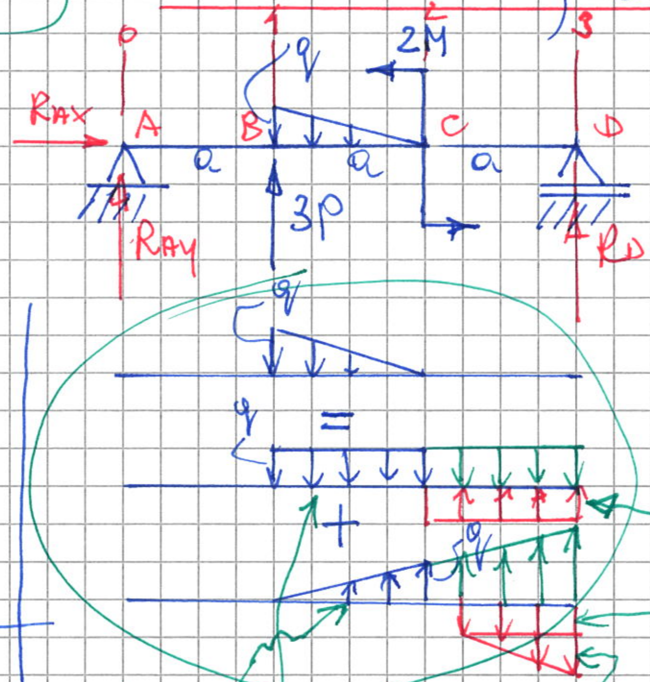
Bending 5 — Clebsch's method, cont. Examples.

Ex. 1. / Ćw. 1.



$q, a, P = qa, M = qa^2$
$EI = const.$

y_C, ϑ_D – ?

static eqs.

(1) $\sum P_{ix} = R_{AX} = 0$

(2) $\sum P_{iy} = R_{AY} + 3P - \frac{qa}{2} + R_D = 0$

(3) $\sum M_i^D = R_{AY}\cdot 3a + 3P\cdot 2a - \frac{1}{2} qa \frac{5}{3} a - 2M = 0$

$\Downarrow$

R_{AX}, R_{AY}, R_D

0 | 1 | 2 | 3

$$M(x) = R_{AY}\cdot x \;\Big|\; + 3P(x-a) - q\frac{(x-a)^2}{2} + q\frac{(x-a)^3}{6a} \;\Big|\; - 2M(x-2a)^0 + \cancel{q\frac{(x-2a)^2}{2}} - \cancel{q\frac{(x-2a)^2}{2}} - q\frac{(x-2a)^3}{6a}$$

$$EI\frac{d^2y}{dx^2} = -M(x) \;\Big|\; -R_{AY}\cdot x \;\Big|\; - 3P(x-a) + q\frac{(x-a)^2}{2} - q\frac{(x-a)^3}{6a} \;\Big|\; + 2M(x-2a)^0 + q\frac{(x-2a)^3}{6a}$$

$$EI\frac{dy}{dx} = C \;\Big|\; -R_A\frac{x^2}{2} \;\Big|\; - 3P\frac{(x-a)^2}{2} + q\frac{(x-a)^3}{6} - q\frac{(x-a)^4}{24a} \;\Big|\; + 2M(x-2a) + q\frac{(x-2a)^4}{24a} \qquad 4*$$

$$EIy = C\cdot x + D \;\Big|\; -R_A\frac{x^3}{6} \;\Big|\; - 3P\frac{(x-a)^3}{6} + q\frac{(x-a)^4}{24} - q\frac{(x-a)^5}{120a} \;\Big|\; + 2M\frac{(x-2a)^2}{2} + q\frac{(x-2a)^5}{120a}$$

(*) (3*) (**)

Boundary conditions

(1) $x = 0$, $y = 0$ (*)

(2) $x = 3a$, $y = 0$ (**)

$\Rightarrow C, D$

$$y_C = y(x=2a) \quad (3*) \quad \text{or} \quad (**)$$

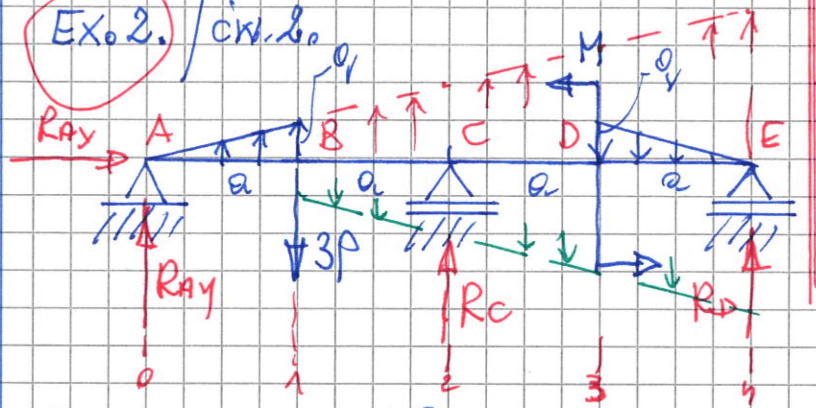
$$\vartheta_D = \vartheta(x=3a) \quad (4*)$$

Homework:

classical procedure of integration

$C_1, C_2, C_3, D_1, D_2, D_3$

Ex. 2. / cw. 2.



$q, a, P = qa, M = qa^2,$
$EI = const$

reactions - 2.

$\underbrace{R_{AX}, R_{AY}, R_C, R_D}_{4}$

I static equations

① $\sum P_{ix} = R_{AX} = 0$

② $\sum P_{iy} = R_{AY} + \frac{1}{2}qa - 3P + R_C - \frac{1}{2}qa + R_D = 0$

③ $\sum M_i^E = R_{AY}\cdot 4a + \frac{1}{2}qa\cdot\frac{10}{3}a - 3P\cdot 3a + R_C\cdot 2a - M - \frac{1}{2}qa\cdot\frac{2}{3}a = 0$

4 reactions - 3 static eqs. $\Rightarrow$ 1x hyperstatic

II Diff. eq.

$$M(x) = \overset{0}{\Big|}\ R_{AY}\cdot x + \frac{qx^3}{6a}\ \overset{1}{\Big|}\ -3P(x-a) - \frac{q(x-a)^2}{2} - \frac{q(x-a)^3}{6a}\ \overset{2}{\Big|}\ R_C(x-2a)\ \overset{3}{\Big|}\ -M(x-3a)^0 - \frac{q(x-3a)^2}{2} + \frac{q(x-3a)^3}{6a}\ \overset{4}{\Big|}$$

$$EI\frac{d^2y}{dx^2} = \Big|\ -R_{AY}\cdot x - \frac{qx^3}{6a}\ \Big|\ +3P(x-a) + \frac{q(x-a)^2}{2} + \frac{q(x-a)^3}{6a}\ \Big|\ -R_C(x-2a)\ \Big|\ +M(x-3a)^0 + \frac{q(x-3a)^2}{2} - \frac{q(x-3a)^3}{6a}$$

$$EI\frac{dy}{dx} = C\ \Big|\ -R_{AY}\frac{x^2}{2} - \frac{qx^4}{24a}\ \Big|\ +3P\frac{(x-a)^2}{2} + \frac{q(x-a)^3}{6} + \frac{q(x-a)^4}{24a}\ \Big|\ -\frac{R_C(x-2a)^2}{2}\ \Big|\ +M(x-3a) + \frac{q(x-3a)^3}{6} - \frac{q(x-3a)^4}{24a}$$

$$EIy = C \cdot x + D \;\Big|^{0}\; - R_A \frac{x^3}{6} - \frac{q x^5}{120a} \;\Big|^{1}\; + 3P\frac{(x-a)^3}{6} + q\frac{(x-a)^4}{24} + q\frac{(x-a)^5}{120a} \;\Big|^{2}\; - R_C\frac{(x-2a)^3}{6} \;\Big|^{3}\; + M\frac{(x-3a)^2}{2} + q\frac{(x-3a)^4}{24} - q\frac{(x-3a)^5}{120a} \;\Big|^{4}$$

$*$ $\quad$ $**$ $\quad$ $3*$

Boundary conditions

① $x = 0$, $y = 0$ $\quad *$

② $x = 2a$, $y = 0$ $\quad **$

③ $x = 4a$, $y = 0$ $\quad 3*$

L = number of reactions + number of constants

$$4 + 2 = 6$$

R = static equations + boundary conditions

$$3 + 3 = 6$$

$$L = R$$

Additional: classical method (Homework)

$R_{AX}, R_{AY}, R_C, R_E \longrightarrow 4$

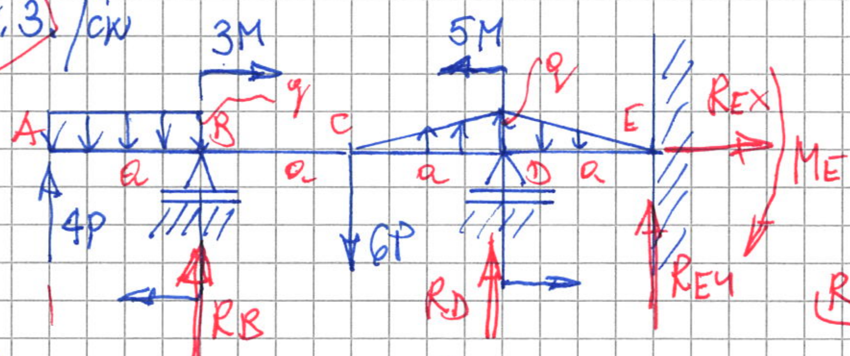
$C_1, C_2, C_3, C_4, D_1, D_2, D_3, D_4 \longrightarrow 8$

12

static eqs $\rightarrow$ 3

boundary conditions $\rightarrow$ 9 !

Ex. 3 /ciw



$q, a, P = qa, M = qa^2$

$EI = const$

$\underbrace{R_B, R_D, R_{EX}, R_{EY}, M_E}$

5 reactions

2x hyperstatic beam !

+ C - D - 2

$\Sigma = 7$

Ⓐ Clebsch's method

L = n. of reactions + n. of constants

$5 + 2 = 7$

R = n. of static eqs. + n. of b. conditions

$3 + 4 = 7$

$L = R$

Boundary conditions

① $x = a, y = 0$

② $x = 3a, y = 0$

③ $x = 4a, \vartheta = 0$

④ $x = 4a, y = 0$

Static eqs.

① $\Sigma P_{ix} = 0$

② $\Sigma P_{iy} = 0$

③ $\Sigma M_i^E = 0$

Ⓑ Classical method of integration

$C_1, C_2, C_3, C_4, D_1, D_2, D_3, D_4 \Rightarrow 8$

$\underline{R_B, R_D, R_{EX}, R_{EY}, M_E \Rightarrow 5}$

$\Sigma = 13$

Static eqs.

① $\Sigma P_{ix} = 0$

② $\Sigma P_{iy} = 0$

③ $\Sigma M_i^E = 0$

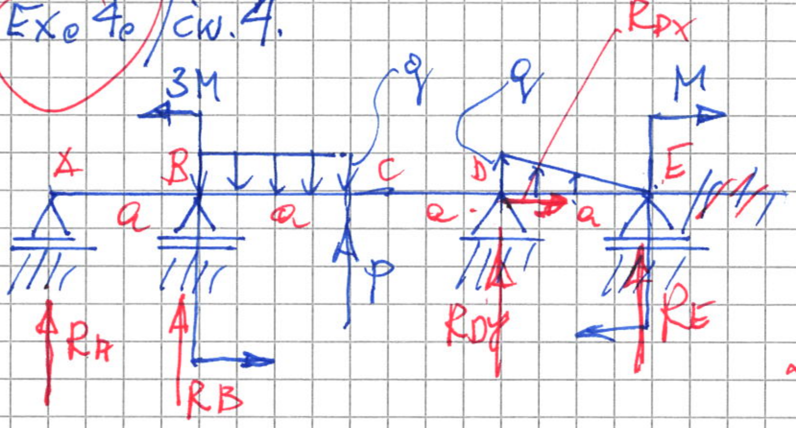
We need 10 b. conditions !

① ⋮ ⑩ } ? homework

Ex. 4. / cw. 4.

$q, a, P = qa, M = qa^2$

$EI = const$



$R_A, R_B, R_{Dx}, R_{Dy}, R_E$

5 reactions

2x hyperstatic beam

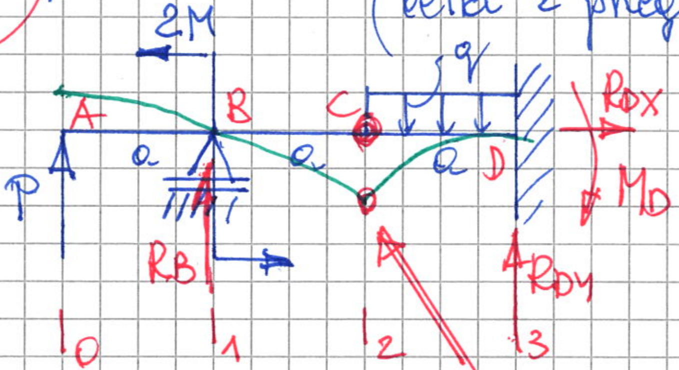
(A) Clebsch's method

(B) classical method

⟹ homework

Ex. 5 / ćw. 5

Beams with hinge (belki z przegubem)



- different slopes
- the same deflections

$q, a, P = qa, M = qa^2$

$EI = const$

$v_B, y_C - ?$

- Clebsch's method
- classical –"–

I static eqs.

① $\sum P_{ix} = R_{DX} = 0$

② $\sum P_{iy} = P + R_B - qa + R_{DY} = 0$

③ $M_C^L = P \cdot 2a - 2M + R_B \cdot a = 0$

④ $M_C^R = \frac{qa^2}{2} - R_{DY} \cdot a + M_D = 0$

$\Downarrow$

R_B, R_{DX}, R_{DY}, M_D

or $\Longrightarrow$

① $\sum P_{ix} = 0$

② $\sum P_{iy} = 0$

③ $\sum M_i^D = 0$

④ $M_C^R = 0$

If different values of C, then also different values of D

It means that:

C_1, D_1 (A–C)

C_2, D_2 (C–D)

II Diff. eq. (Clebsch's method)

a) to the hinge (do przegubu)

	0	1	2
$M(x) =$	$P \cdot x$	$-2M(x-a)^0 + R_B(x-a)$	
$EI\frac{d^2y}{dx^2} =$	$-P \cdot x$	$+2M(x-a)^0 - R_B(x-a)$	
$EI\frac{dy}{dx} = C_1$	$-P\frac{x^2}{2}$	$+2M(x-a) - R_B\frac{(x-a)^2}{2}$	
$EIy = C_1 x + D_1$	$-P\frac{x^3}{6}$	$+2M\frac{(x-a)^2}{2} - R_B\frac{(x-a)^3}{6}$	

5*

*

**

$\Longrightarrow$ C_1, D_1 ?

b) behind the hinge (za przegubem)

	0	1	2	3
$M(x) =$	$p \cdot x$	$-2M(x-a)^0 + R_B(x-a)$	$-q\dfrac{(x-2a)^2}{2}$	
$EJ\dfrac{d^2y}{dx^2} =$	$-p \cdot x$	$+2M(x-a)^0 - R_B(x-a)$	$+q\dfrac{(x-2a)^2}{2}$	
$EJ\dfrac{dy}{dx} = C_2$	$-p \cdot \dfrac{x^2}{2}$	$+2M(x-a) - R_B\dfrac{(x-a)^2}{2}$	$+q\dfrac{(x-2a)^3}{6}$	4* $\Rightarrow$ C_2, D_2 2
$EJy = C_2 \cdot x + D_2$	$-p \cdot \dfrac{x^3}{6}$	$+2M\dfrac{(x-a)^2}{2} - R_B\dfrac{(x-a)^3}{6}$	$+q\dfrac{(x-2a)^4}{24}$	3*

Boundary conditions

1. $x = a$, $y = 0$ (*)
2. $x = 2a$, $y_L = y_R$ (*)(*) = (3*)
3. $x = 3a$, $\vartheta = 0$ ~~(*)(*) = (3*)~~ (4*)
4. $x = 3a$, $y = 0$ (3*)

} C_1, D_1, C_2, D_2

$\vartheta_B = \vartheta(x=a)$ (5*)

$y_C = y(x=2a)$ (*)(*) or (3*)

Diff. eq. (classical procedure)

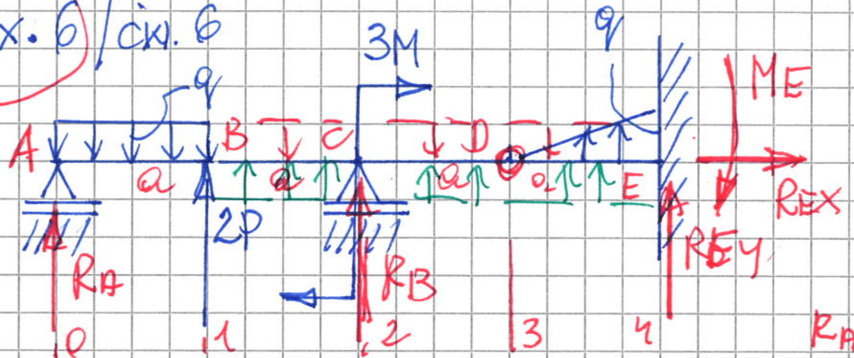
$C_1, D_1, C_2, D_2, C_3, D_3 \Rightarrow 6$

Boundary conditions:

1. $x = a$, $\vartheta_L = \vartheta_R$
2. $x = a$, $y_L = 0$
3. $x = a$, $y_R = 0$
4. $x = 2a$, $y_L = y_R$
5. $x = 3a$, $\vartheta = 0$
6. $x = 3a$, $y = 0$

⇑ as homework

Ex. 6 / ćw. 6



$q, a, P = qa, M = qa^2$

$EI = const$

reactions − ?

$R_A, R_B, R_{EX}, R_{EY}, M_E$ − ?

(I) static eqs.

(1) $\sum P_{ix} = R_{EX} = 0$

(2) $\sum P_{iy} = R_A - qa + 2P + R_B + \frac{1}{2}qa + R_{EY} = 0$

(3) $M_D^L = R_A \cdot 3a - qa \cdot \frac{5}{2}a + 2P \cdot 2a + 3M + R_B \cdot a = 0$

(4) $M_D^R = -\frac{1}{2}qa \cdot \frac{2}{3}a - R_{EY} \cdot a + M_E = 0$

5 reactions − 4 st. eqs. $\Longrightarrow$ 1x hyperstatic beam

(II) Diff. eq. (to the hinge / do przegubu)

	0	1	2 – 3
$M(x) =$	$R_A \cdot x - \frac{qx^2}{2}$	$+2P(x-a) + \frac{q(x-a)^2}{2}$	$+3M(x-2a)^0 + R_B(x-2a)$
$EI\frac{d^2y}{dx^2} =$	$-R_A x + q\frac{x^2}{2}$	$-2P(x-a) - \frac{q(x-a)^2}{2}$	$-3M(x-2a)^0 - R_B(x-2a)$
$EI\frac{dy}{dx} = C_1$	$- R_A\frac{x^2}{2} - \frac{qx^3}{6}$	$-2P\frac{(x-a)^2}{2} - \frac{q(x-a)^3}{6}$	$-3M(x-2a) - R_B\frac{(x-2a)^2}{2}$
$EIy = C_1 x + D_1$	$-R_A\frac{x^3}{6} - \frac{qx^4}{24}$	$-2P\frac{(x-a)^3}{6} - \frac{q(x-a)^4}{24}$	$-3M\frac{(x-2a)^2}{2} - R_B\frac{(x-2a)^3}{6}$

(*) (**) 3*

$\Downarrow$

C_1, D_1 − ?

b) behind the hinge (za pregubom)

	0	1	2	3, 4	
$M(x) =$	$R_A \cdot x - \frac{qx^2}{2}$	$+2P(x-a) + q\frac{(x-a)^2}{2}$	$+3M(x-2a)^0 + R_B(x-2a)$	$+q\frac{(x-3a)^3}{6a}$	
$EI\frac{d^2y}{dx^2} =$	$-R_A \cdot x + \frac{qx^2}{2}$	$-2P(x-a) - q\frac{(x-a)^2}{2}$	$-3M(x-2a)^0 - R_B(x-2a)$	$-q\frac{(x-3a)^3}{6a}$	
$EI\frac{dy}{dx} = C_2$	$-R_A\frac{x^2}{2} + q\frac{x^3}{6}$	$-2P\frac{(x-a)^2}{2} - q\frac{(x-a)^3}{6}$	$-3M(x-2a) - R_B\frac{(x-2a)^2}{2}$	$-q\frac{(x-3a)^4}{24a}$	5*
$EIy = C_2 \cdot x + D_2$	$-R_A\frac{x^3}{6} + \frac{qx^4}{24}$	$-2P\frac{(x-a)^3}{6} - q\frac{(x-a)^4}{24}$	$-3M\frac{(x-2a)^2}{2} - R_B\frac{(x-2a)^3}{6}$	$-q\frac{(x-3a)^5}{120a}$	6*

$\Downarrow$

$C_2, D_2 - ?$ 4*

Boundary conditions

1. $x = 0,\ y = 0$ *
2. $x = 2a,\ y = 0$ * *
3. $x = 3a,\ y_L = y_R$ 3* = 4*
4. $x = 4a,\ \vartheta = 0$ 5*
5. $x = 4a,\ y = 0$ 6*

L = n. of reactions + n. of constants

$5 + 4 = 9$

R = n. static eqs + n. of b. conditions

$4 + 5 = 9$

$L = R$

classical method $\Longrightarrow$ Homework

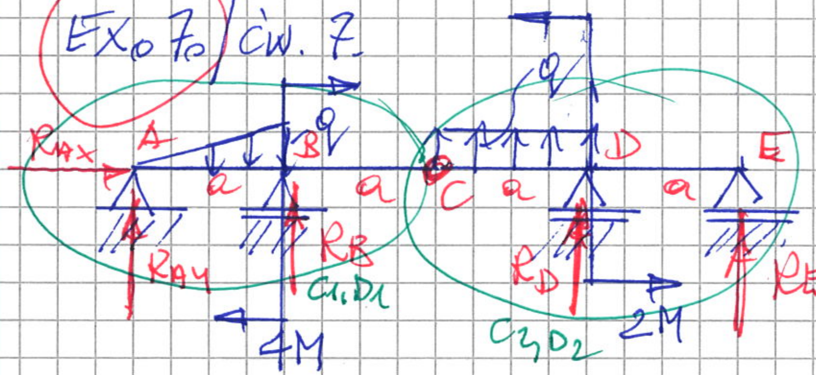
$C_1, D_1, C_2, D_2, C_3, D_3, C_4, D_4$ - 8
reactions - 5

$\Sigma = 5+8 = 13$

n. of static eqs - 4
we need 9 boundary conditions

(1) $x=0$, $y=0$
(2) $x=a$, $\vartheta_L=\vartheta_R$
(3) $x=a$, $y_L=y_R$
(4) $x=2a$, $\vartheta_L=\vartheta_R$
(5) $x=2a$, $y_L \Rightarrow 0$
(6) $x=2a$, $y_R \Rightarrow 0$
(7) $x=3a$, $y_L=y_R$
(8) $x=4a$, $\vartheta=0$
(9) $x=4a$, $y=0$

Ex. 7. / cw. 7.



$q, a, M=qa^2, EI=const$

reactions - ?

$R_{AX}, R_{AY}, R_B, R_D, R_E$ - 5
$5-4 = 1\times$ hyperstatic
$+ C_1, D_1, C_2, D_2$

(I) static eqs.

(1) $\Sigma P_{ix}=0$
(2) $\Sigma P_{iy}=0$
(3) $M_L^C=0$
(4) $M_R^C=0$

Clebsch's method
(boundary conditions)

(1) $x=0$, $y=0$
(2) $x=a$, $y=0$
(3) $x=2a$, $y_L=y_R$
(4) $x=3a$, $y=0$
(5) $x=4a$, $y=0$

L = n. of reactions + n. of constants = 5+4 = 9

R = n. of static eqs + n. b. conditions = 4+5 = 9

L = R

classical procedure

$R_{Ax}, R_{Ay}, R_B, R_D, R_E - 5$

$C_1, D_1, C_2, D_2, C_3, D_3, C_4, D_4 - 8$

$\Sigma = 13$

I static eqs.

1. $\Sigma P_{ix} = 0$
2. $\Sigma P_{iy} = 0$
3. $M_L^C = 0$
4. $M_R^C = 0$

II Boundary conditions

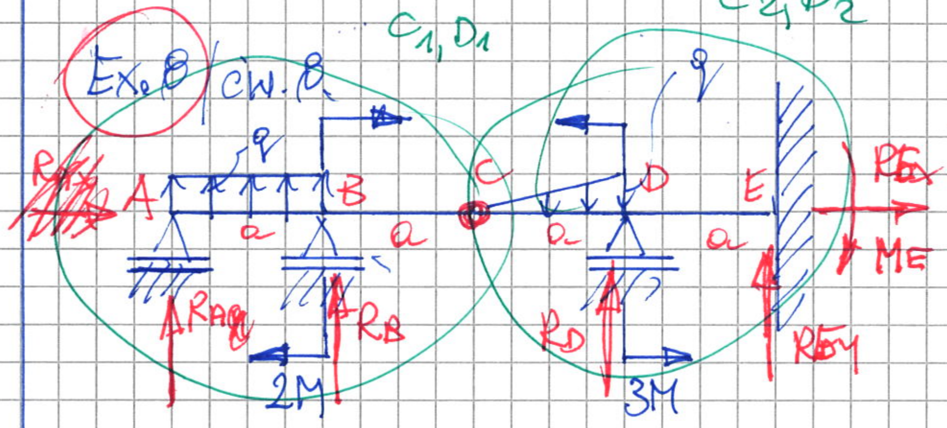
1. $x = 0, \ y = 0$
2. $x = a, \ \vartheta_L = \vartheta_R$
3. $x = a, \ y_L = 0$
4. $x = a, \ y_R = 0$
5. $x = 2a, \ y_L = y_R$
6. $x = 3a, \ \vartheta_L = \vartheta_R$
7. $x = 3a, \ y_L = 0$
8. $x = 3a, \ y_R = 0$
9. $x = 4a, \ y = 0$

L = number of reactions + n. of constants = 5 + 8 = 13

R = number of static eqs. + n. of b. conditions = 4 + 9 = 13

$$L = R$$

Ex. 8 / cw. 8.



q, a, $M = qa^2$, EI = const

reactions - ?

$R_A, R_B, R_D, R_{Ex}, R_{Ey}, M_E - 6$

$C_1, D_1, C_2, D_2 - 4$

A. Clebsch's method

B. Classical procedure

$\Rightarrow$ homework !