Tension-compression, hyperstatic objects / 3
Rozciąganie-ściskanie, przypadki hiperstat.

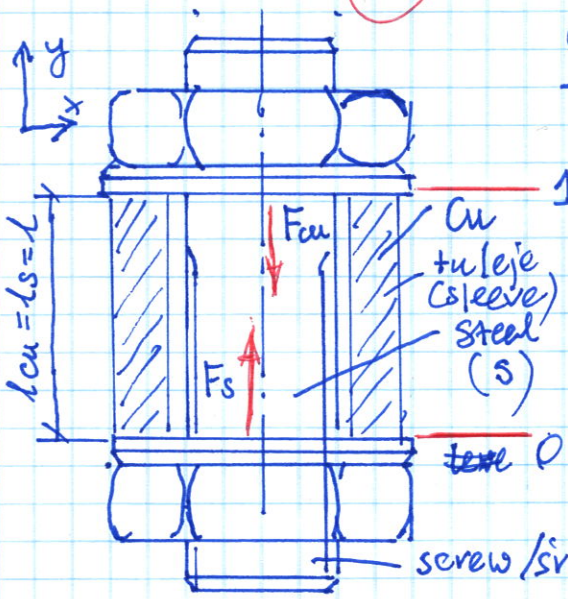
Ex. 1. / ćw. 1.

@

$l_{cu} = l_s = l$, E_{cu}, E_s, A_{cu}, A_s

α_{cu}, α_s, $\Delta T \uparrow$ (heating) podgrzanie

σ_{cu}, σ_s - ?

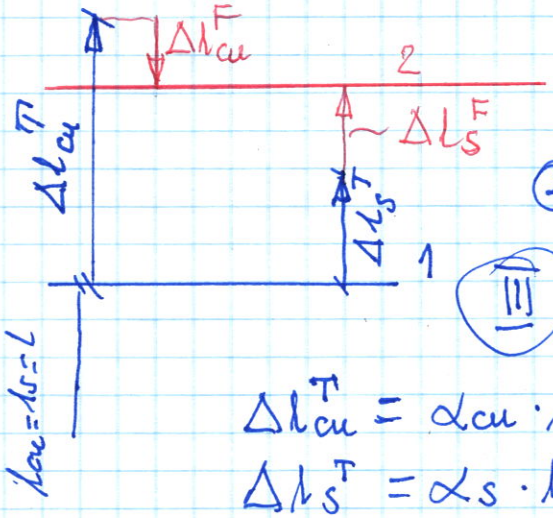


I Static eq.

① $\sum P_{iy} = F_s - F_{cu} = 0$

$F_s = F_{cu} = F$

2 forces - 1 st. eq $\Rightarrow$ 1x hyperstatic



II Geometric eq.

② $\Delta l_{cu}^T - \Delta l_{cu}^F = \Delta l_s^T + \Delta l_s^F$

III Physical conditions

$$\Delta l_{cu}^T = \alpha_{cu} \cdot l \cdot \Delta T \qquad \Delta l_{cu}^F = \frac{F \cdot l}{E_{cu} \cdot A_{cu}}$$

$$\Delta l_s^T = \alpha_s \cdot l \cdot \Delta T \qquad \Delta l_s^F = \frac{F \cdot l}{E_s \cdot A_s}$$

III ~ II

$$\alpha_{cu} \cdot l \cdot \Delta T - \frac{F \cdot l}{E_{cu} \cdot A_{cu}} = \alpha_s \cdot l \cdot \Delta T + \frac{F \cdot l}{E_s \cdot A_s}$$

$$(\alpha_{cu} - \alpha_s) \cdot l \cdot \Delta T = F \cdot l \left(\frac{1}{E_{cu} A_{cu}} + \frac{1}{E_s \cdot A_s} \right)$$

$$(\alpha_{cu} - \alpha_s) \cdot l \cdot \Delta T = F \cdot l \, \frac{(E_s \cdot A_s + E_{cu} A_{cu})}{A_{cu} A_s E_{cu} E_s}$$

$$F = \frac{(\alpha_{cu} - \alpha_s) \cdot l \cdot \Delta T \cdot A_{cu} \cdot A_s E_{cu} E_s}{l (E_s A_s + E_{cu} A_{cu})}$$

$$\sigma_{cu} = \frac{F}{A_{cu}} = \frac{(\alpha_{cu} - \alpha_s) \cdot l \cdot \Delta T \cdot A_s E_{cu} \cdot E_s}{l (E_s A_s + E_{cu} A_{cu})}$$

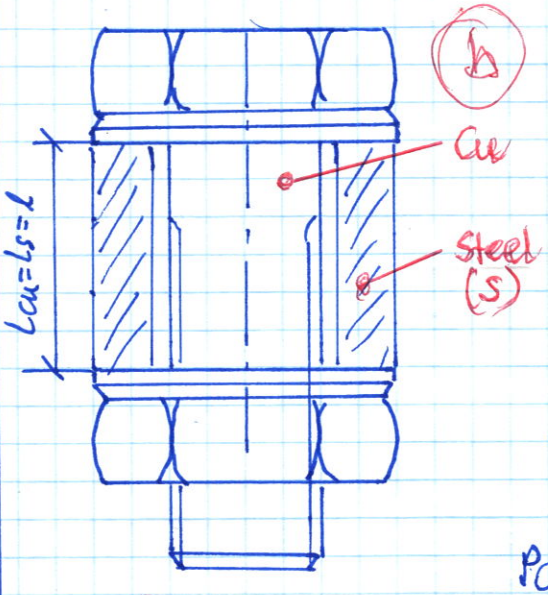
$$\sigma_{cu} = (\alpha_{cu} - \alpha_s)\cdot \Delta T \frac{A_s \cdot E_{cu} \cdot E_s}{(E_s A_s + E_{cu} A_{cu})}$$

$$\sigma_s = (\alpha_{cu} - \alpha_s)\cdot \Delta T \frac{A_{cu} \cdot E_{cu} \cdot E_s}{(E_s A_s + E_{cu} A_{cu})}$$

Ex. 2. / ćw. 2.

$l_{cu} = l_s = l$, E_{cu}, E_s, A_{cu}, A_s

α_{cu}, α_s, $\Delta T \downarrow$ (cooling) ochłodzenie



σ_{cu}, σ_s - ?

(I) Static eq - ?

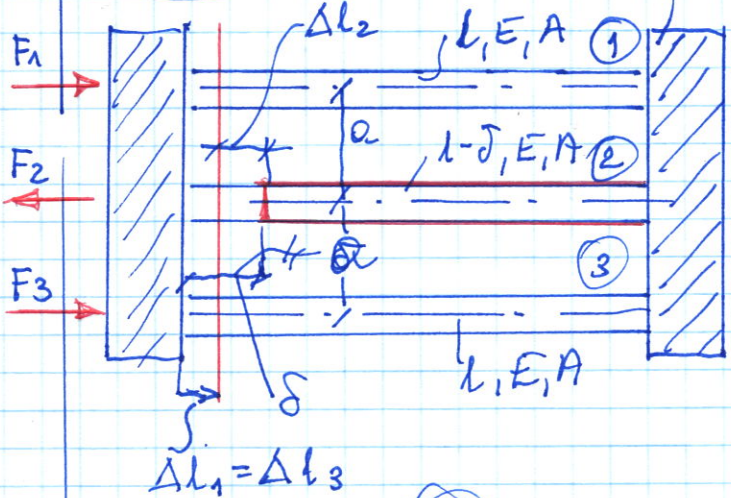
(II) Geometric eq - ?

(III) Physical conditions - ?

Proszę wykonać samodzielnie (homework)

Porównać wyniki dla (a) i (b)

Ex. 3. / ćw. 3. (a)



nieodkształc. (solid)

l, E, A, δ, a $\quad \delta \ll l$

$\sigma_{①}, \sigma_{②}, \sigma_{③}$ - ?

(I) Static eqs.

① $\sum P_{iX} = F_1 - F_2 + F_3 = 0$

② $\sum M_{i②} = F_1 \cdot a - F_3 \cdot a = 0 \Rightarrow$

$\Rightarrow F_1 = F_3$

(II) Geometric eq

$$|\Delta l_1| + |\Delta l_2| = \delta$$

(III) Physical eqs (Hooke's law)

$$\Delta l_1 = \Delta l_3 = \frac{F_1 \cdot l}{EA}$$

$$\Delta l_2 = \frac{F_2 \cdot (l-\delta)}{EA} \simeq \frac{F_2 \cdot l}{E \cdot A}$$

III → II

$$|\Delta l_1| + |\Delta l_2| = \delta$$

$$\frac{F_1 \cdot l}{EA} + \frac{F_2 \cdot l}{EA} = \delta \quad \Big| \cdot \frac{EA}{l}$$

$$F_1 + F_2 = \frac{\delta \cdot E \cdot A}{l} \Rightarrow F_2 = \frac{\delta EA}{l} - F_1$$

$\Rightarrow$ (I)

$$F_1 - F_2 + F_3 = F_1 - \left(\frac{\delta EA}{l} - F_1\right) + F_1 = 0$$

$$F_1 - \frac{\delta EA}{l} + F_1 + F_1 = 0 \qquad 3F_1 = \frac{\delta EA}{l}$$

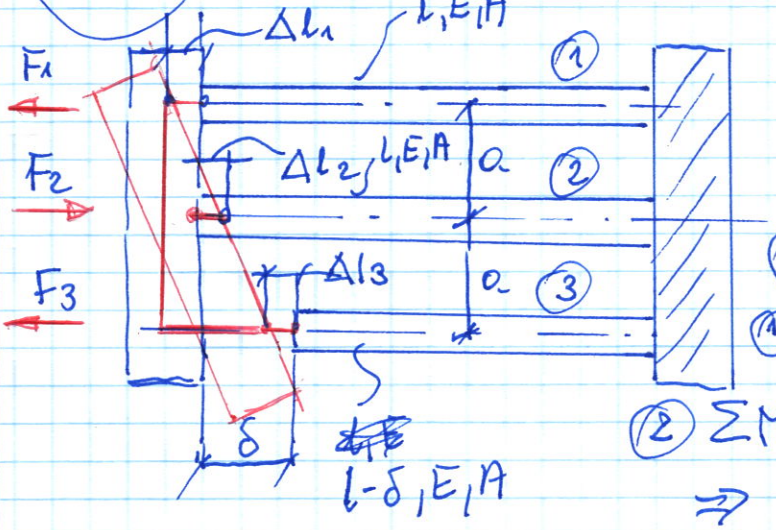
$$F_1 = F_3 = \frac{\delta EA}{3l} \; ; \; F_2 = \frac{\delta EA}{l} - \frac{\delta EA}{3l} = \frac{2}{3}\frac{EA}{l}$$

$$\sigma_1 = \sigma_3 = \frac{\delta E}{3l} \qquad \sigma_2 = \frac{2 \cdot \delta E}{3l}$$

Ex. 3. / cw. 3. (b)

$l, E, A, \delta, a \qquad \delta \ll l$

$\sigma_{①}, \sigma_{②}, \sigma_{③} - ?$



(I) Static eqs.

① $\Sigma P_{ix} = -F_1 + F_2 - F_3 = 0$

② $\Sigma M_i^{②} = -F_1 \cdot a + F_3 \cdot a = 0 \Rightarrow$

$\Rightarrow F_1 = F_3$

(II) Geometric eq.

$$\frac{|\Delta l_1| + |\Delta l_2|}{a} = \frac{|\Delta l_1| + (\delta - \Delta l_3)}{2a} \quad \Big| \cdot 2a$$

$$2(\Delta l_1 + \Delta l_2) = \Delta l_1 + (\delta - \Delta l_3)$$

$$2\Delta l_1 + 2\Delta l_2 = \Delta l_1 + \delta - \Delta l_3$$

$$\Delta l_1 + 2\Delta l_2 + \Delta l_3 = \delta$$

(III) Physical conditions

$$\Delta l_1 = \frac{F_1 \cdot l}{EA}, \quad \Delta l_2 = \frac{F_2 \cdot l}{E \cdot A}$$

$$\Delta l_3 = \frac{F_3 \cdot (l-\delta)}{EA} \approx \frac{F_3 \cdot l}{EA}$$

(III) ↔ (II)

$$\frac{F_1 \cdot l}{EA} + \frac{2F_2 \cdot l}{EA} + \frac{F_3 \cdot l}{EA} = \delta \quad \Big| \cdot \frac{EA}{l}$$

$$F_1 + 2F_2 + F_3 = \frac{\delta EA}{l} \qquad (\text{de fapt } F_1 = F_3)$$

$$F_1 + 2F_2 + F_1 = \frac{\delta EA}{l}$$

$$2F_1 + 2F_2 = \frac{\delta EA}{l} \quad | :2$$

$$F_1 + F_2 = \frac{\delta EA}{2l} \Rightarrow \boxed{F_2 = \frac{\delta EA}{2l} - F_1}$$

$\Rightarrow$ (I)

$$-F_1 + F_2 - F_3 = 0 \qquad -F_1 + F_2 - F_1 = 0$$

$$-2F_1 + F_2 = 0$$

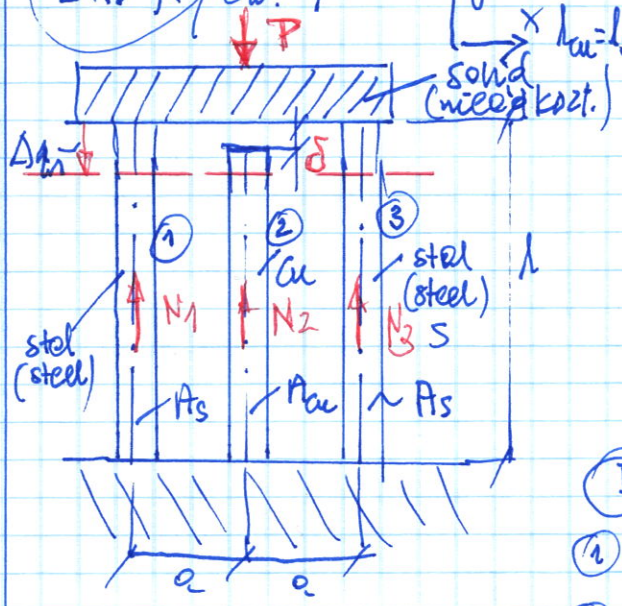
$$-2F_1 + \frac{\delta EA}{2l} - F_1 = 0 \Rightarrow 3F_1 = \frac{\delta EA}{2l}$$

$$\boxed{F_1 = \frac{\delta EA}{6l}} \qquad \boxed{F_2 = \frac{\delta EA}{2l} - \frac{\delta EA}{6l} = \frac{\delta EA}{3l}}$$

$$\delta_1 = \delta_3 = \frac{\delta EA}{6l}, \quad \delta_2 = \frac{\delta E}{3l}$$

$$\delta_{1@} = 2\delta_{1b} \qquad \delta_{2@} = 2\delta_{2b}$$

Ex. 4. / ćw. 4



$A_s = A_{cu} = A$

$l_{cu} = l_s = l$, a, A_s, A_{cu}, E_s, E_{cu}, δ, P

$\sigma_{①}$, $\sigma_{②}$, $\sigma_{③}$ – ?

$\Downarrow$

$\sigma_{①} = \frac{N_1}{A}$, $\sigma_{②} = \frac{N_2}{A}$, $\sigma_{③} = \frac{N_3}{A}$

1 × hyperstatic

(I) static eqs.

① $\sum P_{iy} = N_1 + N_2 + N_3 - P = 0$

② $\sum M_i^2 = N_1 \cdot a - N_3 \cdot a = 0$

$\Rightarrow N_1 = N_3$

(II) Geometric eq.

③ $\Delta l_1 - \Delta l_2 = \delta$ $\qquad \Delta l_1 = \Delta l_3$

(III) Physical conditions

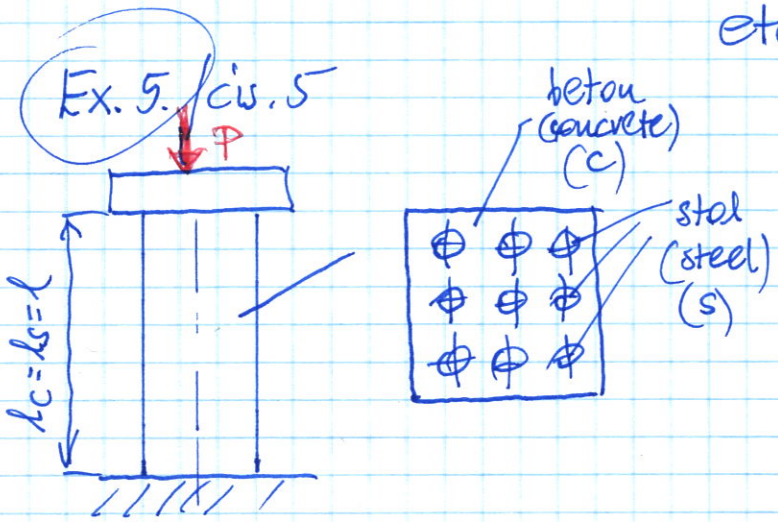
$\Delta l_1 = \frac{N_1 \cdot l}{E_s A}$; $\Delta l_2 = \frac{N_2 (l - \delta)}{E_{cu} \cdot A} \simeq \frac{N_2 \cdot l}{E_{cu} \cdot A}$

(III) ⇝ (II) $\Rightarrow N_2 = f(N_1) \Rightarrow$ (I)

$\rightarrow N_1 \Rightarrow \sigma_{①} = \sigma_{③}$, $\sigma_{②}$

etc.

Ex. 5. / ćw. 5



P, A_c, A_c, $l_s = l_c = l$

E_c, E_s

σ_s – ? $\quad \sigma_c$ – ?