WROCŁAW UNIVERSITY OF TECHNOLOGY

Identyfication of chaos phenomenon in mechanics of construction problems

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OUTLINE OF THE PRESENTATION

- Identyfication methods of chaotic systems - classification
- Example systems
- Tools for identification chaos phenomenon
- Examples of chaotic vibrations in selected problems of bar structures
- Summary and conlusions

TOOLS FOR IDENTYFICATION OF CHAOTIC SYSTEMS

QUALITATIVE ANALYSIS

- POINCARÉ SECTIONS
- BIFURCATION DIAGRAMS
- FOURIER POWER SPECTRUM ANALYSIS
- WAVELET ANALYSIS

TOOLS FOR IDENTYFICATION OF CHAOTIC SYSTEMS

QUANTITATIVE ANALYSIS

- ATTRACTOR DIMENSION ANALYSIS
- LYAPUNOV CHARACTERISTIC EXPONENTS

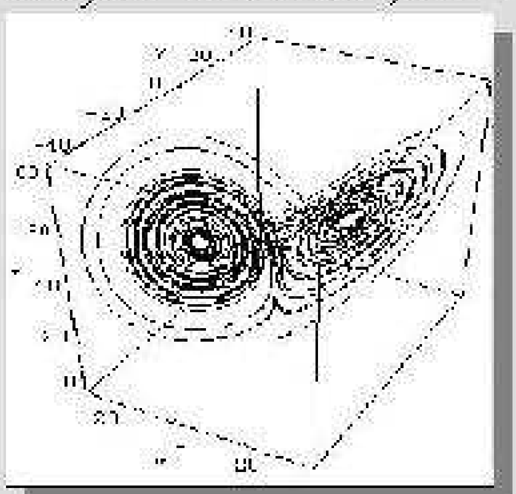
EXAMPLE SYSTEMS

LORENZ SYSTEM

$$\dot{x} = -\sigma \cdot (x - y)$$
$$\dot{y} = -x \cdot z + r \cdot x - y$$
$$\dot{z} = x \cdot y - b \cdot z$$

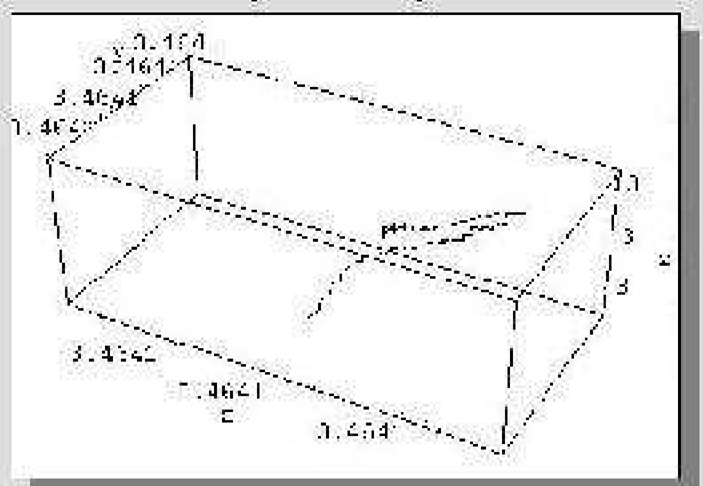
CHAOTIC STATE

$$\sigma = 16, r = 45.92, b = 4$$



NON-CHAOTIC STATE

$$\sigma = 16, r = 4, b = 4$$



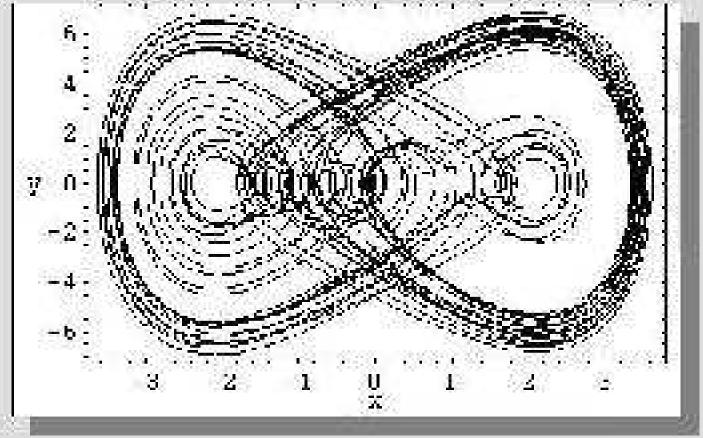
EXAMPLE SYSTEMS

DUFFING SYSTEM

$$\dot{x} = y$$
$$\dot{y} = -\alpha \cdot y - x^3 + \beta \cdot \cos(\omega \cdot z)$$
$$\dot{z} = 1$$

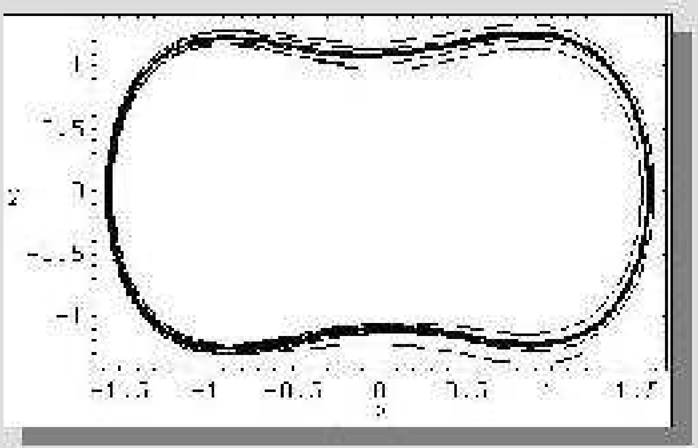
CHAOTIC STATE

$$\alpha = 0.1, \beta = 10, \omega = 1$$



NON-CHAOTIC STATE

$$\alpha = 0.1, \beta = 1, \omega = 1$$



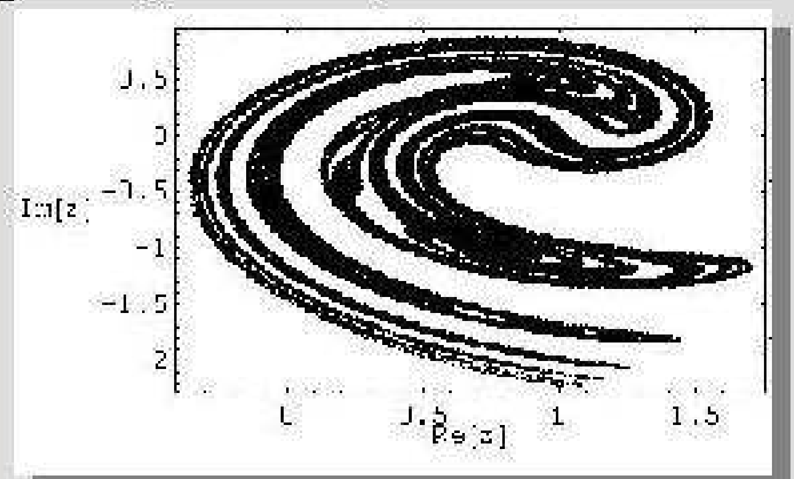
EXAMPLE SYSTEMS

IKEDA MAP

$$z(n+1) = p + b\cdot z(n)\cdot \exp(\boldsymbol{I}\cdot(\kappa-\alpha)/1+|z(n)|^2)$$

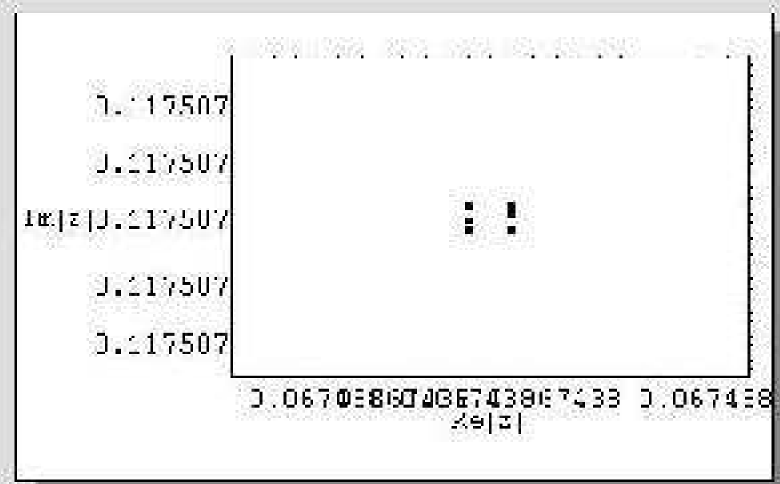
CHAOTIC STATE

$p=1, b=0.9, \kappa=0.4 \alpha=6$



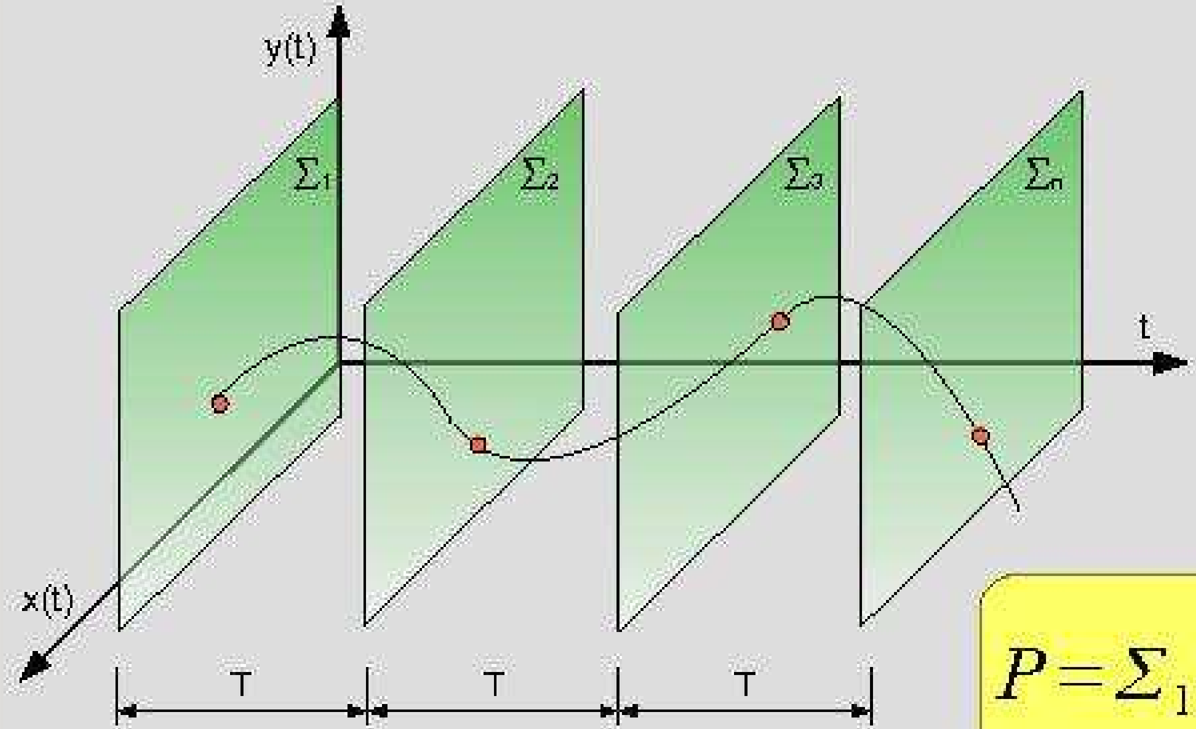
NON-CHAOTIC STATE

$p=0.1, b=0.9, \kappa=0.4 \alpha=6$



POINCARÉ SECTIONS

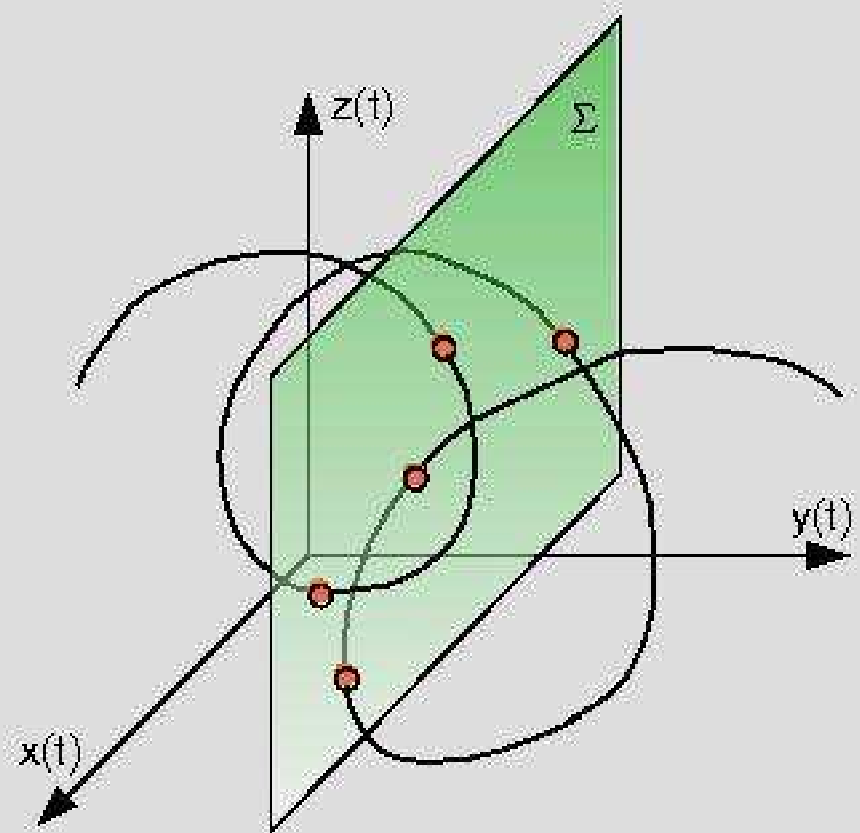
NON-AUTONOMOUS SYSTEMS: $\dot{\bar{x}} = f(\bar{x}, t)$



$$P = \Sigma_1 + \Sigma_2 + \ldots + \Sigma_n$$

POINCARÉ SECTIONS

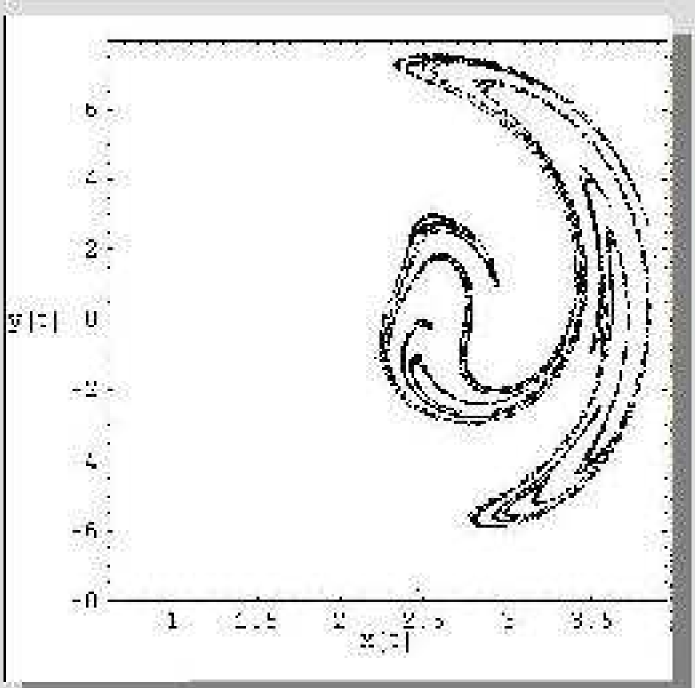
AUTONOMOUS SYSTEMS: $\dot{\mathbf{x}} = f(\overline{\mathbf{x}})$



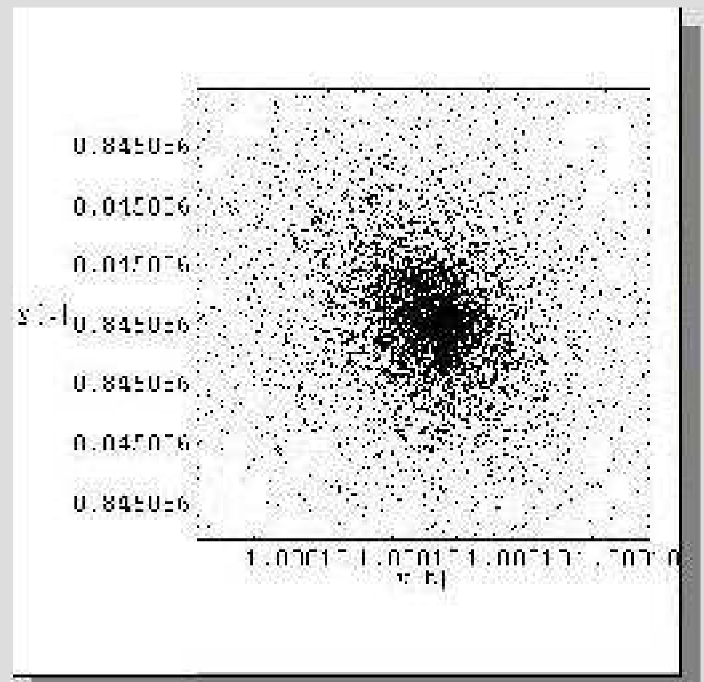
$$P = \Sigma$$

POINCARÉ SECTIONS DUFFING SYSTEM

CHAOTIC STATE

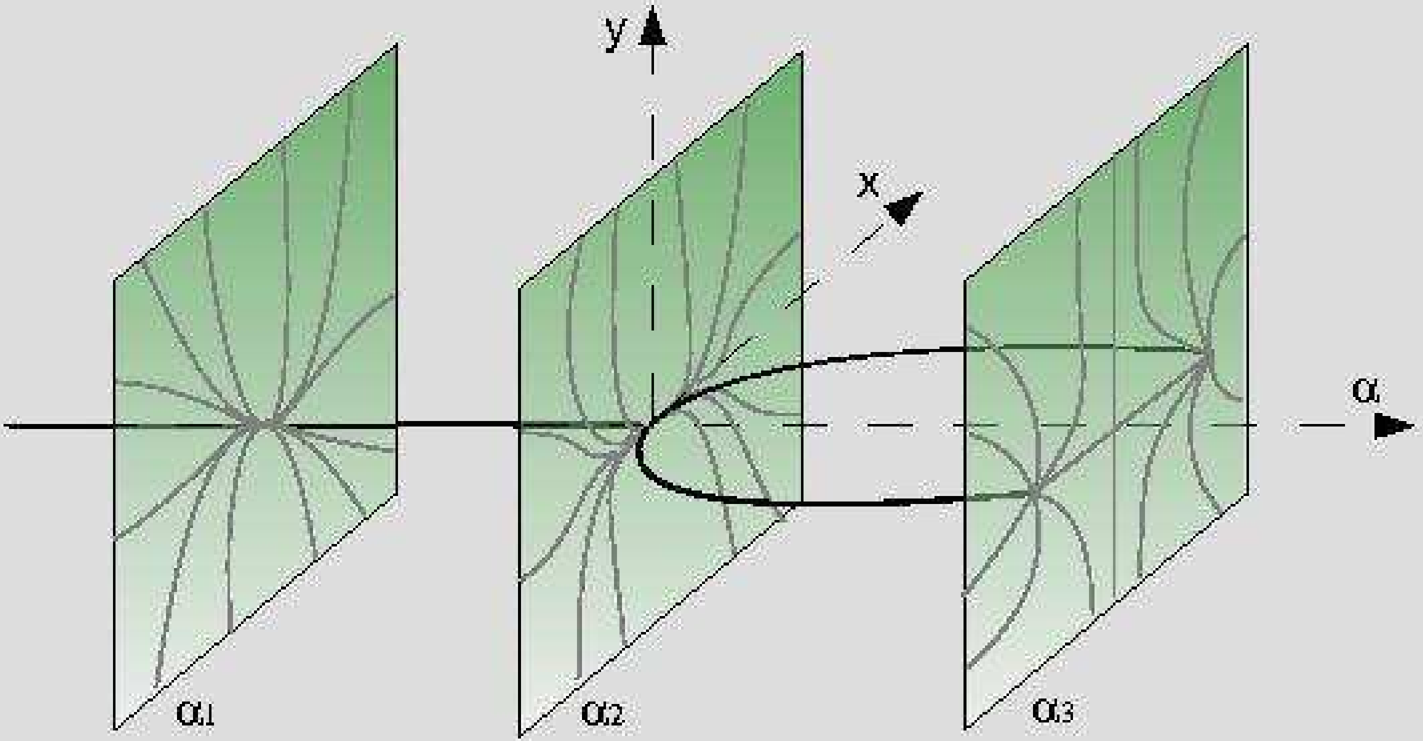


NON-CHAOTIC STATE



BIFURCATION DIAGRAMS

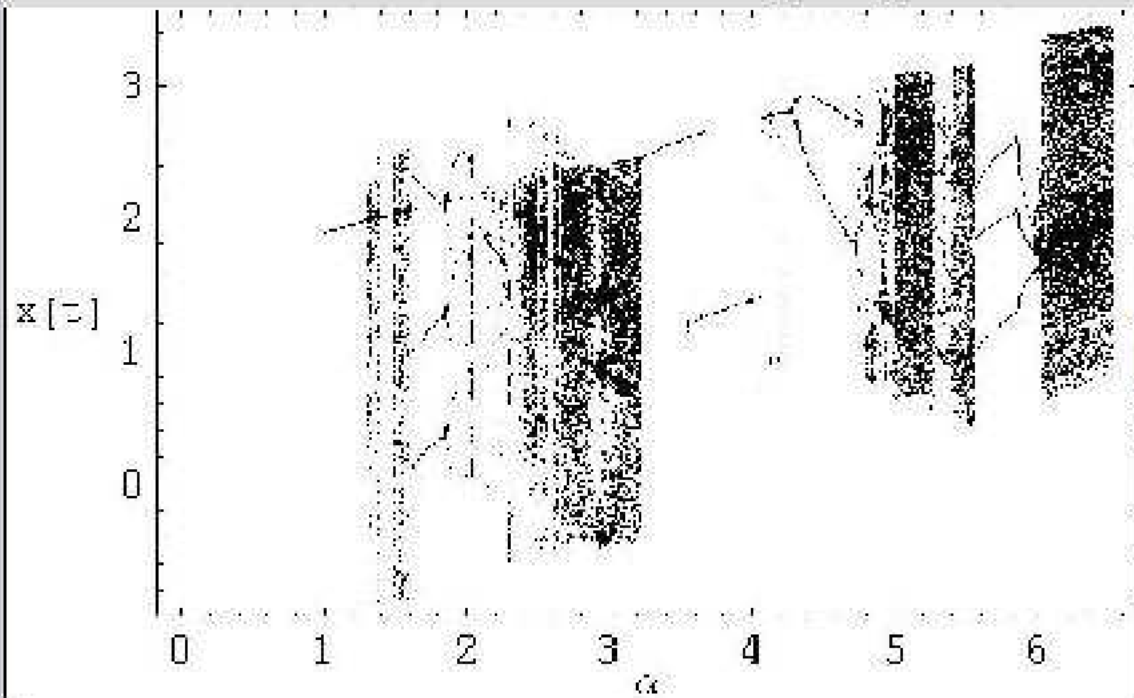
$$\dot{\mathbf{x}} = f(\alpha, \overline{\mathbf{x}})$$



BIFURCATION DIAGRAMS DUFFING SYSTEM

$\dot{\mathbf{x}} = f(\alpha, \mathbf{x})$

$$\dot{x} = y$$
$$\dot{y} = -0.1 \cdot y - x^3 + \alpha \cdot \cos(1.0 \cdot z)$$
$$\dot{z} = 1$$

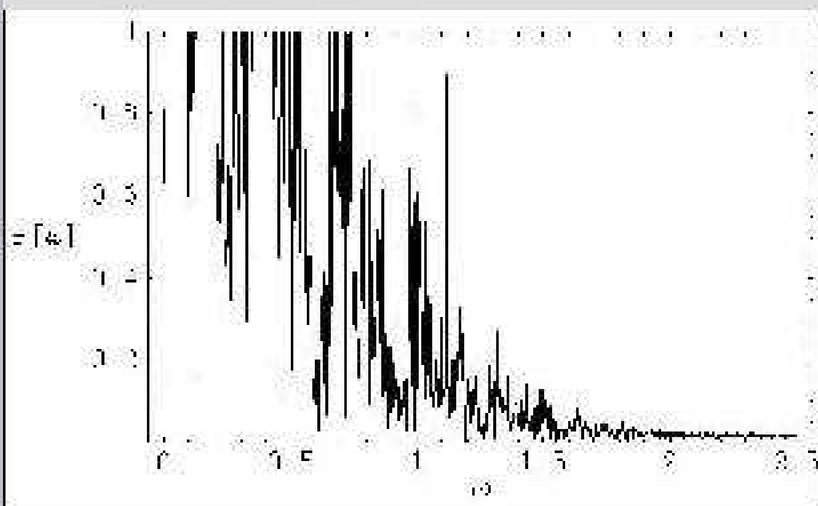


$\alpha \in (1; 6.3)$

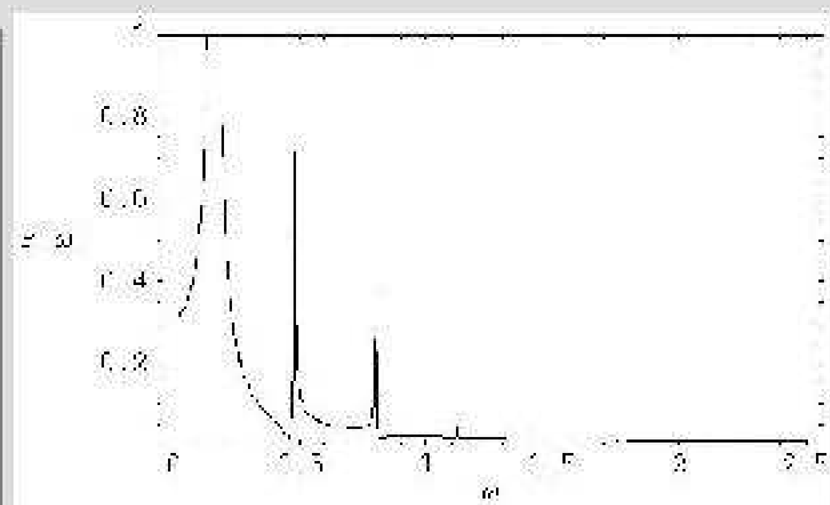
FOURIER POWER SPECTRUM DUFFING SYSTEM

$$P(\omega)=\frac{1}{\sqrt{(n)}}\sum_{i=1}^{n}\exp(2\pi \boldsymbol{I})\cdot(i-1)\cdot(\omega-1)/n$$

CHAOTIC STATE



NON-CHAOTIC STATE



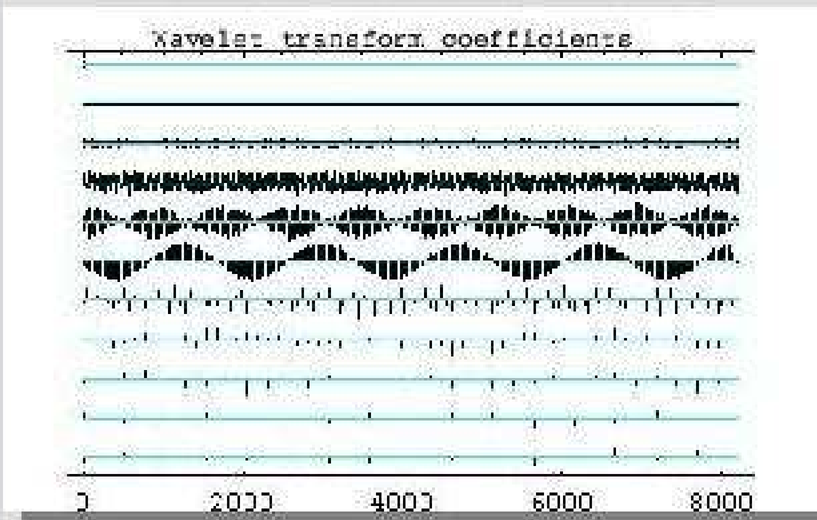
WAVELET ANALYSIS
DUFFING SYSTEM

$$f_j(t) = f_{j-1}(t) + d_{j-1}(t)$$

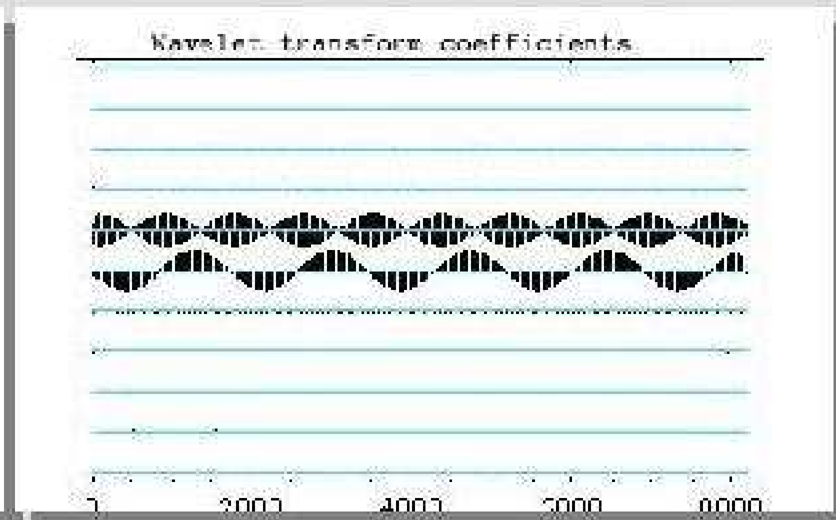
$$f_j(t) = \sum_k f_k^{j-1} \cdot \Phi_{j-1,k}(t) + \sum_k d_k^{j-1} \cdot \Psi_{j-1,k}(t)$$

$$f_k^j = \langle \Phi_{jk}, f_j \rangle, \quad d_k^j = \langle \Psi_{jk}, f_j \rangle$$

CHAOTIC STATE



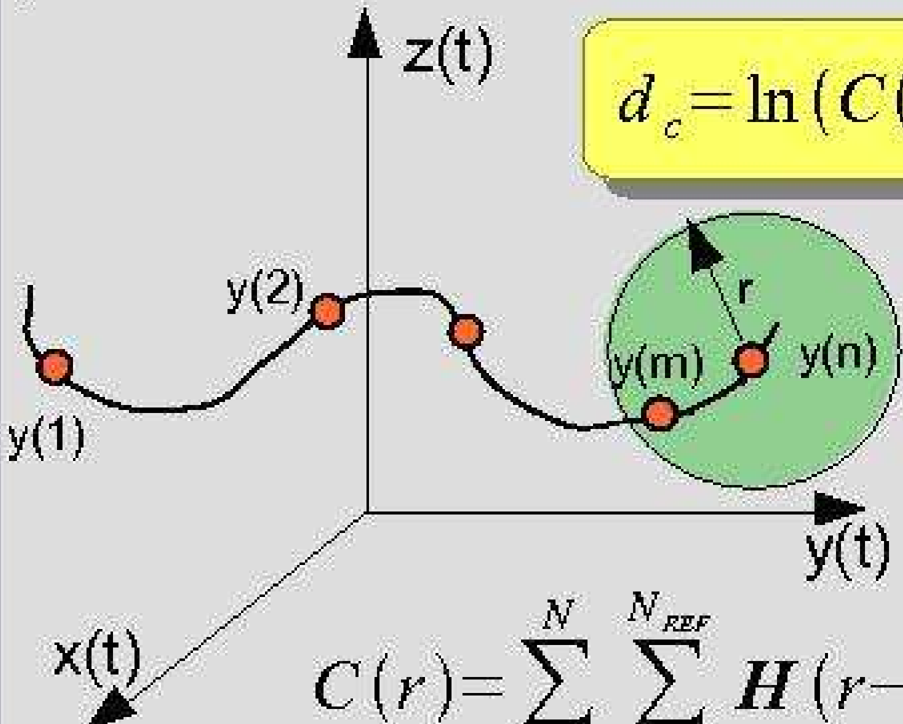
NON-CHAOTIC STATE



ATTRACTOR DIMENSION GEOMETRY FROM TIME SERIES

$$x(n) = x(t_0 + n \cdot \Delta t)$$

$$y(n) = [x(n), x(n+T), \ldots, x(n+(d-1)T)]$$

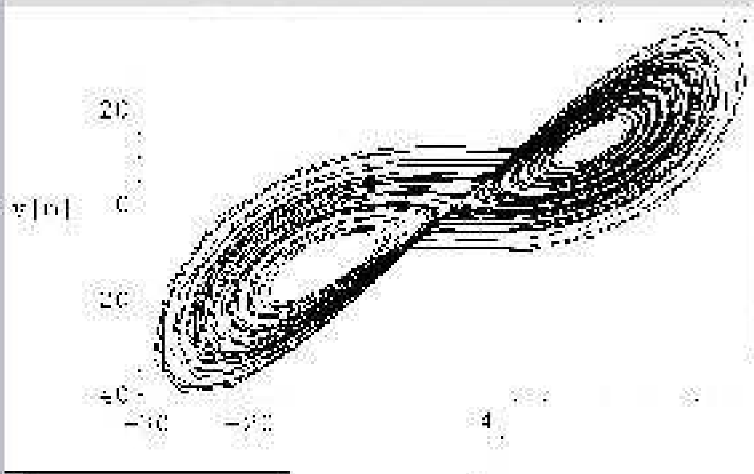


$$d_c = \ln(C(r))/\ln(r), \quad d = \boldsymbol{E}(d_c) + 1$$

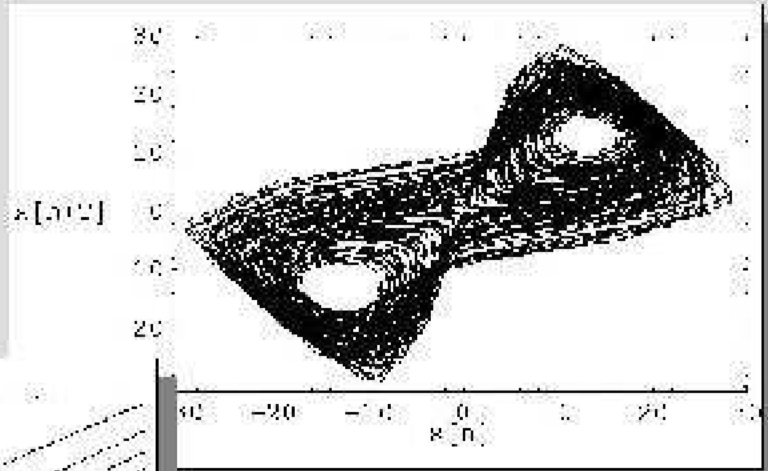
$$C(r) = \sum_{n=1}^{N} \sum_{m=1}^{N_{REF}} \boldsymbol{H}(r - \|y(n) - y(m)\|) / N \cdot N_{REF}$$

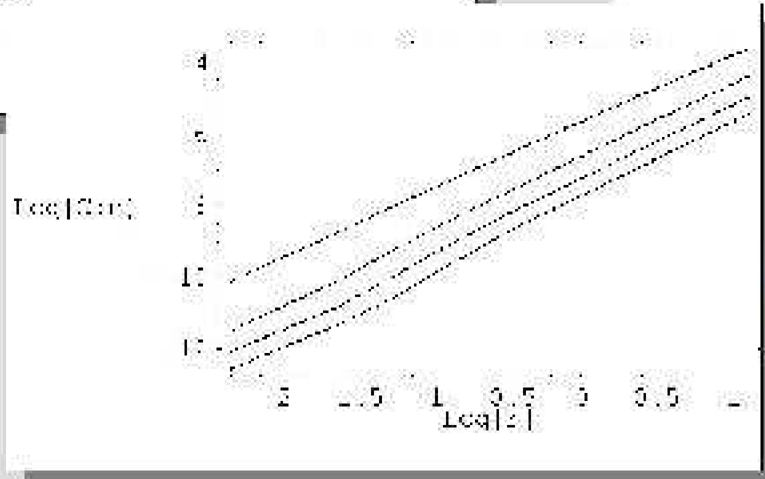
ATTRACTOR DIMENSION GEOMETRY FROM TIME SERIES – LORENZ SYSTEM

PHASE PORTRAIT



RECONSTRUCTION

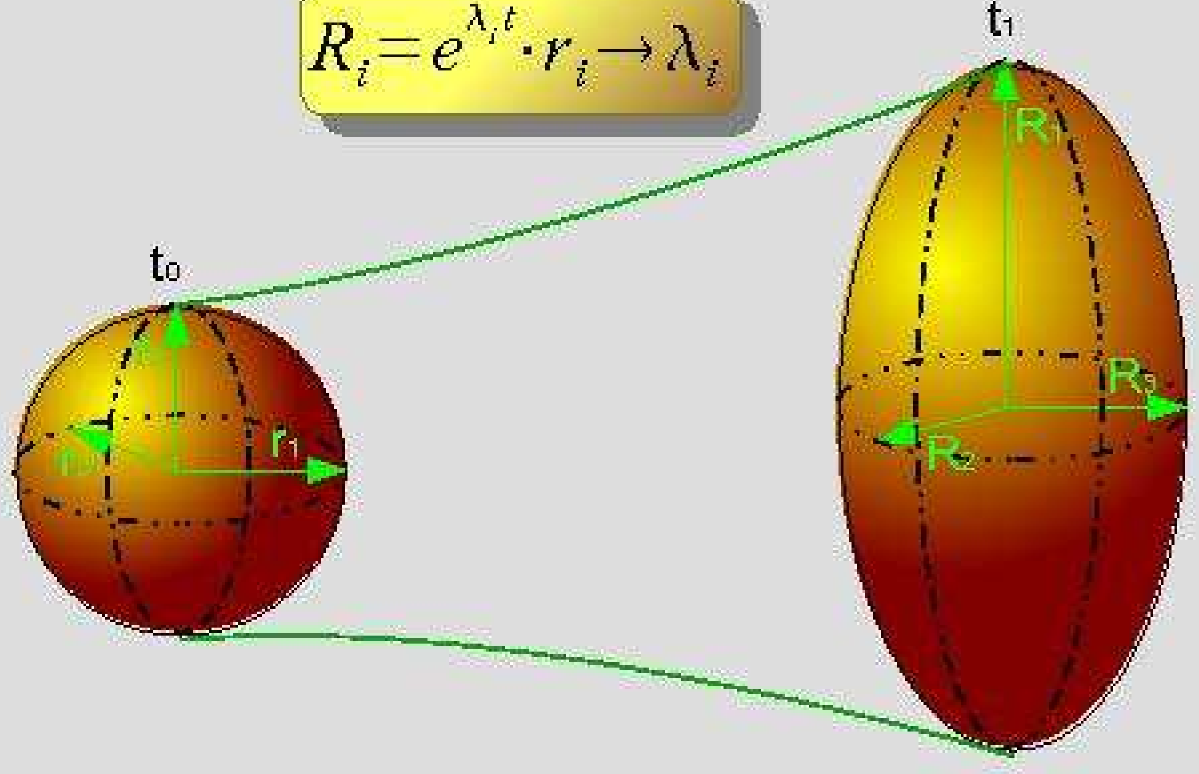




$$d_c = 2.06$$

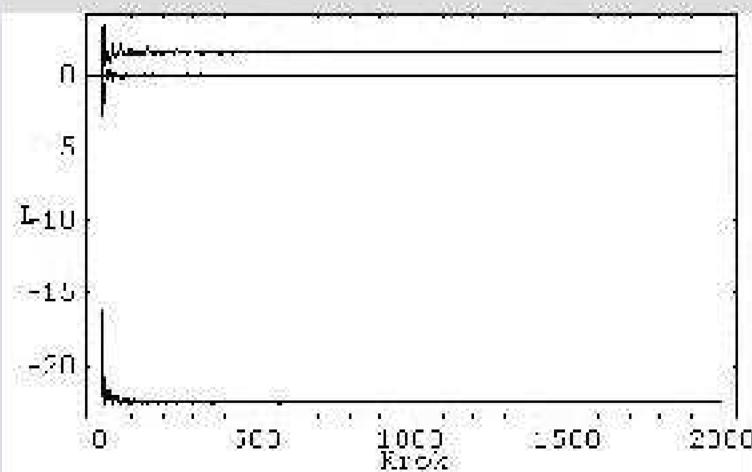
LYAPUNOV EXPONENTS

$$R_i = e^{\lambda_i t} \cdot r_i \rightarrow \lambda_i$$



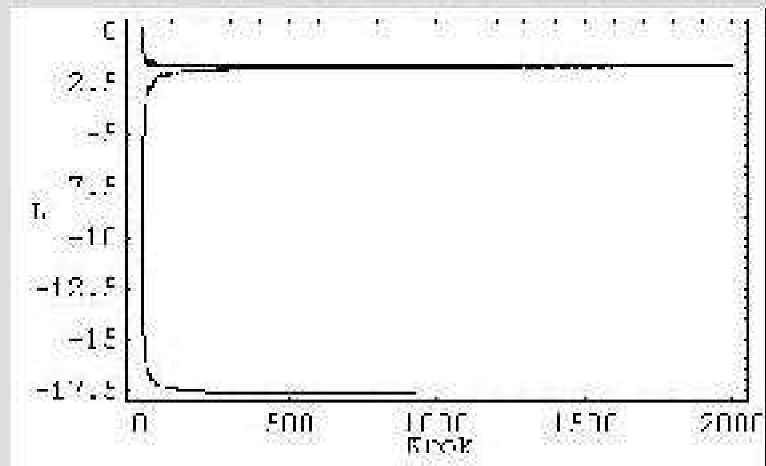
LYAPUNOV EXPONENTS LORENZ SYSTEM

CHAOTIC STATE



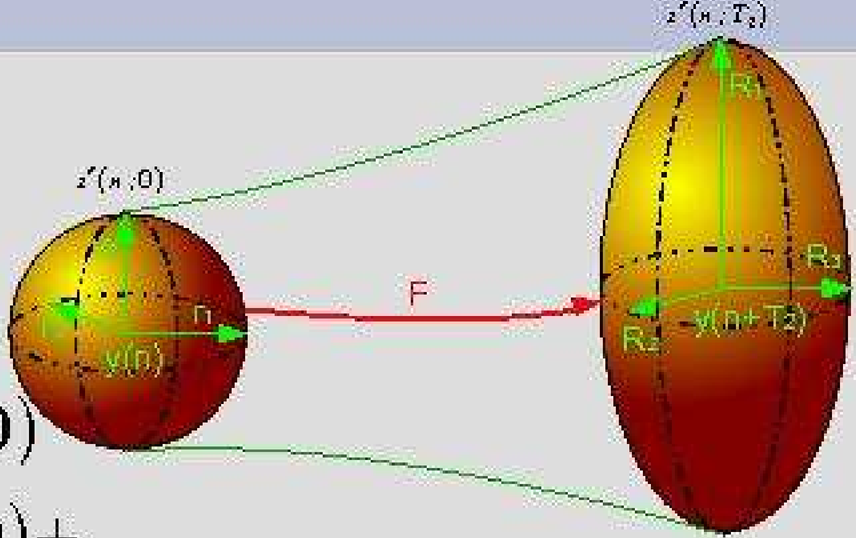
$$\lambda_1 = 1.5084 \,, \lambda_2 = -0.005 \,, \lambda_3 = -22.5022$$

NON-CHAOTIC STATE



$$\lambda_1 = -1.6437 \,, \lambda_2 = -1.6627 \,, \lambda_3 = -17{,}6934$$

LYAPUNOV EXPONENTS FROM TIME SERIES



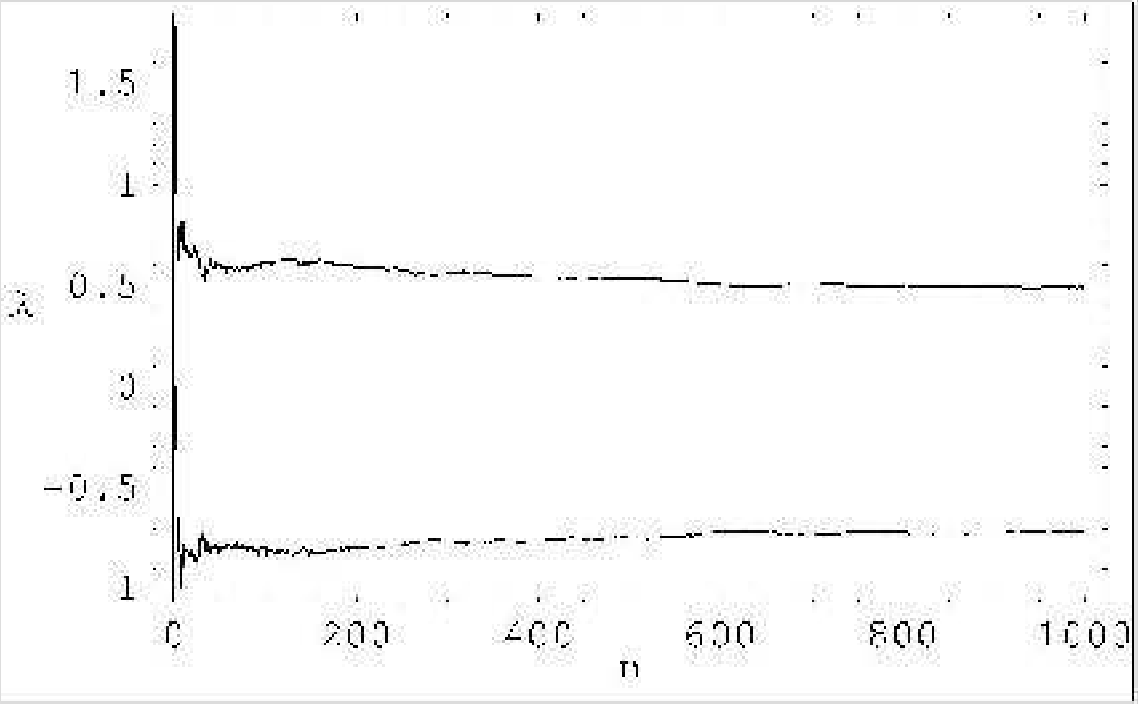
$$y(n+T_2)=F(y(n))$$

$$z^r_\alpha(n,T_2)=DF_{\alpha\beta}\,z^r_\beta(n;0)$$
$$+DF^{(2)}_{\alpha\beta\gamma}(n)\,z^r_\beta(n;0)\,z^r_\gamma(n;0)+\ldots$$

$$\boldsymbol{DF}(m+1)\cdot\boldsymbol{Q}(m)=\boldsymbol{Q}(m+1)\cdot\boldsymbol{R}(m+1)$$

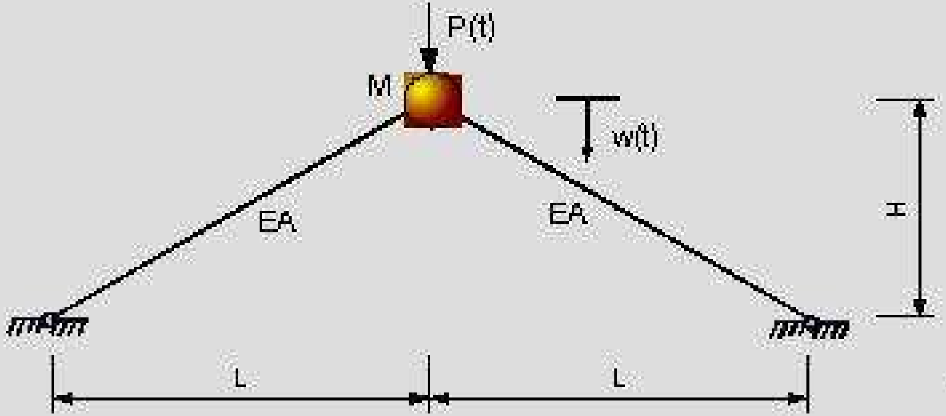
$$\lambda_i=\frac{1}{KT_2}\sum_{k=1}^{K}\ln R_{ii}(k),\ \ i=1,2,\ldots,d$$

LYAPUNOV EXPONETS FROM TIME SERIES – IKEDA MAP



$$\lambda_1 = 0.505\,, \lambda_2 = -0.719$$

CHAOTIC VIBRATIONS – MISES TRUSS



$$M = 0.01; \quad L = 2.5; \quad c = 0.001;$$

$$H = 0.5; \quad \sin\alpha_0 = \frac{H}{\sqrt{H^2 + L^2}};$$

$$P(t) = \sin(0.99 \cdot t)$$

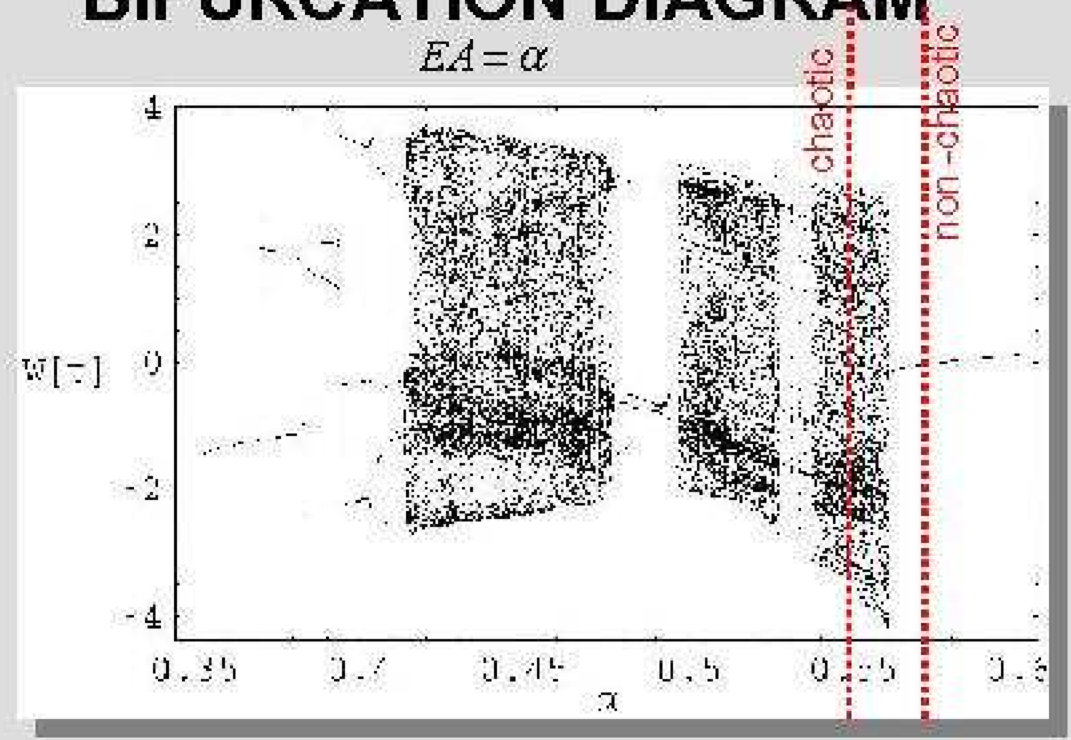
$$m\ddot{w} + c\dot{w} + \beta w = P(t) \quad \text{where} \quad \dot{(\,)} = \frac{\partial}{\partial t}$$

Initial conditions $\quad w(0) = 0 \quad \dot{w}(0) = 0$

$$\beta = -\frac{2EA}{L}(H - w)\sin\alpha_0 \frac{\sqrt{L^2 + (H - w)^2} - \sqrt{L^2 + H^2}}{\sqrt{L^2 + (H - w)^2}}$$

CHAOTIC VIBRATIONS – MISES TRUSS

BIFURCATION DIAGRAM

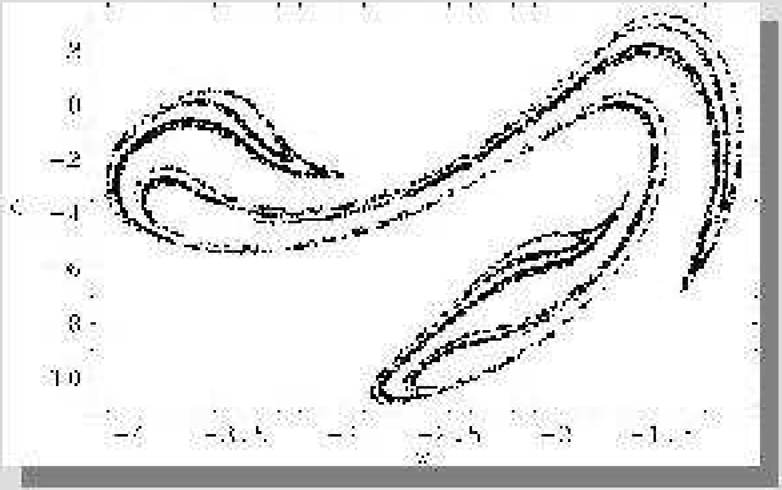


CHAOTIC STATE EA=0.55

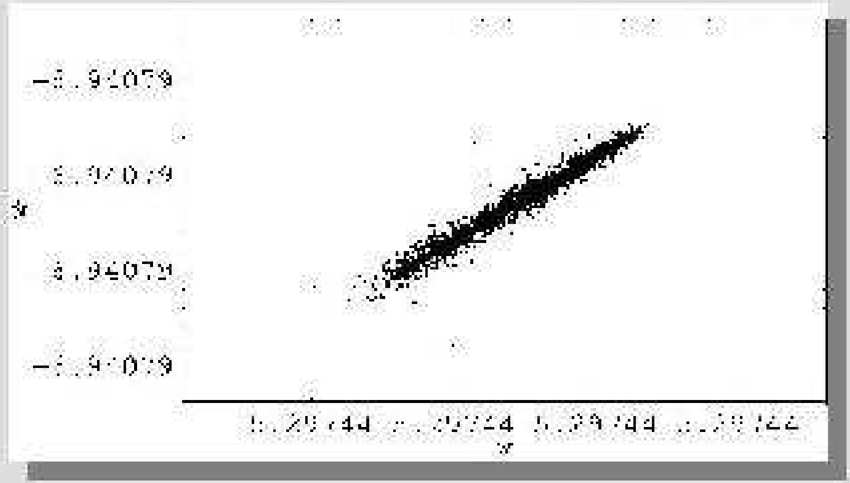
NON-CHAOTIC STATE EA=0.57

CHAOTIC VIBRATIONS – MISES TRUSS

POINCARE SECTION



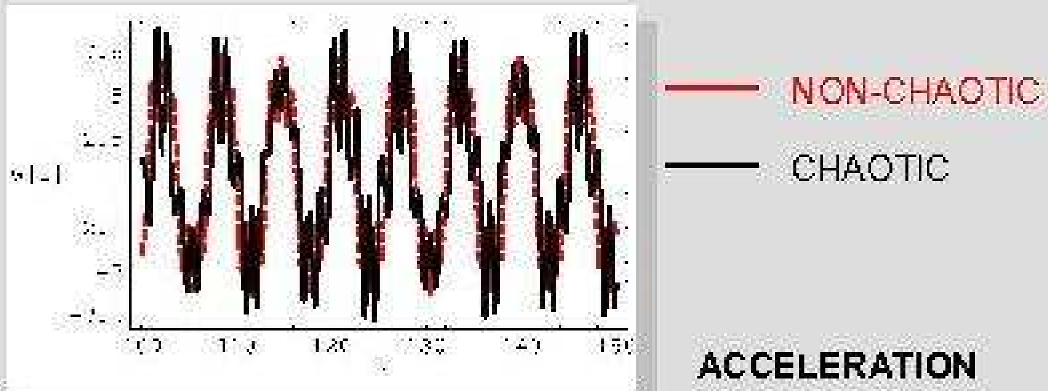
CHAOTIC STATE



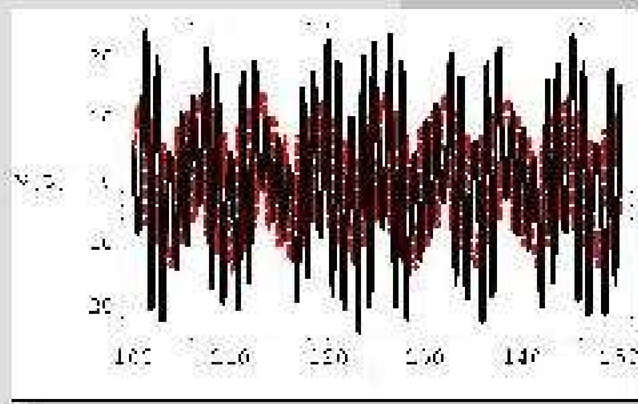
NON-CHAOTIC STATE

CHAOTIC VIBRATIONS – MISES TRUSS

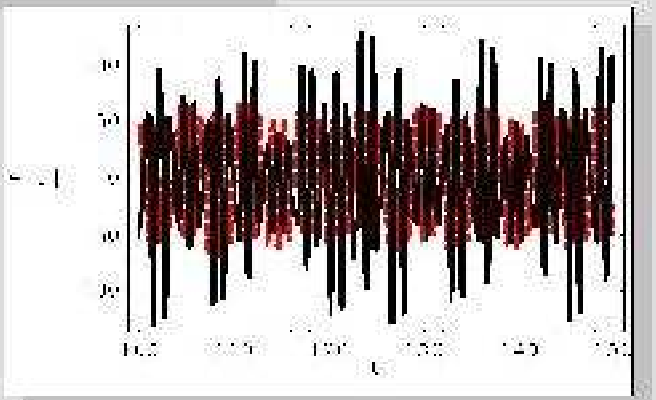
DISPLACEMENT



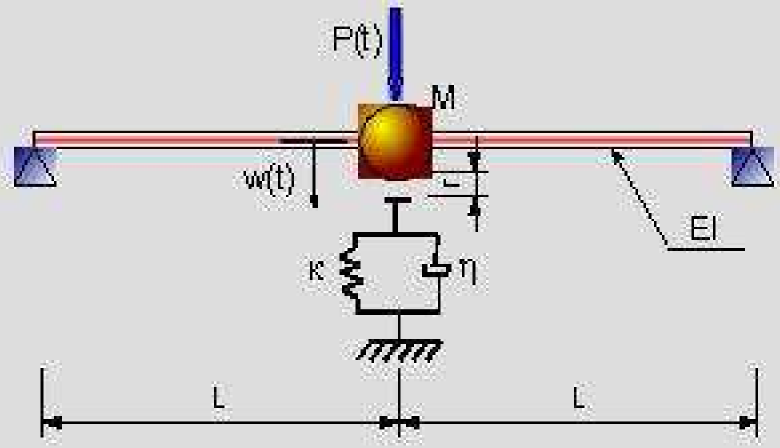
VELOCITY



ACCELERATION



CHAOTIC VIBRATIONS – CONTACT PROBLEM



$M=0.5;\quad L=1;\quad \eta=0.25;$

$\kappa=850;\quad p(t)=\frac{6}{\pi}\sin(4\pi t);\quad r=0$

chaotic state $EI = 3.0$

non-chaotic state $EI = 3.3$

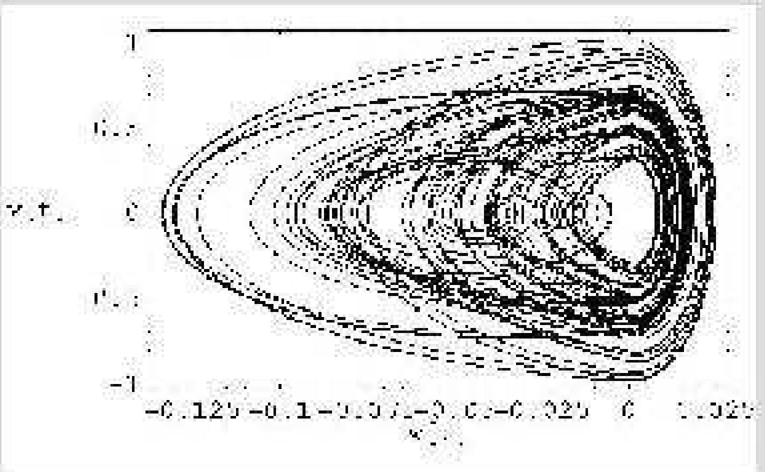
$$m\ddot{w}+\eta H(w-r)\dot{w}+12\frac{EI}{L^3}w+\kappa H(w-r)w=P(t)$$

Initial conditions $w(0)=0\quad \dot{w}(0)=0$

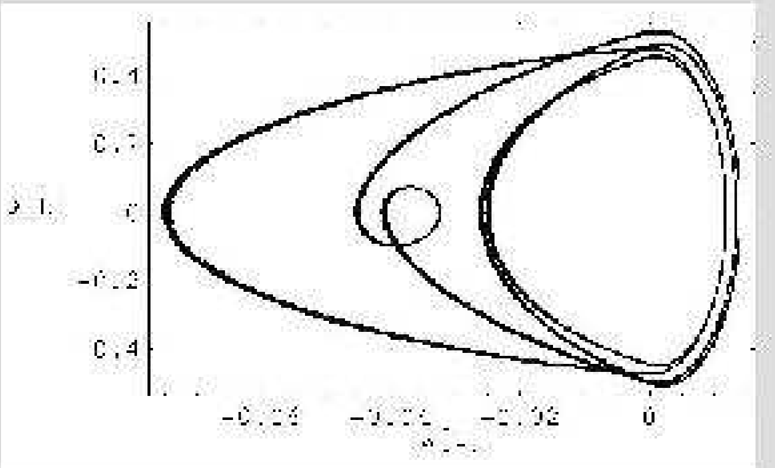
where $\dot{(\,)}=\frac{\partial}{\partial t}$, $H(\,)$ Heaviside function

CHAOTIC VIBRATIONS – CONTACT PROBLEM

PHASE PORTRAITS



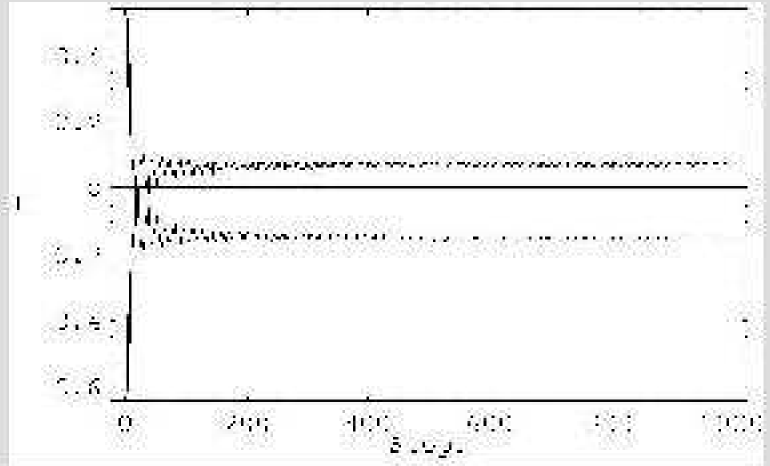
CHAOTIC STATE



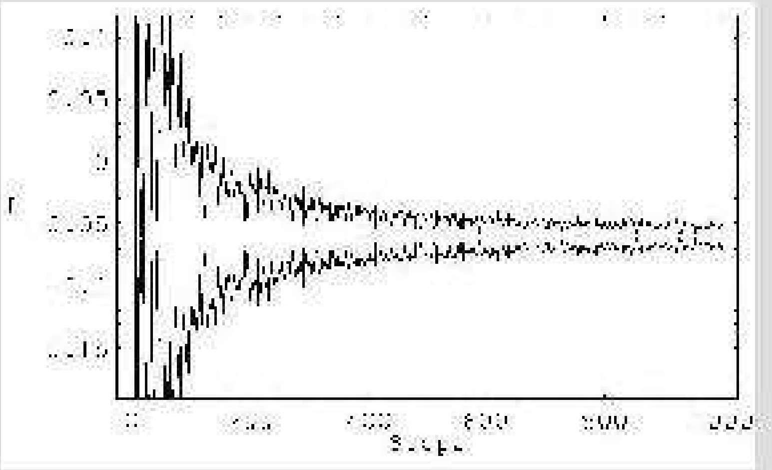
NON-CHAOTIC STATE

CHAOTIC VIBRATIONS – CONTACT PROBLEM

LYAPUNOV EXPONETS



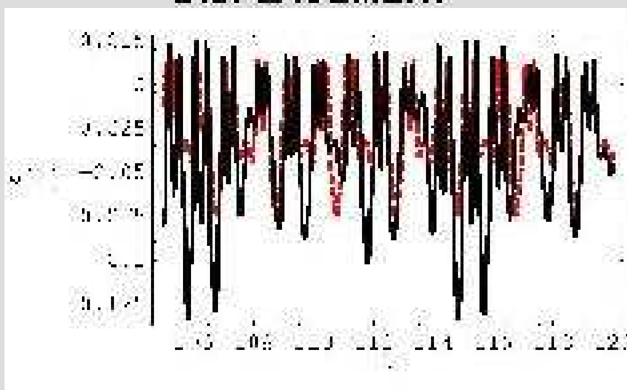
CHAOTIC STATE



NON-CHAOTIC STATE

CHAOTIC VIBRATIONS – CONTACT PROBLEM

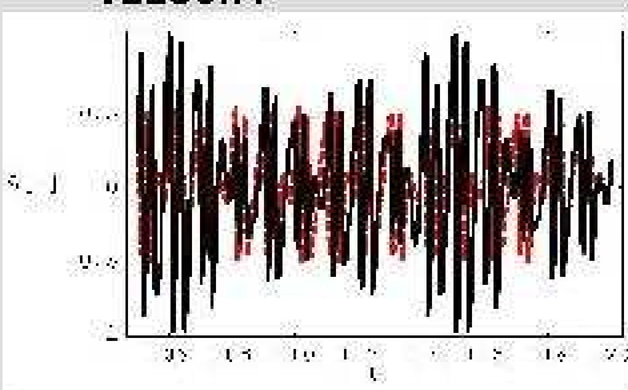
DISPLACEMENT



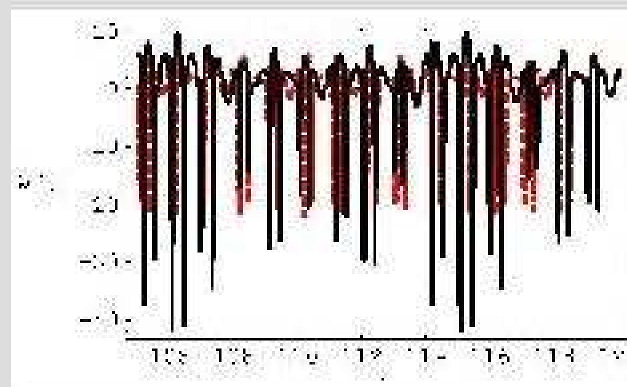
NON-CHAOTIC

CHAOTIC

VELOCITY



ACCELERATION



SUMMARY AND CONCLUSIONS

- Qualitative and quantitative tools for identification of chaos phenomenon in dynamical systems were shown
- The examples of chaotic vibrations in nonlinear problems of bar dynamics were investigated
- Transitions from chaotic to non-chaotic vibrations causes a large increase of the amplitudes of displacements, velocities, accelerations
- For specific values of equation parameters chaotic motions could be dangerous in the engineering sense.

THANK YOU FOR ATTENTION