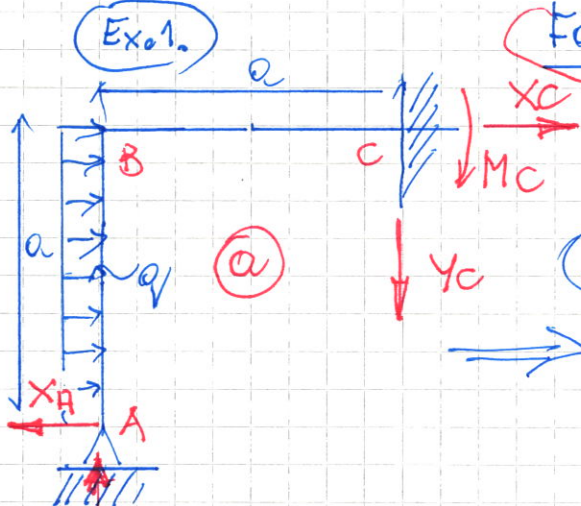


Exo 1.

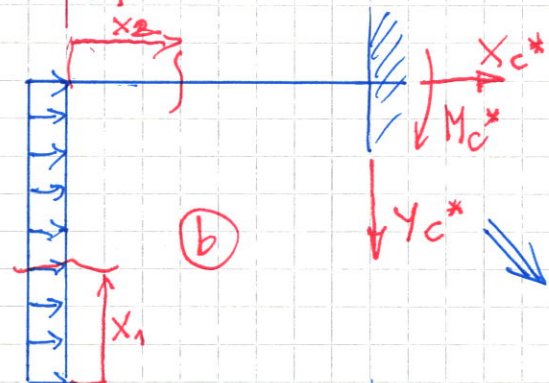
Force Method, frames

q, EI, EA, EA (bending + tension/comp.)
reactions - 2



(I) static eqs.
 ① $\sum P_{ix} = -X_A + X_c + qa = 0$
 ② $\sum P_{iy} = Y_A - Y_c = 0 \Rightarrow Y_A = Y_c$
 ③ $\sum M_{ic} = X_A \cdot a + Y_A \cdot a - \frac{qa^2}{2} = 0$

5 reaction - 3 static eqs. \Rightarrow 2x hyperstatic frame
 $X_A = X_1, Y_A = X_2$ hyperstatic!



(I) static eqs.
 ① $\sum P_x = qa + X_c^* = 0 \Rightarrow X_c^* = -qa$
 ② $\sum P_{iy} = -Y_c^* = 0$
 ③ $\sum M_{ic} = -\frac{qa^2}{2} + M_c^* = 0 \Rightarrow M_c^* = \frac{qa^2}{2}$

(II) bending moments + normal
 $0 \leq x_1 \leq a$ | $0 \leq x_2 \leq a$

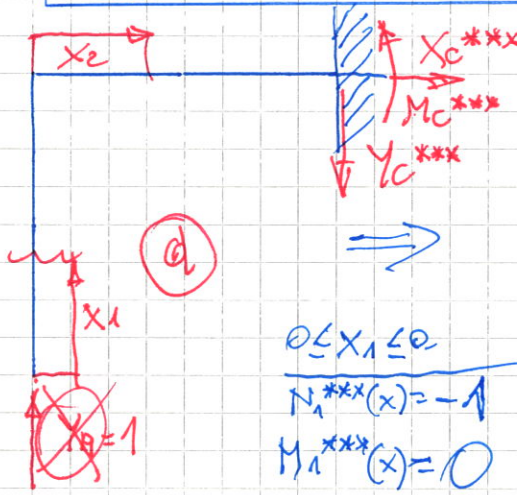
$N_1(x) = 0$	$N_2(x) = -qa$
$M_1(x) = -\frac{qx^2}{2}$	$M_2(x) = -\frac{qa^2}{2}$
$M_1(0) = 0$	
$M_1(a) = -\frac{qa^2}{2}$	



(I) static eqs.
 ① $\sum P_{ix} = -X_A + X_c^{**} = 0 \Rightarrow X_A = X_c^{**} = 1$
 ② $\sum P_{iy} = -Y_c^{**} = 0$
 ③ $\sum M_{ic} = 1 \cdot a - M_c^{**} = 0 \Rightarrow M_c^{**} = a$

(II) bending moments + normal
 $0 \leq x_1 \leq a$ | $0 \leq x_2 \leq a$

$N_1^{**} = 0$	$N_2^{**}(x) = 1$
$M_1^{**}(x) = 1 \cdot x = x$	$M_2^{**}(x) = 1 \cdot a = a$
$M_1^{**}(0) = 0$	
$M_1^{**}(a) = 1 \cdot a = a$	



(I) static eqs.
 ① $\sum P_{ix} = X_c^{***} = 0$
 ② $\sum P_{iy} = 1 - Y_c^{***} = 0 \Rightarrow Y_c^{***} = 1$
 ③ $\sum M_{ic} = 1 \cdot a - M_c^{***} = 0 \Rightarrow M_c^{***} = a$

(II) Bending moments + normal force
 $0 \leq x_1 \leq a$ | $0 \leq x_2 \leq a$

$N_1^{***}(x) = -1$	$N_2^{***}(x) = 0$
$M_1^{***}(x) = 0$	$M_2^{***}(x) = 1 \cdot x = x$
	$M_2^{***}(0) = 0, M_2^{***}(a) = a$

(-5) = (1)

Canonical eqs.

$$(4) X_1 \delta_{11} + X_2 \delta_{12} + \Delta_{1P} = 0$$

$$(5) X_1 \delta_{21} + X_2 \delta_{22} + \Delta_{2P} = 0$$

$$\Delta_{1P} \Rightarrow (b) + (c)$$

$$\Delta_{1P} = \frac{1}{EI} \left[\int_0^a M_1(x) \cdot M_1^{**}(x) dx + \int_0^a M_2(x) \cdot M_2^{**}(x) dx \right] + \frac{1}{EA} \left[\int_0^a N_1(x) \cdot N_1^{**}(x) dx + \int_0^a N_2(x) \cdot N_2^{**}(x) dx \right] = ?$$

$$\Delta_{2P} \Rightarrow (b) + (d)$$

$$\Delta_{2P} = \frac{1}{EI} \left[\int_0^a M_1(x) \cdot M_1^{***}(x) dx + \int_0^a M_2(x) \cdot M_2^{***}(x) dx \right] + \frac{1}{EA} \left[\int_0^a N_1(x) \cdot N_1^{***}(x) dx + \int_0^a N_2(x) \cdot N_2^{***}(x) dx \right] = ?$$

$$\delta_{11} \Rightarrow (c) + (c)$$

$$\delta_{11} = \frac{1}{EI} \left[\int_0^a M_1^{**}(x) \cdot M_1^{**}(x) dx + \int_0^a M_2^{**}(x) \cdot M_2^{**}(x) dx \right] + \frac{1}{EA} \left[\int_0^a N_1^{**}(x) \cdot N_1^{**}(x) dx + \int_b^a N_2^{**}(x) \cdot N_2^{**}(x) dx \right] = ?$$

$$\delta_{22} \Rightarrow (d) + (d)$$

$$\delta_{22} = \frac{1}{EI} \left[\int_0^a M_1^{***}(x) \cdot M_1^{***}(x) dx + \int_0^a M_2^{***}(x) \cdot M_2^{***}(x) dx \right] + \frac{1}{EA} \left[\int_0^a N_1^{***}(x) \cdot N_1^{***}(x) dx + \int_0^a N_2^{***}(x) \cdot N_2^{***}(x) dx \right] = ?$$

$$-7 = 2$$

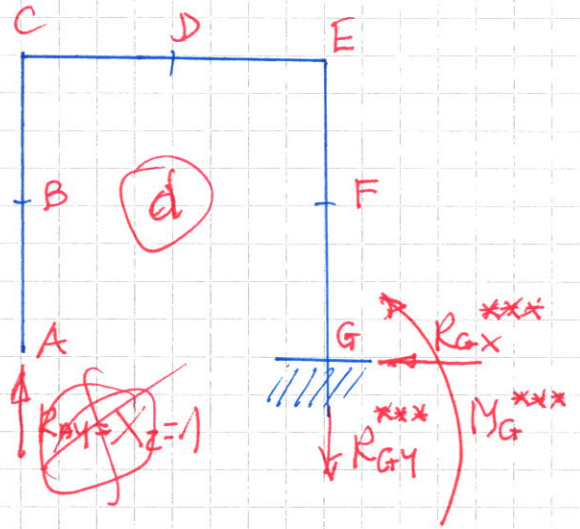
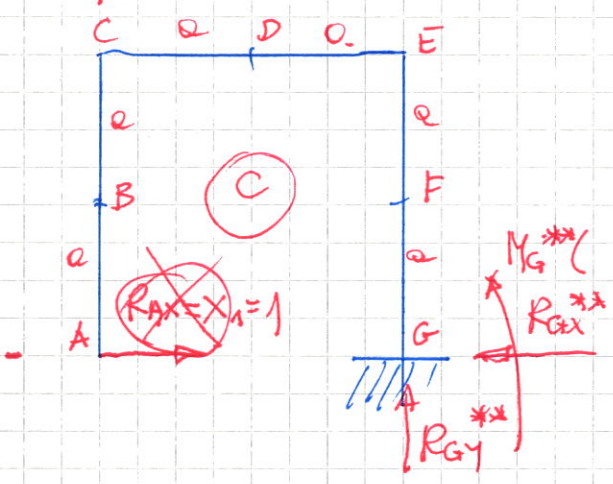
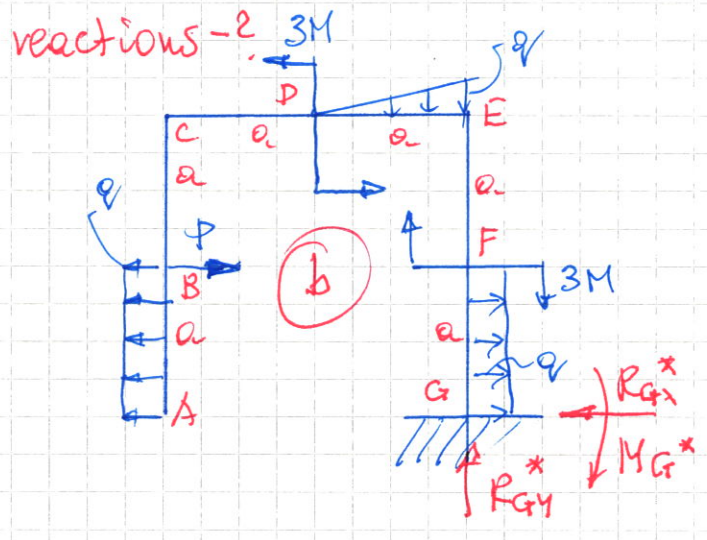
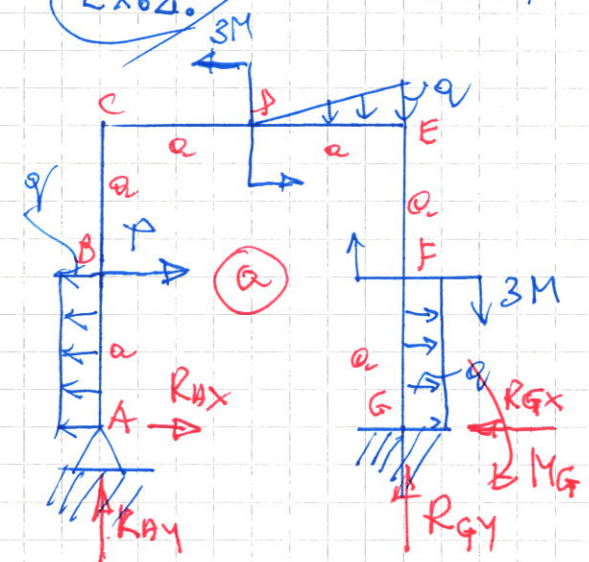
$$\delta_{12} = \delta_{21} \Rightarrow \textcircled{c} + \textcircled{d}$$

$$\delta_{12} = \delta_{21} \approx \frac{1}{EI} \left[\int_0^a M_1^{**}(x) \cdot M_1^{***}(x) dx + \int_0^a M_2^{**}(x) \cdot M_2^{***}(x) dx \right] + \frac{1}{EA} \left[\int_0^a N_1^{**}(x) \cdot N_1^{***}(x) dx + \int_0^a N_2^{**}(x) \cdot N_2^{***}(x) dx \right] = \textcircled{e}$$

$$\left. \begin{array}{l} \textcircled{4} \quad X_1 \delta_{11} + X_2 \delta_{12} + \Delta_{1P} = 0 \\ \textcircled{5} \quad X_1 \delta_{21} + X_2 \delta_{22} + \Delta_{2P} = 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} X_1 = X_A \\ X_2 = Y_A \end{array} \right\} \Rightarrow \begin{array}{l} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array}$$

$$\text{from } \begin{array}{l} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array} \Rightarrow X_C, Y_C, M_C$$

Ex. 2. $q, a, P=qa, M=qa^2, EI=const, EA=const$



(I) static eqs

(1) $\sum P_{ix} = R_{Ax} - qa + P + qa - R_{Gx} = 0$

(2) $\sum P_{iy} = R_{Ay} - \frac{1}{2}qa + R_{Gy} = 0$

(3) $\sum M_{iG} = +R_{Ay} \cdot 2a - \frac{qa^2}{2} + P \cdot a - 3M - \frac{1}{2}qa \cdot \frac{1}{2}a + 3M + \frac{qa^2}{2} - M_G = 0$

5 reactions - 3 st. eqs \Rightarrow 2x hyperstatic frame
 $R_{Ax} = X_1$
 $R_{Ay} = X_2$ } \Rightarrow hyperstatic reactions

(4) $X_1 \delta_{11} + X_2 \delta_{12} + \Delta_{1P} = 0$
 $X_1 \delta_{21} + X_2 \delta_{22} + \Delta_{2P} = 0$ } canonical eqs.
= geometrical eqs

$$\Delta_{1p} \Rightarrow b + c$$

$$\Delta_{2p} \Rightarrow b + d$$

$$\delta_{11} \Rightarrow c + c$$

$$\delta_{12} = \delta_{21} \Rightarrow c + d$$

$$\delta_{22} \Rightarrow d + d$$