$F_x = k_p x(t)$ - elasticity force

$F_g = Mg$ - gravity force

$T = c\,\dot{x}(t)$ - damping force

$L = J(t)\dot{\varphi}(t)$ - moment of momentum

PARAMETRIC NONLINEAR EQUATION OF MOTION

$$J(t)\ddot{\varphi}(t)+\dot{J}(t)\dot{\varphi}(t)+c\,h(t)\dot{x}(t)+k_p h(t)x(t)+Mg\,x(t)=0$$

$$l(t)=l_o+\Delta\cos n\omega t$$
$$h(t)=l(t)\cos\varphi(t)$$
$$x(t)=l(t)\sin\varphi(t)$$

$$\Delta=\frac{S}{M}=\frac{\mu\Delta h}{2M}$$

LINEARIZATION OF EQUATION

$$h(t) = l(t)\cos\varphi(t) \approx l(t)$$

$$x(t) = l(t)\sin\varphi(t) \approx l(t)\varphi(t)$$

$$\dot{x}(t) = \dot{l}(t)\varphi(t) + l(t)\dot{\varphi}(t)$$

$$J(t)\ddot{\varphi}(t) + [\dot{J}(t) + c\,l^2(t)]\dot{\varphi}(t) + l(t)[k_p l(t) + Mg]\varphi(t) = 0$$

$$J(t) = Ml^2(t)$$

$$\dot{J}(t) = 2Ml(t)\dot{l}(t)$$

$$l(t) = l_o + \Delta l(t)$$

$$\Delta l(t) = -\Delta\cos n\omega t$$

$$\dot{l}(t) = n\omega\Delta\sin n\omega t$$

PARAMETRIC EQUATION OF MOTION linear with coefficients variable in time

Equation of motion for the variable length pendulum

$$\ddot{\varphi}(t) + 2[\alpha\omega + \frac{\dot{l}(t)}{l(t)}]\dot{\varphi}(t) + \frac{l_o}{l(t)}[\omega^2 + \frac{k_p}{M}\frac{\Delta l(t)}{l_o}]\varphi(t) = 0$$

$$\omega^2 = \frac{k}{M} = \frac{k_p + k_w}{M} = \frac{k_p}{M} + \frac{g}{l_o}$$

STABILITY ANALYSIS

- Equation was numerically integrated
- Monodromy matrix was evaluated
- Multiplicators of monodromy matrix were evaluated
- Stability of the equation was analysed using multiplicators

SCHEME OF PRESENTATION

RESULTS AND CONCLUSION

For given data the greatest modulus of multiplicator is grater than one (loss of dynamic stability) when mobile mass exceeds value.

$$\boxed{\mu_{\min} = 4056\,\mathrm{kg}}$$

$$\boxed{\mu_{\min} = 12832\,\mathrm{kg}}$$

This value of mass is equivalent to mass of 50 or 160 people. Synchronized vertical motion of those people with amplitude approximately 15cm will cause parametric resonant horizontal vibration of footbridge.

RESULTS LIST

μ_{min}= 3724 kg

μ_{min}= 10381 kg

μ_{min}= 3869 kg

μ_{min}= 12159 kg

$$\mu_{\min} = 4056 \text{ kg}$$

$$\mu_{\min} = 12832 \text{ kg}$$

The exact results are only 5% greater than the ones obtained by simplified analysis (energy balance)

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SUMMARY

- The analysis carried out shows that peoples' movement on a cable-stayed footbridge can cause its parametric resonant horizontal vibration.
- The exact analysis confirms results of the simplified analysis (on the basis of energy balance)
- Such parametric resonant states can reduce comfort of footbridge operational use, and could even cause real hazard for safe and non-failure use.

THE END

Thank you!