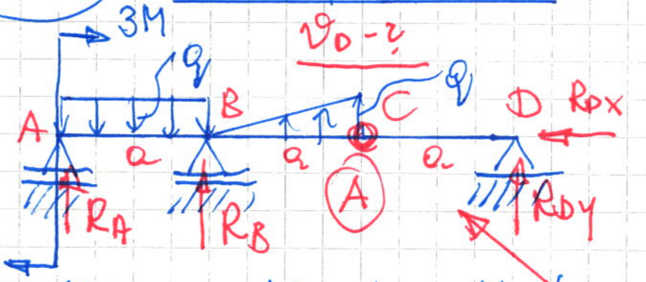
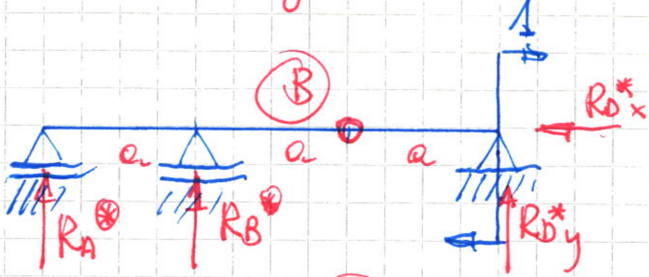
Ex. 3. $q, M, a, EI = const$

$v_D - ?$



Maxwell–Mohr's method

statically determinate



for beam (A)

(I) static eqs.

(1) $\sum P_{ix} = -R_{Dx} = 0$

(2) $\sum P_{iy} = R_A - qa + R_B + \frac{1}{2}qa + R_{Dy} = 0$

(3) $M_C^L = 3M + R_A \cdot 2a + R_B a + \frac{1}{2}qa \cdot \frac{1}{3}a = 0$ $\quad - qa \cdot \frac{3}{2}a$

(4) $M_C^R = R_{DY} \cdot a = 0$

$\Downarrow$

R_A, R_B, R_{Dx}, R_{Dy}

$0 \le x_1 \le a$

$M_1(x) = 3M + R_A x - q\frac{x^2}{2}$

$a \le x_2 \le 2a$

$M_2(x) = 3M + R_A \cdot x - qa\left(x - \frac{a}{2}\right) + \frac{q(x-a)^3}{6a}$

$2a \le x_3 \le 3a$

$M_3(x) = 3M + R_A \cdot x - qa\left(x - \frac{a}{2}\right) + \frac{1}{2}qa\left(x - \frac{5}{3}a\right)$

for beam (B)

(I) static eqs.

(1) $\sum P_{ix} = -R_{Dx}^* = 0$

(2) $\sum P_{iy} = R_A^* + R_B^* + R_{Dy}^* = 0$

(3) $M_C^L = R_A^* \cdot 2a + R_B^* \cdot a = 0$

(4) $M_C^R = -1 + R_{Dy}^* \cdot a = 0$

$\Downarrow$

$R_A^*, R_B^*, R_{Dx}^*, R_{Dy}^*$

$0 \le x_1 \le a$

$M_1^\circ(x) = R_A^* \cdot x$

$a \le x_2 \le 2a$

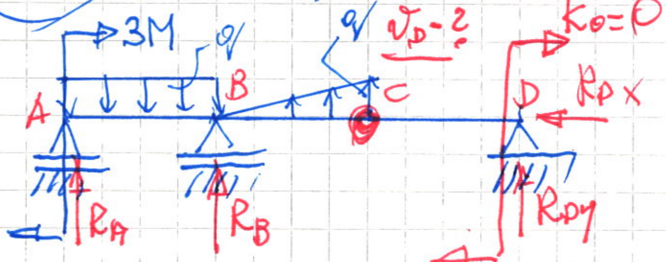
$M_2^\circ(x) = R_A^* \cdot x + R_B^*(x-a)$

$2a \le x_3 \le 3a$

$M_3^\circ(x) = R_A^* \cdot x + R_B^*(x-a)$

$$v_D = \frac{1}{EI}\left[\int_0^a \underset{(A)}{M_1(x)} \cdot \underset{(B)}{M_1^\circ(x)}\,dx + \int_a^{2a} \underset{(A)}{M_2(x)} \cdot \underset{(B)}{M_2^\circ(x)}\,dx + \int_{2a}^{3a} \underset{(A)}{M_3(x)} \cdot \underset{(B)}{M_3^\circ(x)}\,dx\right] = ?$$

Ex. 3 cont. $q, M, a, EI = \text{const}$



Castigliano's method

statically determinate

I) Static eqs.

1) $\sum P_{ix} = -R_{Dx} = 0$

2) $\sum P_{iy} = R_A - qa + R_B + \frac{1}{2}qa + R_{Dy} = 0$

3) $M_C^L = 3M + R_A \cdot 2a - qa \cdot \frac{3}{2}a + R_B \cdot a + \frac{1}{2}qa \cdot \frac{1}{3}a = 0$

4) $M_C^R = K_D - R_{Dy} \cdot a = 0$

$R_A = f(q, M, K_D)$

$R_B = f^*(q, M, K_D)$!

II) Bending moments

$0 \le x_1 \le a$

$$M_1(x) = 3M - \frac{qx^2}{2} + R_A \cdot x$$

$$\frac{\partial M_1(x)}{\partial K_D} \neq 0$$ because $R_A = f(K_D)$

$a \le x_2 \le 2a$

$$M_2(x) = 3M - qa\left(x - \frac{a}{2}\right) + R_A \cdot x + \frac{q(x-a)^3}{6a} + R_B(x-a)$$

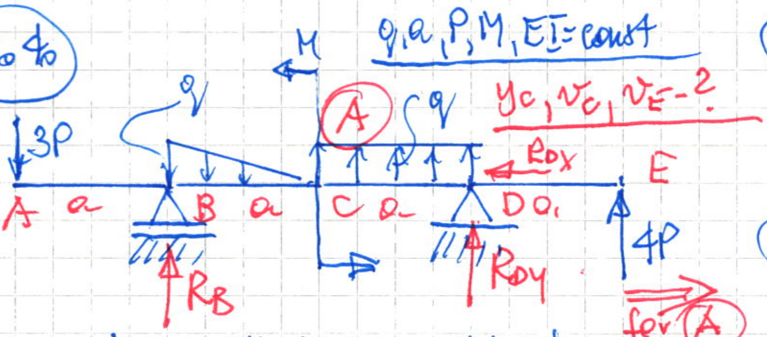
$$\frac{\partial M_2(x)}{\partial K_D} \neq 0$$ because $R_A = f(K_D)$ and $R_B^* = f(K_D)$

$$v_D = \frac{1}{EI}\left[\int_0^a M_1(x) \cdot \frac{\partial M_1(x)}{\partial K_D}dx + \int_a^{2a} M_2(x) \cdot \frac{\partial M_2(x)}{\partial K_D}dx + \int_{2a}^{3a} M_3(x) \cdot \frac{\partial M_3(x)}{\partial K_D}dx\right] = ?$$

Ex. 4

$q, a, P, M, EI = const$

y_C, v_C, v_E - ?



Maxwell-Mohr method

statically determinate

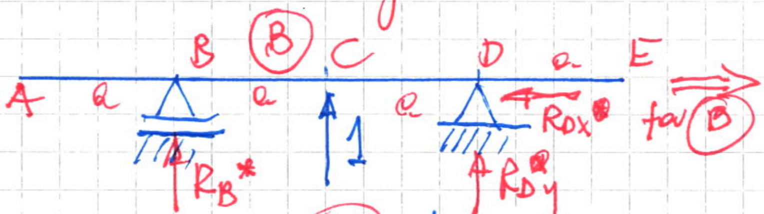
I Static eqs

① $\sum P_{ix} = 0$
② $\sum P_{iy} = 0$
③ $\sum M_i^E = 0$

$\Rightarrow R_B, R_{Dy}, R_{Dx}$

II Bending moments

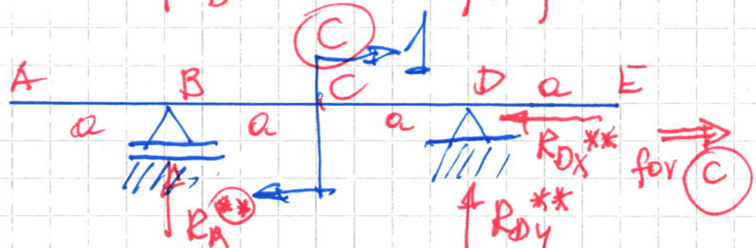
$M_1(x) = ?$
$M_2(x) = ?$
$M_3(x) = ?$
$M_4(x) = ?$



$M_1^o(x) = ?$
$M_2^o(x) = ?$
$M_3^o(x) = ?$
$M_4^o(x) = ?$

① $\sum P_{ix} = 0$
② $\sum P_{iy} = 0$
③ $\sum M_i^E = 0$

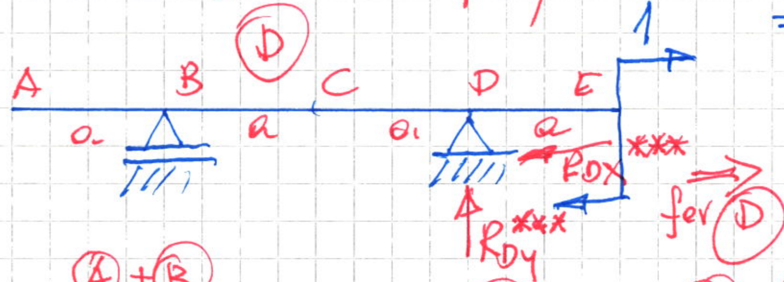
$\underbrace{\qquad}_{R_B^*, R_{Dx}^*, R_{Dy}^*}$



$M_1^o(x) = ?$
$M_2^o(x) = ?$
$M_3^o(x) = ?$
$M_4^o(x) = ?$

① $\sum P_{ix} = 0$
② $\sum P_{iy} = 0$
③ $\sum M_i^E = 0$

$\underbrace{\qquad}_{R_B^{**}, R_{Dx}^{**}, R_{Dy}^{**}}$



A + B

$$y_C = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot M_1^o(x)\,dx + \int_a^{2a} M_2(x)\cdot M_2^o(x)\,dx + \int_{2a}^{3a} M_3(x)\cdot M_3^o(x)\,dx + \int_{3a}^{4a} M_4(x)\cdot M_4^o(x)\,dx\right] = ?$$

A + C

$$v_C = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot M_1^o(x)\,dx + \int_a^{2a} M_2(x)\cdot M_2^o(x)\,dx + \int_{2a}^{3a} M_3(x)\cdot M_3^o(x)\,dx + \int_{3a}^{4a} M_4(x)\cdot M_4^o(x)\,dx\right] = ?$$

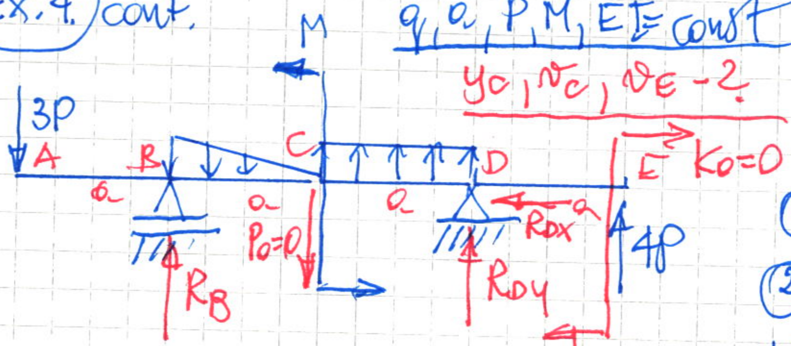
A + D

$$v_E = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot M_1^o(x)\,dx + \int_a^{2a} M_2(x)\cdot M_2^o(x)\,dx + \int_{2a}^{3a} M_3(x)\cdot M_3^o(x)\,dx + \int_{3a}^{4a} M_4(x)\cdot M_4^o(x)\,dx\right] = ?$$

Ex. 4. cont.

$q, a, P, M, EI = const$

$y_C, \vartheta_C, \vartheta_E - ?$



Castigliano method

statically determinate

(I) static eqs.

(1) $\sum P_{ix} = -R_{Dx} = 0$

(2) $\sum P_{iy} = -3P + R_B - \frac{1}{2}qa - P_0 + qa + R_{Dy} + 4P = 0$

(3) $\sum M_i^E = -3P \cdot 4a + R_B \cdot a - \frac{1}{2}qa\frac{8}{3}a - P_0 \cdot 2a - M + qa \cdot \frac{3}{2}a + R_{Dy} \cdot a + \cancel{K_0} = 0$

$R_B = f(P, M, q, P_0, K_0)$

$R_{Dy} = f(P, M, q, P_0, K_0)$!

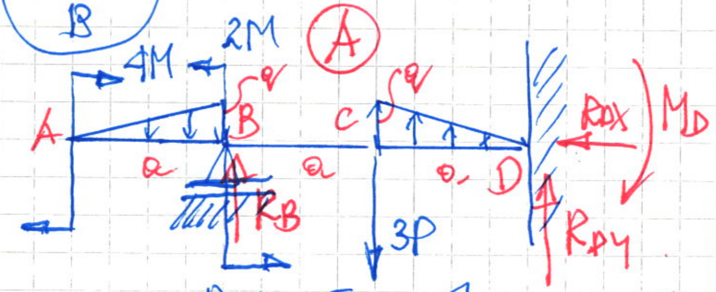
$$y_C = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot\frac{\partial M_1(x)}{\partial P_0}dx + \int_a^{2a} M_2(x)\cdot\frac{\partial M_2(x)}{\partial P_0}dx + \int_{2a}^{3a} M_3(x)\cdot\frac{\partial M_3(x)}{\partial P_0}dx + \int_{3a}^{4a} M_4(x)\cdot\frac{\partial M_4(x)}{\partial P_0}dx\right] = ?$$

$$\vartheta_C = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot\frac{\partial M_1(x)}{\partial M}dx + \int_a^{2a} M_2(x)\frac{\partial M_2(x)}{\partial M}dx + \int_{2a}^{3a} M_3(x)\frac{\partial M_3(x)}{\partial M}dx + \int_{3a}^{4a} M_4(x)\cdot\frac{\partial M_4(x)}{\partial M}dx\right] = ?$$

$$\vartheta_E = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot\frac{\partial M_1(x)}{\partial K_0}dx + \dots + \int_{3a}^{4a} M_4(x)\frac{\partial M_4(x)}{\partial K_0}dx\right] = ?$$

$$\begin{cases} M_1(x), & \frac{\partial M_1(x)}{\partial P_0}, & \frac{\partial M_1(x)}{\partial M}, & \frac{\partial M_1(x)}{\partial K_0} \\ M_2(x) -, & \frac{\partial M_2(x)}{\partial P_0}, & \frac{\partial M_2(x)}{\partial M}, & \frac{\partial M_2(x)}{\partial K_0} \\ M_3(x), & \frac{\partial M_3(x)}{\partial P_0}, & \frac{\partial M_3(x)}{\partial M}, & \frac{\partial M_3(x)}{\partial K_0} \\ M_4(x), & \frac{\partial M_4(x)}{\partial P_0}, & \frac{\partial M_4(x)}{\partial M}, & \frac{\partial M_4(x)}{\partial K_0} \end{cases}$$

Ex. 2. B



$q, a, P, M, EI = const$

reactions - ?

Maxwell-Mohr method

hyperstatic beam

for ⇒ A

Ⓘ Static eqs.

① $\sum P_{ix} = 0$

② $\sum P_{iy} = 0$

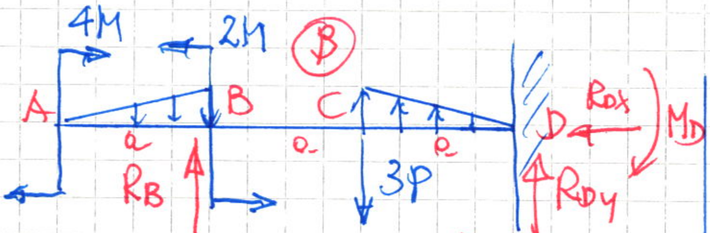
③ $\sum M_i^D = 0$

4 reactions - 3 static eqs ⇒ 1x hyperstatic

R_B - hyperstatic reaction

④ geometrical eq.

$y_B = 0$

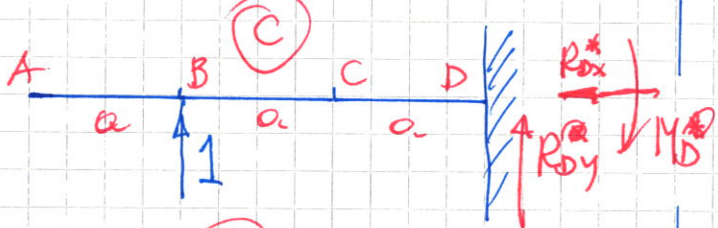


statically determinate

$M_1(x) = 4M - \frac{qx^3}{6a}$

$M_2(x) = 4M - \frac{1}{2}qa\left(x - \frac{2}{3}a\right) - 2M + R_B(x-a)$

$M_3(x) = 4M - \frac{1}{2}qa\left(x - \frac{2}{3}a\right) - 2M + R_B(x-a)$
$- 3P(x-2a) + \underbrace{q\frac{(x-2a)^2}{2} - q\frac{(x-2a)^3}{6a}}_{\text{trapeze}}$



$M_1^o(x) = 0$

$M_2^o(x) = 1(x-a) = x-a$

$M_3^o(x) = 1(x-a) = x-a$

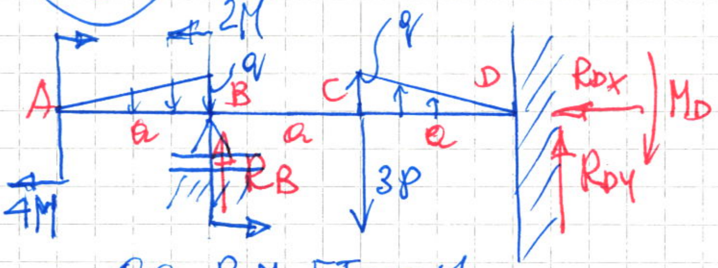
B + C

$$\underbrace{y_B = \frac{1}{EI}\left[\int_0^a \overset{B}{M_1(x)} \cdot \overset{C}{M_1^o(x)}\,dx + \int_a^{2a} \overset{B}{M_2(x)} \cdot \overset{C}{M_2^o(x)}\,dx + \int_{2a}^{3a} \overset{B}{M_3(x)} \cdot \overset{C}{M_3^o(x)}\,dx\right]}_{} = 0\,!$$

R_B ⇒ ①, ②, ③ ⇒ R_{Dx}, R_{Dy}, M_D

static eqs

Ex. 2. B cont.



$q, a, P, M, EI = const$

reactions - ?

Menabrea-Castigliano m.

hyperstatic beam

(I) static eqs.

(1) $\sum P_{ix} = 0$

(2) $\sum P_{iy} = 0$

(3) $\sum M_{iD} = 0$

4 reactions - 3 static eqs $\Rightarrow$

1x hyperstatic beam

R_B - hyperstatic ~~eq~~ reaction

(4) geometrical eq.

$y_B = 0$ $\quad \frac{\partial U}{\partial R_B} = y_B = 0$

M-C method

geometrical eq.

$$(4)\quad y_B = 0 = \frac{\partial U}{\partial R_B} = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot\frac{\partial M_1(x)}{\partial R_B}dx + \int_a^{2a} M_2(x)\cdot\frac{\partial M_2(x)}{\partial R_B}dx + \int_{2a}^{3a} M_3(x)\cdot\frac{\partial M_3(x)}{\partial R_B}dx\right] = 0 \; !$$

$$\begin{cases} M_1(x), & \frac{\partial M_1(x)}{\partial R_B} \\ M_2(x), & \frac{\partial M_2(x)}{\partial R_B} \\ M_3(x), & \frac{\partial M_3(x)}{\partial R_B} \end{cases}$$

$$R_B \Rightarrow \left.\begin{matrix} (1) \\ (2) \\ (3) \end{matrix}\right\} \Rightarrow R_{DX}, R_{DY}, M_D$$