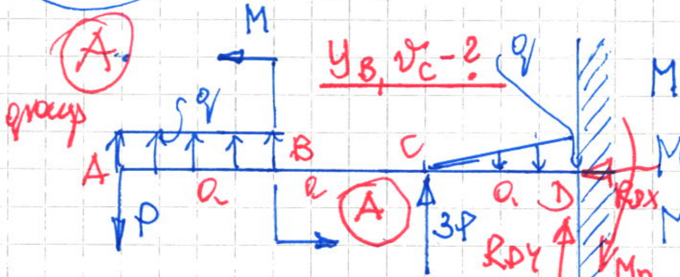
Ex. 1.

$q, P, M, a, EI = \text{const}$

$y_B, \vartheta_C - ?$



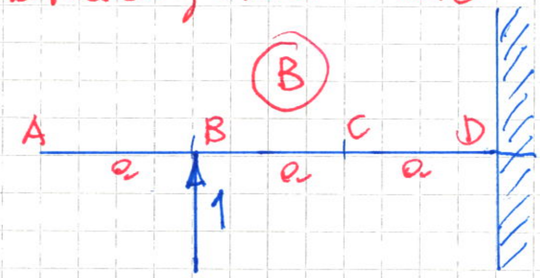
$$M_1(x) = -P\cdot x + \frac{qx^2}{2}$$

$$M_2(x) = -P\cdot x + qa\left(x-\frac{a}{2}\right) - M$$

$$M_3(x) = -P\cdot x + qa\left(x-\frac{a}{2}\right) - M + 3P(x-2a) - q\frac{(x-2a)^3}{6a}$$

Maxwell-Mohr's method

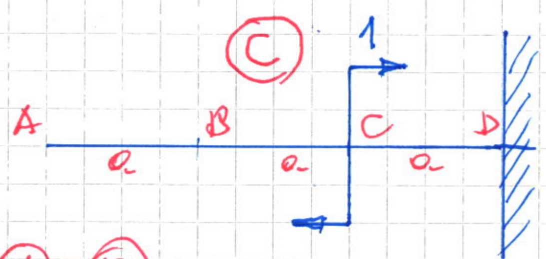
statically determinate



$$M_1^{*}(x) = 0$$

$$M_2^{*}(x) = 1\cdot(x-a) = x-a$$

$$M_3^{*}(x) = 1(x-a) = x-a$$



$$M_1^{**}(x) = 0$$

$$M_2^{**}(x) = 0$$

$$M_3^{**}(x) = 1$$

A + B

$$y_B = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot M_1^{*}(x)\,dx + \int_a^{2a} M_2(x)\cdot M_2^{*}(x)\,dx + \int_{2a}^{3a} M_3(x)\cdot M_3^{*}(x)\,dx\right] =$$

$$= \frac{1}{EI}\left[\int_0^a \underbrace{\cancel{\left(-P\cdot x + \frac{qx^2}{2}\right)\cdot 0\cdot dx}}_{=0} + \int_a^{2a}\left[-P\cdot x + qa\left(x-\frac{a}{2}\right) - M\right](x-a)\,dx + \int_{2a}^{3a}\left[-P\cdot x + qa\left(x-\frac{a}{2}\right) - M + 3P(x-2a)\right]\cdot(x-a)\,dx\right.$$

A + C

$$\vartheta_C = \frac{1}{EI}\left[\int_0^a M_1(x)\cdot M_1^{**}(x)\,dx + \int_a^{2a} M_2(x)\cdot M_2^{**}(x)\,dx + \int_{2a}^{3a} M_3(x)\cdot M_3^{**}(x)\,dx\right] =$$

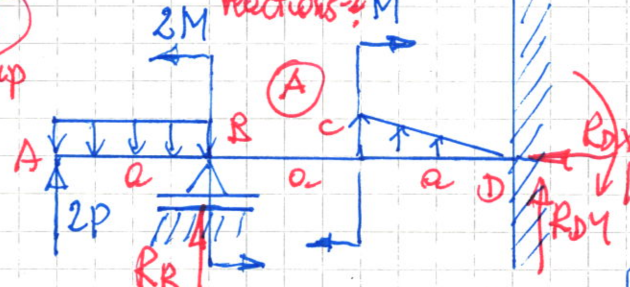
$$\frac{1}{EI}\left[\int_0^a \underbrace{\cancel{\left(-P\cdot x + \frac{qx^2}{2}\right)\cdot 0\,dx}}_{=0} + \int_a^{2a} \underbrace{\cancel{\left[-P\cdot x + qa\left(x-\frac{a}{2}\right) - M\right]\cdot 0\,dx}}_{=0} + \int_{2a}^{3a}\left[-P\cdot x + qa\left(x-\frac{a}{2}\right) - M + 3P(x-2a) - q\frac{(x-2a)^3}{6a}\right]\cdot 1\,dx\right]$$

Ex. 1.

B group

$q, P, M, a, EI = const$

reactions - 4



I static eqs.

① $\sum P_{ix} = -R_{DX} = 0$

② $\sum P_{iy} = 2P - qa + R_B + \frac{qa}{2} + R_{DY} = 0$

③ $\sum M_i^D = 2P \cdot 3a - qa\left(\frac{5}{2}a\right) - 2M + R_B \cdot 2a + M + \frac{1}{2}qa\left(\frac{2}{3}a\right) + M_D = 0$

R_B, R_{DX}, R_{DY}, M_D - 4 reactions

4 reactions - 3 static eqs. ⇒ 1x hyperst.

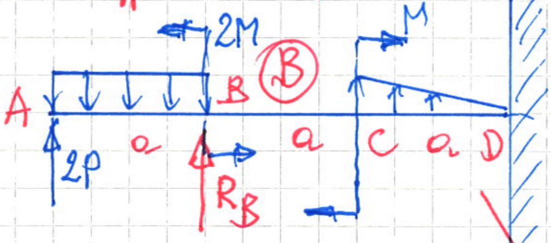
R_B - hyperstatic reaction

II Geometrical eq.

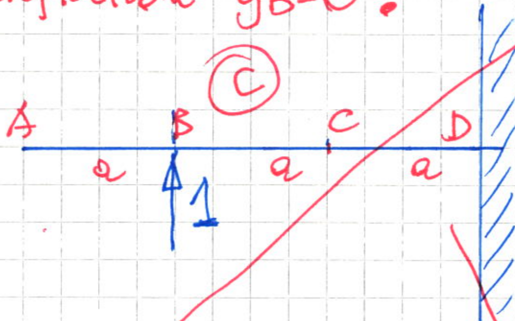
④ $y_B = 0$

Maxwell-Mohr's method (analytical version)

1x hyperstatic beam



Note: now statically determinate, but with deflection $y_B = 0$!



Bending moments for beam B

$M_1(x) = 2P \cdot x - q\frac{x^2}{2}$

$M_2(x) = 2P \cdot x - qa\left(x - \frac{a}{2}\right) - 2M + R_B(x-a)$

$M_3(x) = 2P \cdot x - qa\left(x - \frac{a}{2}\right) - 2M + R_B(x-a) + M + \underbrace{q\frac{(x-2a)^2}{2} - q\frac{(x-2a)^3}{6a}}_{\text{trapeze}}$

Bending moments for beam C

$M_1^*(x) = 0$

$M_2^*(x) = 1(x-a) = (x-a)$

$M_3^*(x) = 1(x-a) = x-a$

Geometrical equation again

B + C

$$y_B = \frac{1}{EI}\left[\int_0^a M_1(x) \cdot M_1^*(x)\,dx + \int_a^{2a} M_2(x) \cdot M_2^*(x)\,dx + \int_{2a}^{3a} M_3(x) \cdot M_3^*(x)\,dx\right] = 0!$$

$$y_B = \frac{1}{EI}\left[\int_0^a \cancel{\left(2P \cdot x - \frac{qx^2}{2}\right) \cdot 0\,dx} + \int_a^{2a}\left[2Px - qa\left(x - \frac{a}{2}\right) - 2M + R_B(x-a)\right](x-a)\,dx + \int_{2a}^{3a}\left[2P \cdot x - qa\left(x - \frac{a}{2}\right) - 2M + R_B(x-a) + M + q\frac{(x-2a)^2}{2} - \frac{q(x-2a)^3}{6a}\right] \cdot (x-a)\,dx\right] = 0!$$

$\Longrightarrow R_B \Rightarrow$ ① ② ③ static eqs $\Rightarrow R_{DX}, R_{DY}, M_D$