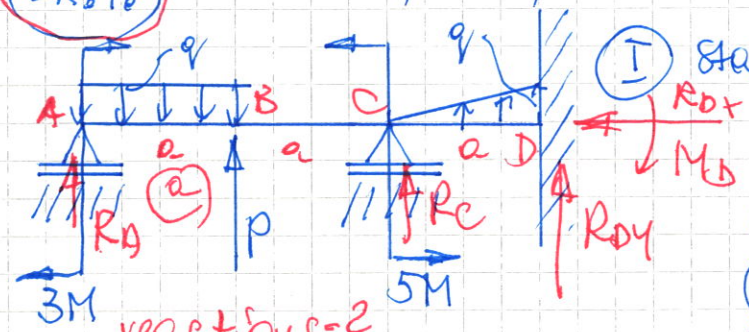
Force Method (beams) – 1

$q, a, P = qa, M = qa^2, EI = const$

Ex. 1.



reactions – ?

$R_A = X_1, R_C = X_2$

I Static eqs.

1. $\sum P_{ix} = 0$
2. $\sum P_{iy} = 0$
3. $\sum M_i^B = 0$

II Bending moments

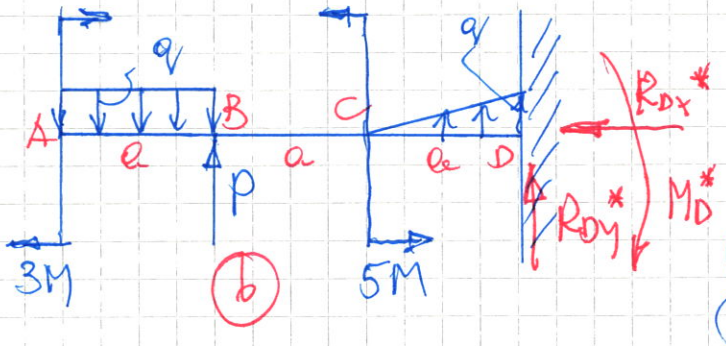
$M_1(x) =$

$M_2(x) =$

$M_3(x) =$

5 reactions – 3 st. eqs. $\Rightarrow$ 2x hyperstatic

R_A, R_C – hyperstatic



I Static eqs.

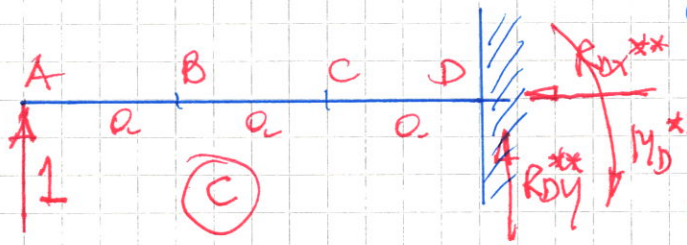
1. $\sum P_{ix} = 0$
2. $\sum P_{iy} = 0$
3. $\sum M_i^B = 0$

II Bending moments

$M_1^*(x) =$

$M_2^*(x) =$

$M_3^*(x) =$



I Static eqs.

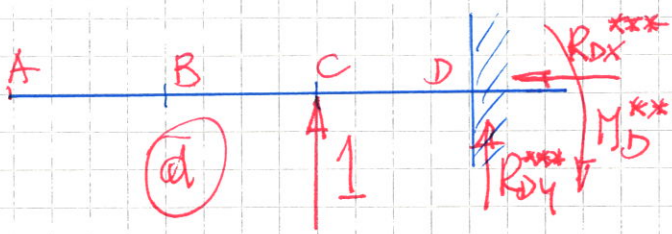
1. $\sum P_{ix} = 0$
2. $\sum P_{iy} = 0$
3. $\sum M_i^B = 0$

II Bending moments

$M_1^{**}(x) =$

$M_2^{**}(x) =$

$M_3^{**}(x) =$



I Static eqs.

1. $\sum P_{ix} = 0$
2. $\sum P_{iy} = 0$
3. $\sum P_{iy} = 0$

II Bending moments

$M_1^{***}(x) =$

$M_2^{***}(x) =$

$M_3^{***}(x) =$

2x hyperstatic beam

$\Downarrow$

(4) $X_1 \delta_{11} + X_2 \delta_{12} + \Delta_{1P} = 0$

(5) $X_1 \delta_{21} + X_2 \delta_{22} + \Delta_{2P} = 0$

} canonical eqs. (geometrical eqs.)

$\Delta_{1P} \Rightarrow$ (b) + (c)

$\Delta_{2P} \Rightarrow$ (b) + (d)

$\delta_{11} \Rightarrow$ (c) + (c)

$\delta_{12} = \delta_{21} \Rightarrow$ (c) + (d)

$\delta_{22} \Rightarrow$ (d) + (d)

(4) $X_1\delta_{11} + X_2\delta_{12} + \Delta_{1P} = 0$

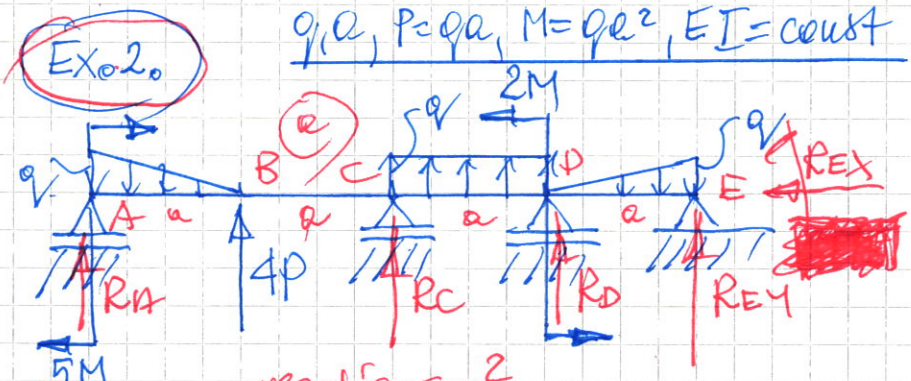
(5) $X_1\delta_{21} + X_2\delta_{22} + \Delta_{2P} = 0$

$\Rightarrow X_1 = R_A$, $X_2 = R_C$

$\Rightarrow$ (1), (2), (3) for beam (a) $\Rightarrow$ R_{Dx}, R_{Dy}, M_D

Ex. 2.

$q, a, P = qa, M = qa^2, EI = const$ (only bending)



reactions - 2 (Force Method)

R_A, R_C – hyperstatic reactions

(I) Static eqs

(1) $\Sigma P_{ix} = 0$

(2) $\Sigma P_{iy} = 0$

(3) $\Sigma M_{iE} = 0$

5 reactions 3 static eqs $\Rightarrow$ 2x hyperstatic

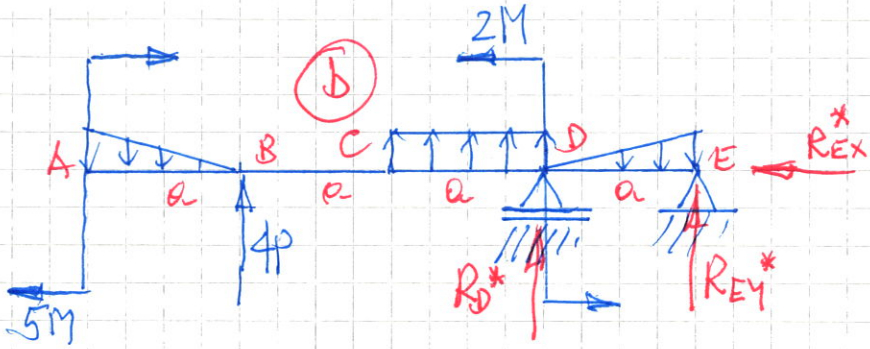
$R_A = X_1$

$R_C = X_2$

(4) $X_1\delta_{11} + X_2\delta_{12} + \Delta_{1P} = 0$

(5) $X_1\delta_{21} + X_2\delta_{12} + \Delta_{2P} = 0$

(II) Bending moments $M_i(x) = ?$



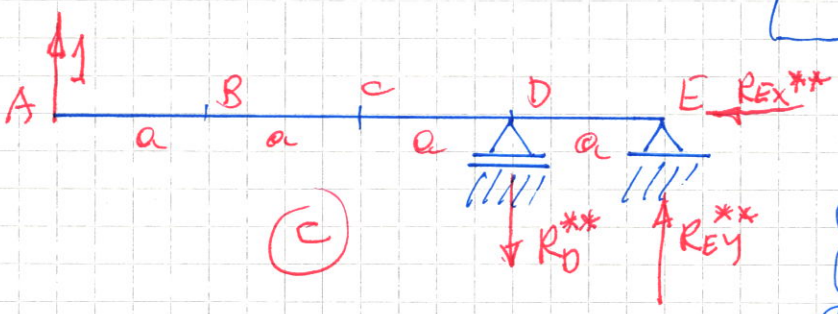
I Static eqs.

1. $\Sigma P_{ix} = 0$
2. $\Sigma P_{iy} = 0$
3. $\Sigma M_i^E = 0$

$\Rightarrow R_D^*, R_{EY}^*, R_{EX}^*$

II Bending moments

$M_i^*(x) = ?$



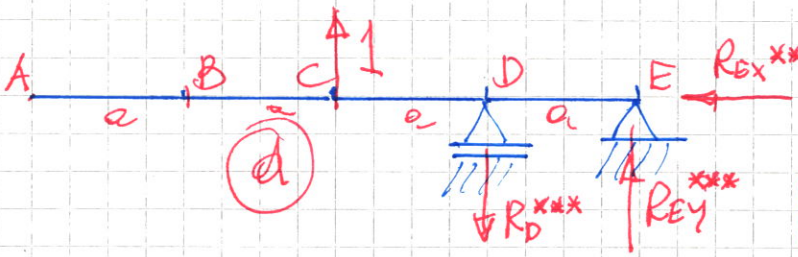
I Static eqs.

1. $\Sigma P_{ix} = 0$
2. $\Sigma P_{iy} = 0$
3. $\Sigma M_i^E = 0$

$\Rightarrow R_{EX}^{**}, R_{EY}^{**}, R_D^{**}$

II Bending moments

$M_i^{**}(x) = ?$



I static eqs.

1. $\Sigma P_{ix} = 0$
2. $\Sigma P_{iy} = 0$
3. $\Sigma M_i^E = 0$

$\Rightarrow R_{EX}^{***}, R_{EY}^{***}, R_D^{***}$

II Bending moments

$M_i^{***}(x) = ?$

$\Delta_{1P} \Rightarrow$ b + c

$\Delta_{2P} \Rightarrow$ b + d

$\delta_{11} \Rightarrow$ c + c

$\delta_{12} = \delta_{21} \Rightarrow$ c + d

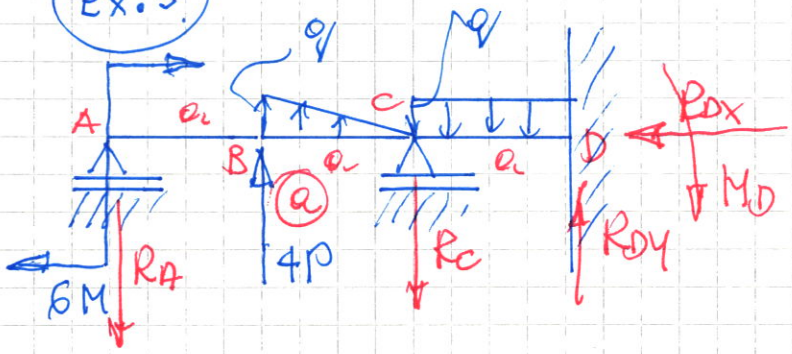
$\delta_{22} \Rightarrow$ d + d

4. $X_1\delta_{11} + X_2\delta_{12} + \Delta_{1P} = 0$
5. $X_1\delta_{21} + X_2\delta_{22} + \Delta_{2P} = 0$

$\Rightarrow X_1 = R_A, \; X_2 = R_C$

1, 2, 3 } for beam a $\Rightarrow$ R_D, R_{EX}, R_{EY}

Ex. 3.



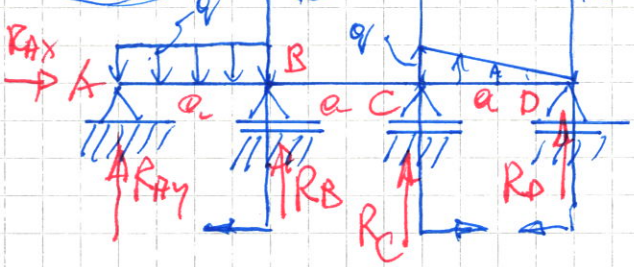
(only bending)

$q, a, P = qa, M = qa^2, EI = \text{const}$

reactions - ?

(Force Method)

Ex. 4.



(only bending)

$q, a, M = qa^2, EI = \text{const}$

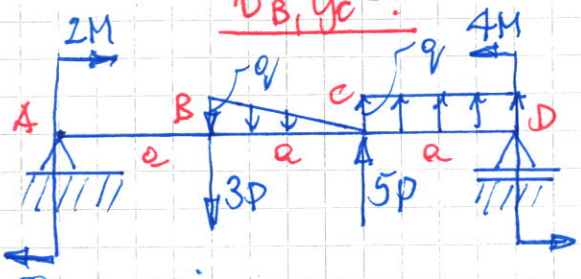
reactions - ?

(Force Method)

Ex. 1.

$q, a,\ P = qa,\ M = qa^2,\ EI = \text{const}$

$\vartheta_B, y_C - ?$

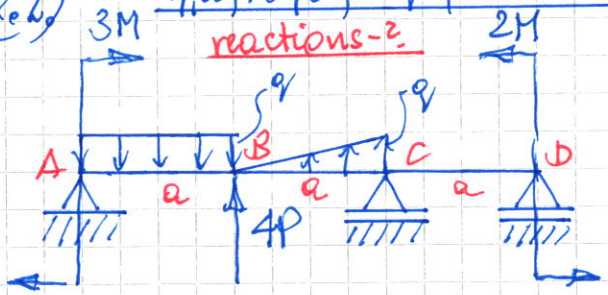


① Castigliano theorem
② Maxwell-Mohr method (analytical)

Ex. 2.

$q, a,\ P = qa,\ M = qa^2,\ EI = \text{const}$

reactions - ?

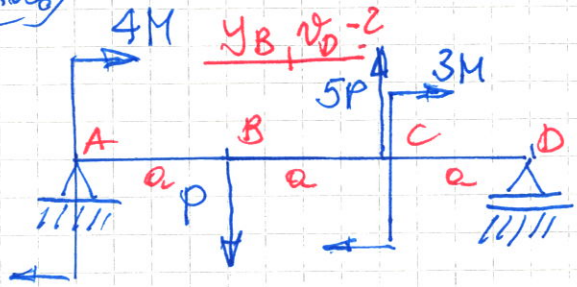


① Menabrea - Castigliano th.
② Maxwell - Mohr method (analytical)

Ex. 3.

$P, a,\ M = P \cdot a,\ EI = \text{const}$

$y_B, \vartheta_D - ?$



M-M / Vereshchagin procedure (graph multiplication method)