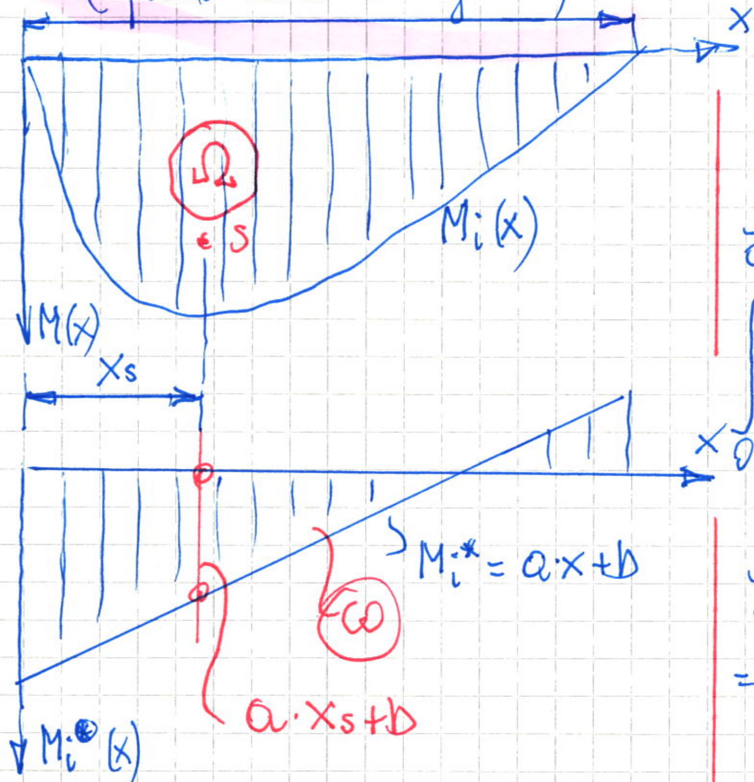
Vereshchagin procedure
(sposób Wereszczagina)



$$\int_0^l M_i(x)\cdot M_i^*(x)\,dx =$$

$$\int_0^l M_i(x)\cdot(a\cdot x + b)\,dx =$$

$$\int_0^l M_i(x)\cdot a\cdot x\cdot dx + b\int_0^l M_i(x)\,d\cdot x$$

$$= a\int_0^l M_i(x)\cdot x\cdot dx + b\int_0^l M_i(x)\,dx =$$

$$\int_0^l M_i(x)\,dx = \Omega$$

$$\int_0^l M_i(x)\cdot x\,dx = \Omega\cdot x_s \quad \Rightarrow$$

$$\int_0^l M_i(x)\cdot M_i^*(x)\,dx =$$

$$a\cdot\Omega\cdot x_s + b\cdot\Omega$$

$$\Omega\,(a\cdot x_s + b)$$

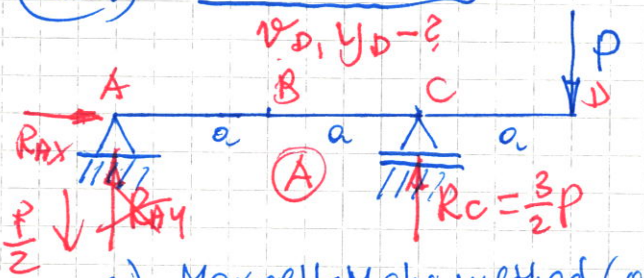
$$\int_0^l M_i(x)\cdot M_i^*(x)\,dx = \Omega\,(a\cdot x_s + b)$$

$$\begin{matrix} y(x) \\ \vartheta(x) \end{matrix} = \frac{1}{EJ}\left[\int_0 M_{\bar{A}}(x)\cdot M_i^*(x)\,dx + \int_{l_i}^{l_{i+1}} M_i(x)\cdot M_i^*(x)\,dx\right] =$$

$$= \frac{1}{EJ}\sum_{i=1}^{n}\left[\Omega_i\,(a_i x_{si} + b_i)\right]$$

Ex. 1. $P, a, EI = const$

$v_D, y_D - ?$



a) Maxwell-Mohr method (analyt.)

b) M-M, Vereshchagin procedure

(I) static eqs.

(1) $\sum P_{ix} = R_{AX} = 0$

(2) $\sum P_{iy} = R_{AY} + R_C - P = 0$

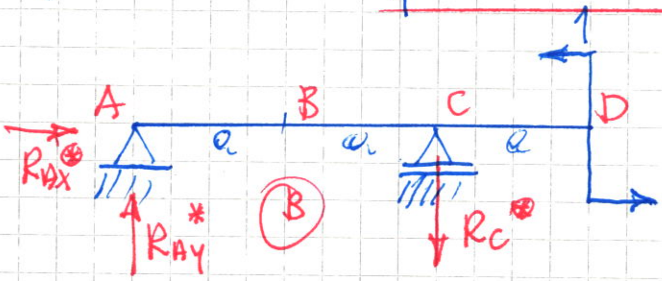
(3) $\sum M_i^C = R_{AY} \cdot 2a + P \cdot a = 0 \;|: a$

$R_{AY} = -\frac{P}{2}$

(2) $-\frac{P}{2} + R_C - P = 0 \quad R_C = \frac{3}{2}P$

(II) Bending m.

$0 \le x_1 \le a$	$a \le x_2 \le 2a$	$2a \le x_3 \le 3a$
$M_1(x) = -\frac{P}{2} \cdot x$	$M_2(x) = -\frac{P}{2} \cdot x$	$M_3(x) = -\frac{P}{2} \cdot x + \frac{3}{2}P(x-2a) = -\frac{Px}{2} + \frac{3}{2}P \cdot x - 3Pa = \frac{Px}{\cancel{2}} - 3Pa = P \cdot x - 3Pa$



(I) static eqs.

(1) $\sum P_{ix} = R_{AX}^{*} = 0$

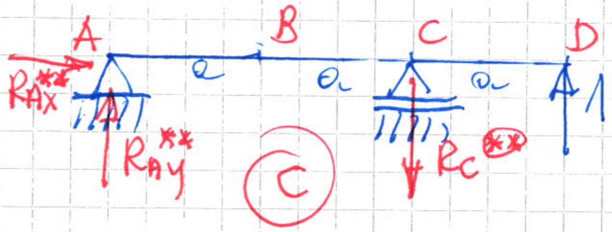
(2) $\sum P_{iy} = R_{AY}^{*} - R_C^{*} = 0 \Rightarrow R_{AY}^{*} = R_C^{*}$

(3) $\sum M_i^C = R_{AY}^{*} \cdot 2a - 1 = 0$

$R_{AY}^{*} = \frac{1}{2a} = R_C^{*}$

(II) Bending moments

$0 \le x_1 \le a$	$a \le x_2 \le 2a$	$2a \le x_3 \le 3a$
$M_1^{*}(x) = R_{AY}^{*} \cdot x = \frac{x}{2a}$	$M_2^{*}(x) = R_{AY}^{*} \cdot x = \frac{x}{2a}$	$M_3^{*}(x) = R_{AY}^{*} \cdot x - R_C^{*}(x-2a) = \frac{x}{2a} - \frac{1}{2a}(x-2a) = \frac{x}{2a} - \frac{x}{2a} + 1 = \underline{1}$



(I) static eqs.

(1) $\sum P_{ix} = R_{AX}^{**} = 0$

(2) $\sum P_{iy} = R_{AY}^{**} - R_C^{**} + 1 = 0$

(3) $\sum M_i^C = R_{AY}^{**} \cdot 2\cancel{a} - 1 \cdot \cancel{a} = 0 \quad R_{AY}^{**} = \frac{1}{2}$

from (2) $\frac{1}{2} - R_C^{**} + 1 = 0 \quad R_C^{**} = \frac{3}{2}$

(II) Bending moments

$0 \le x_1 \le a$	$a \le x_2 \le 2a$	$2a \le x_3 \le 3a$
$M_1^{**}(x) = R_{AY}^{**} \cdot x = \frac{x}{2}$	$M_2^{**}(x) = \frac{x}{2}$	$M_3^{**}(x) = \frac{x}{2} - \frac{3}{2}(x-2a) = -x + 3a$

Metoda M-M (analyt.)

A + B

$$\vartheta_B = \frac{1}{EI}\left[\int_0^a\left(-\frac{P}{2}\cdot x\right)\cdot\frac{x}{2a}dx + \int_a^{2a}\left(-\frac{P}{2}\cdot x\right)\cdot\frac{x}{2a}dx + \int_{2a}^{3a}(P\cdot x-3Pa)\cdot 1\,dx\right] =$$

$$= \frac{1}{EI}\left[\int_0^a -\frac{Px^2}{4a}dx + \int_a^{2a} -\frac{Px^2}{4a}dx + \int_{2a}^{3a}(Px-3Pa)\,dx\right] =$$

$$= \frac{1}{EI}\left[-\frac{Px^3}{12a}\Big|_0^a - \frac{Px^3}{12a}\Big|_a^{2a} + \left(P\frac{x^2}{2}-3Pax\right)\Big|_{2a}^{3a}\right] =$$

$$= \frac{1}{EI}\left[-\frac{Pa^3}{12a} - \frac{P}{12a}\left(8a^3-a^3\right) + \left(\frac{P}{2}9a^2-3Pa\cdot 3a-\frac{P}{2}4a^2+3Pa\cdot 2a\right)\right] =$$

$$= \frac{1}{EI}\left[-\frac{Pa^2}{12} - \cancel{\frac{2}{3}Pa^2} - \frac{7}{12}Pa^2 + \left(\frac{9Pa^2}{2} - 9Pa^2 - \underbrace{2Pa^2+6Pa^2}_{4Pa^2}\right)\right] =$$

$$= \frac{1}{EI}\left[-\frac{2}{3}Pa^2 - \frac{Pa^2}{2}\right] = -\frac{7Pa^2}{6EI}$$

A + C

$$\cancel{y_C = \frac{1}{EI}\left[\int_0^a\left(-\frac{P}{2}x\right)\frac{x}{2}dx + \int_a^{2a}\left(-\frac{P}{2}x\right)\frac{x}{2}dx + \int_{2a}^{3a}(P\cdot x-3Pa)\cdot 1\,dx\right] =}$$

$$\cancel{= \frac{1}{EI}\left[\int_0^a -\frac{Px^2}{4}dx + \int_a^{2a} -\frac{Px^2}{4}dx + \int_{2a}^{3a}(Px-3Pa)dx\right] =}$$

$$\cancel{= \frac{1}{EI}\left[-\frac{Px^3}{12}\Big|_0^a - \frac{Px^3}{12}\Big|_a^{2a} + \left(P\frac{x^2}{2}-3Pa\cdot x\right)\Big|_{2a}^{3a}\right] =}$$

$$\cancel{= \frac{1}{EI}\left[-\frac{Pa^3}{12} - \frac{P}{12}\left(8a^3-a^3\right) + \left(\frac{P}{2}9a^2-3Pa\cdot 3a-\frac{P}{2}\cdot 4a^2+3Pa\cdot 2a\right)\right] =}$$

$$\cancel{= \frac{1}{EI}\left[-\frac{Pa^3}{12} - \frac{7Pa^3}{12} + Pa^2\left(\frac{9}{2} - \right.\right.}$$

$$y_C = \frac{1}{EI}\left[\int_0^a\left(-\frac{P}{2}x\right)\frac{x}{2}dx + \int_a^{2a}\left(-\frac{P}{2}\cdot x\right)\cdot\frac{x}{2}dx + \int_{2a}^{3a}(P\cdot x-3Pa)(-x+3a)dx\right] =$$

$$= \frac{1}{EI}\left[\int_0^a -\frac{Px^2}{4}dx + \int_a^{2a} -\frac{Px^2}{4}dx + \int_{2a}^{3a}\left(-Px^2+3Pxa+3Pax-9Pa^2\right)dx\right] =$$

$$y_C = \frac{1}{EI}\left[-\frac{Px^3}{12}\Big|_0^a - \frac{Px^3}{12}\Big|_a^{2a} + \left(-\frac{Px^3}{3} + 3Pa\frac{x^2}{2} + 3Pa\frac{x^2}{2} - 9Pa^2x\right)\Big|_{2a}^{3a}\right] =$$

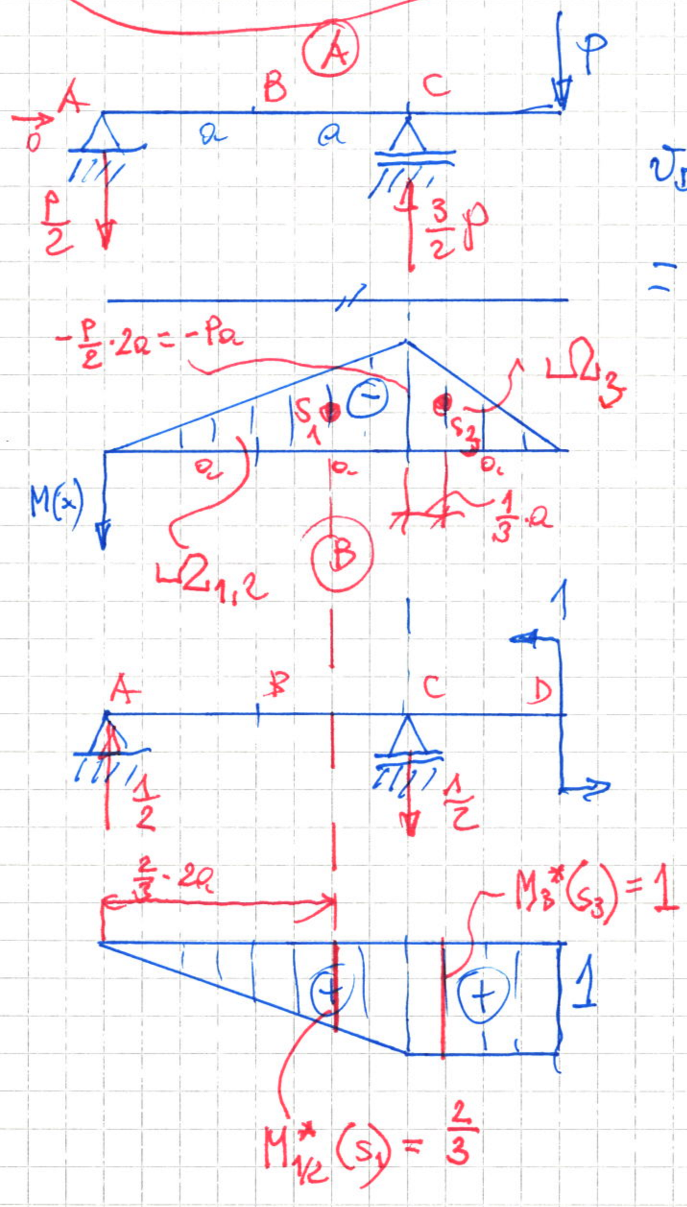
$$= \frac{1}{EI}\left[-\frac{Pa^3}{12} - \frac{P}{12}\left(8a^3 - a^3\right) + \left(-\frac{P}{3}\cdot 27a^3 + \frac{3Pa}{2}\cdot 9a^2 + \frac{3Pa}{2}\cdot 9a^2 - 9Pa^2\cdot 3a\right.\right.$$

$$\left.\left. + \frac{P}{3}8a^3 - \frac{3Pa}{2}4a^2 - \frac{3Pa}{2}4a^2 + 9Pa^2\cdot 2a\right)\right] =$$

$$= \frac{1}{EI}\left[-\frac{2}{3}Pa^3 + Pa^3\left(\cancel{-9} + \cancel{\frac{27}{2}} + \cancel{\frac{27}{2}} - \cancel{27} + \frac{8}{3} - 6 - 6 + \cancel{18}\right)\right] =$$

$$= \frac{1}{EI}\left(-\frac{2}{3}Pa^3 - 3Pa^3 + \frac{8}{3}Pa^3\right) = -\frac{Pa^3}{EI}$$

v_D procedure



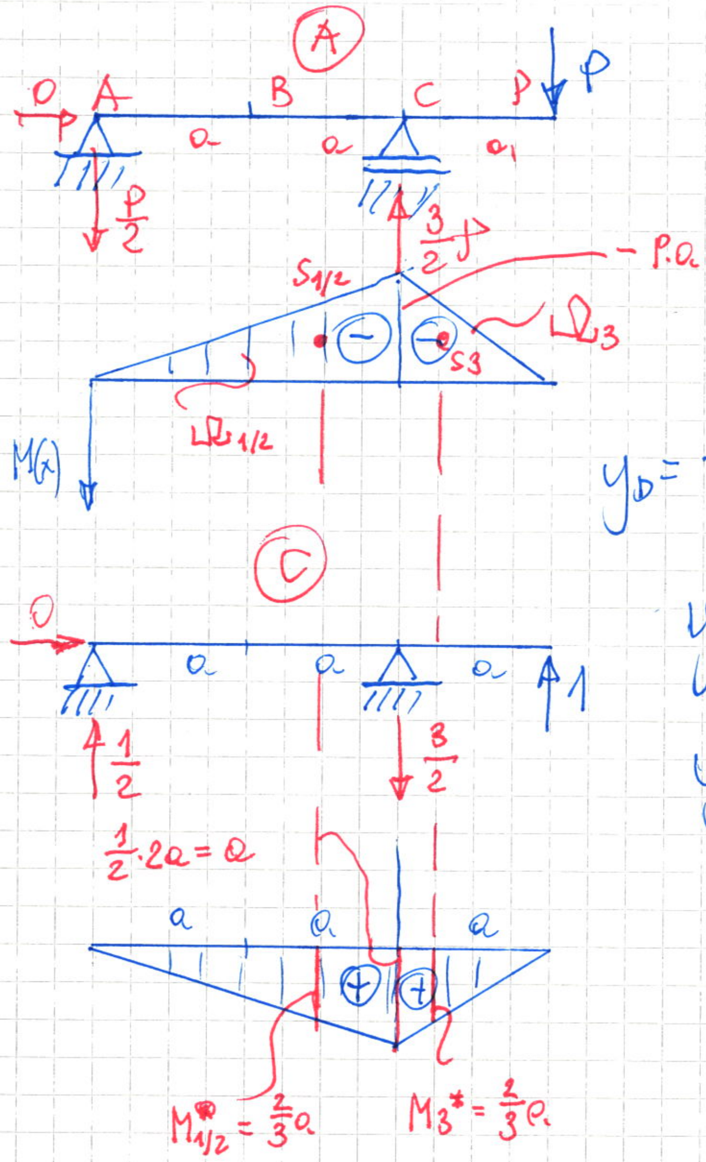
$$v_D = \frac{1}{EI}\left[\Omega_{1/2}\cdot\frac{2}{3} + \Omega_3\cdot 1\right] =$$

$$= \frac{1}{EI}\left[(-Pa^2)\cdot\frac{2}{3} + \left(-\frac{Pa^2}{2}\right)\cdot 1\right] = -\frac{Pa^2}{EI}\left(\frac{2}{3}+\frac{1}{2}\right)\Big] =$$

$$= -\frac{7}{6}\frac{Pa^2}{EI}$$

$$\Omega_{1/2} = \frac{1}{2}\cdot 2a\cdot(-P\cdot a) = -Pa^2$$

$$\Omega_3 = \frac{1}{2}a\cdot(-Pa) = -\frac{Pa^2}{2}$$



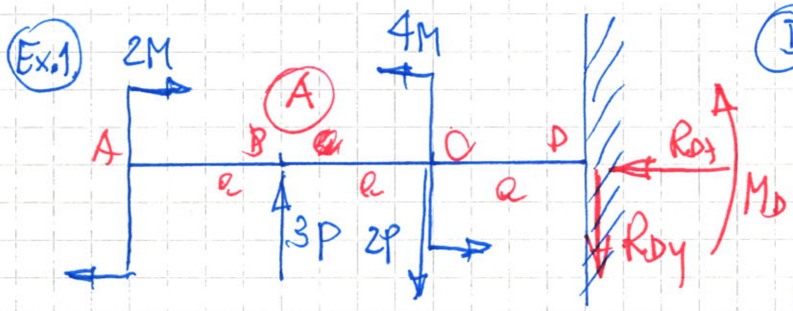
$$y_D = \frac{1}{EI}\left[\Omega_{1/2}\cdot\frac{2}{3}a + \Omega_3\cdot\frac{2}{3}a\right] =$$

$$y_D = \frac{1}{EI}\left[\left(-Pa^2\right)\cdot\frac{2}{3}a + \left(-\frac{Pa^2}{2}\right)\cdot\frac{2}{3}a\right]$$

$$y_D = \frac{-Pa^2}{EI}\left(\frac{2}{3}a + \frac{1}{2}\cdot\frac{2}{3}a\right) =$$

$$y_D = \frac{-Pa^2}{EI}\cdot\frac{\cancel{2}}{\cancel{3}}a\cdot\frac{\cancel{3}}{\cancel{2}} = -\frac{Pa^3}{EI}$$

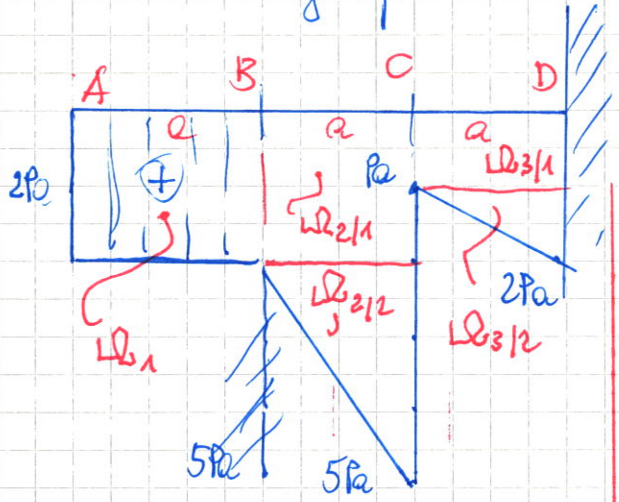
Ex.1



$P, a, M = Pa, EI = const$

$\vartheta_B, y_C - ?$

Vereshchagin procedure



I Static eqc.

(1) $\sum P_{ix} = -R_{Dx} = 0$

(2) $\sum P_{iy} = 3P - 2P - R_{Dy} = 0$

$P - R_{Dy} = 0 \Rightarrow R_{Dy} = P$

(3) $\sum M_{iD} = 2M + 3P \cdot 2a - 2Pa - 4M - M_D = 0$

$2Pa + 6Pa - 2Pa - 4Pa - M_D = 0$

$M_D = 2Pa$

II Bending moments

$0 \leq x_1 \leq a$

$M_1(x) = 2M = 2Pa$

$a \leq x_2 \leq 2a$

$M_2(x) = 2M + 3P(x - a)$

$M_2(a) = 2Pa + 3P(a - a) = 2Pa$

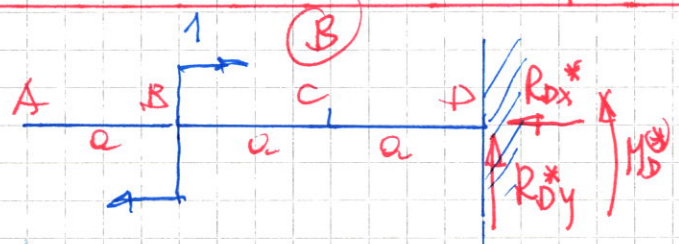
$M_2(2a) = 2Pa + 3P \cdot a = 5Pa$

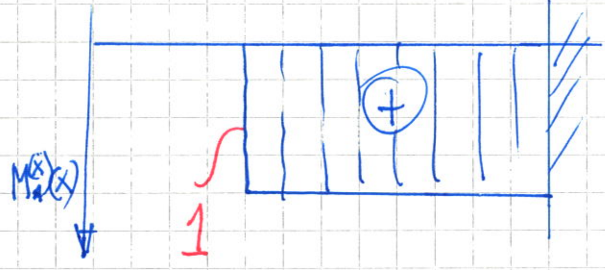
$2a \leq x_3 \leq 3a$

$M_3(2a) = 5Pa - 4Pa = Pa$

$M_3(3a) = 2M + 3P \cdot 2a - 2P \cdot a - 4Pa =$

$2Pa + 6Pa - 2Pa - 4Pa = 2Pa$



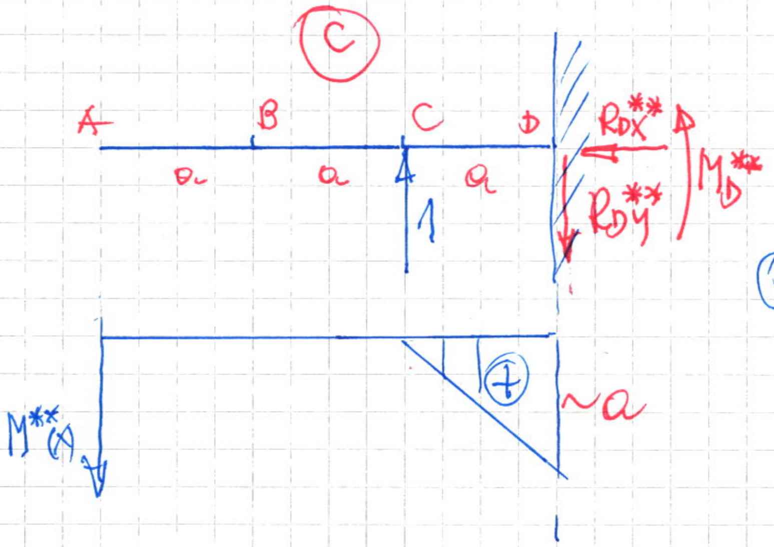


$M_1^*(x) = 0$

$M_2^*(x) = 1$

$M_3^*(x) = 1$

C



I Static eqs.

1. $\sum P_{ix} = R_{DX} = 0$
2. $\sum P_{iy} = 1 - R_{DY}^{**} = 0 \Rightarrow R_{DY}^{**} = 1$
3. $\sum M_i^D = 1 \cdot a - M_D^{**} = 0 \quad M_D^{**} = 1a$
$= a$

$\Omega_1 = 2Pa \cdot a = 2Pa^2$

$\Omega_{2/1} = 2Pa^2$

$\Omega_{2/2} = \frac{1}{2} a \cdot 3Pa = \frac{3}{2} Pa^2$

$\Omega_{3/1} = Pa^2$

$\Omega_{3/2} = \frac{1}{2} Pa^2$

A + B

$$\vartheta_B = \frac{1}{EI}\left[\Omega_1 \cdot 0 + \Omega_{2/1} \cdot 1 + \Omega_{2/2} \cdot 1 + \Omega_{3/1} \cdot 1 + \Omega_{3/2} \cdot 1\right]$$

$$\vartheta_B = \frac{1}{EI}\left[2Pa^2 \cdot 1 + \frac{3}{2}Pa^2 \cdot 1 + Pa^2 \cdot 1 + \frac{1}{2}Pa^2 \cdot 1\right] = \frac{5Pa^2}{EI}$$

$$y_C = \frac{1}{EI}\left[0 + 0 + \Omega_{3/1} \cdot \frac{a}{2} + \Omega_{3/2} \cdot \frac{2}{3}a\right] =$$

$$y_C = \frac{1}{EI}\left[Pa^2 \cdot \frac{a}{2} + \frac{Pa^2}{2} \cdot \frac{2}{3}a\right] = \frac{1}{EI}\left[\frac{Pa^3}{2} + \frac{Pa^3}{3}\right]$$

$$y_C = \frac{5}{6EI} Pa^3$$