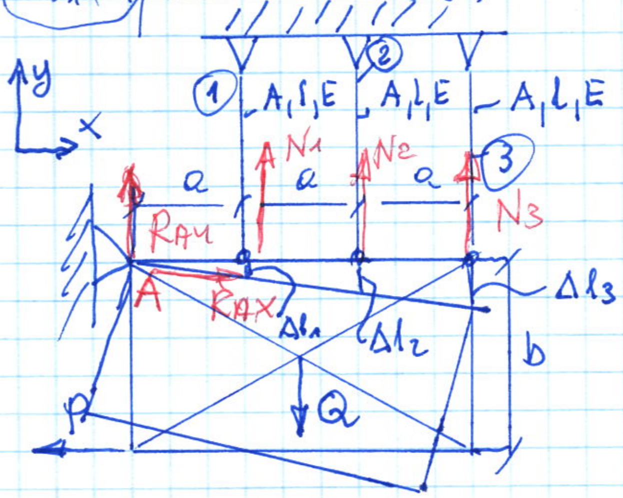
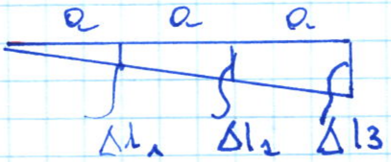
Tension-compression, hyperstatic objects
Rozciąganie-ściskanie, przypadki hiperst. /2

Ex. 1. ćw 1





A, l, E, a, b, Q, P

$\sigma_{(1)}, \sigma_{(2)}, \sigma_{(3)}$ - ?

$\sigma_{(1)} = \frac{N_1}{A}, \sigma_{(2)} = \frac{N_2}{A}, \sigma_{(3)} = \frac{N_3}{A}$

I Static eqs.

① $\sum P_{ix} = R_{Ax} - P = 0$

② $\sum P_{iy} = R_{Ay} + N_1 + N_2 + N_3 - Q = 0$

③ $\sum M_i^A = -N_1 \cdot a - N_2 \cdot 2a - N_3 \cdot 3a + Q \cdot \frac{3}{2}a + P \cdot b = 0$

5 reactions - 3 st. eqs $\Rightarrow$ 2× hyperstatic

Or

3 reactions - 1 st. eq = 2× hyperstatic

5 reactions: ~~R_{Ax}~~, ~~R_{Ay}~~, N_1, N_2, N_3

3 reactions: N_1, N_2, N_3

We need 2 add. equations

similarity of triangles
(podobieństwo trójkątów)

II Geometric eqs.

$$\frac{\Delta l_1}{a} = \frac{\Delta l_2}{2a} = \frac{\Delta l_3}{3a}$$

④ $\frac{\Delta l_1}{a} = \frac{\Delta l_2}{2a}$

⑤ $\frac{\Delta l_1}{a} = \frac{\Delta l_3}{3a}$

III Physical conditions

$$\Delta l_1 = \frac{N_1 \cdot l}{EA}; \quad \Delta l_2 = \frac{N_2 \cdot l}{E \cdot A}; \quad \Delta l_3 = \frac{N_3 \cdot l}{E \cdot A}$$

III → II

④ $\frac{\Delta l_1}{a} = \frac{\Delta l_2}{2a} \Rightarrow \frac{N_1 \cdot l}{a\,EA} = \frac{N_2 \cdot l}{2a\,EA} \quad \Big| \cdot \frac{EA \cdot a}{l}$

$N_1 = \frac{N_2}{2} \qquad N_2 = 2N_1$

⑤ $\frac{\Delta l_1}{a} = \frac{\Delta l_3}{3a} \Rightarrow \frac{N_1 \cdot l}{a\,EA} = \frac{N_3 \cdot l}{3a\,EA} \quad \Big| \cdot \frac{EA \cdot a}{l}$

$N_3 = 3N_1$

II → I

(3) $\sum M_i^A = -N_1 \cdot a - N_2 \cdot 2a - N_3 \cdot 3a + Q \cdot \frac{3}{2}a + P \cdot b = 0$

$$-N_1 \cdot a - (2N_1) \cdot 2a - (3N_1) \cdot 3a + Q \cdot \frac{3}{2}a + P \cdot b = 0$$

$$-N_1 \cdot a - 4N_1 \cdot a - 9N_1 \cdot a + Q\frac{3}{2}a + P \cdot b = 0$$

$$-14N_1 \cdot a + Q\frac{3}{2}a + P \cdot b = 0$$

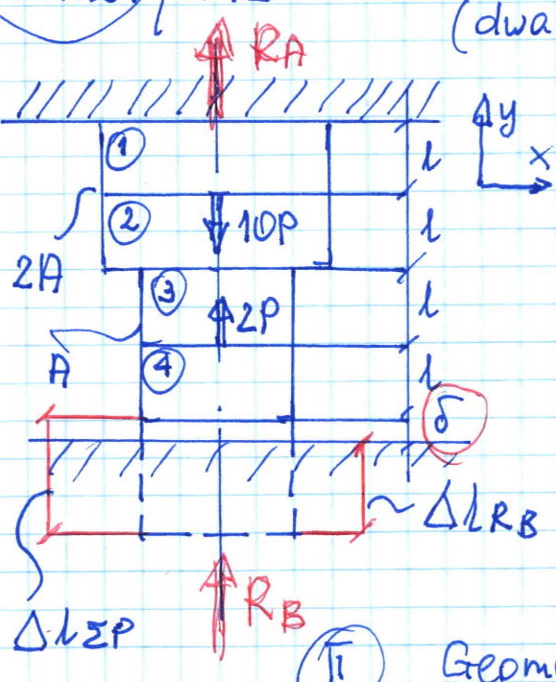
$$N_1 = \frac{3Q}{28} + P\frac{b}{14a}$$

$$\sigma_1 = \frac{N_1}{A} = \frac{3Q}{28A} + \frac{b}{14a}\frac{P}{A}$$

$$\sigma_2 = 2\sigma_1 \quad , \quad \sigma_3 = 3\sigma_1$$

Ex. 2. / ćw. 2

two ways to calculate stresses (dwa sposoby wyznaczenia naprężeń)



$$\frac{P, A, l, E, \delta}{\sigma_1, \sigma_2, \sigma_3, \sigma_4 - ?}$$

1st method

I Static eq.

(1) $\sum P_{iy} = R_A - 10P + 2P + R_B = 0$

1x hiperstatic

II Geometric eq.

(2) $\Delta l_{\Sigma P} - \Delta l_{R_B} = \delta$

III Physical conditions (Hooke's law)

$$\Delta l_{\Sigma P} = \frac{10P \cdot l}{E \cdot 2A} - \frac{2P \cdot l}{E \cdot A} - \frac{2P \cdot 2l}{2EA} = \frac{10Pl}{2EA} - \frac{4Pl}{EA} =$$

$$= \frac{5Pl}{EA} - \frac{4Pl}{EA} = \frac{P \cdot l}{EA}$$ (wartość dodatnia)

~~III~~

$$\Delta l_{RB} = \frac{R_B \cdot 2l}{EA} + \frac{R_B \cdot 2l}{2EA} = \frac{R_B \cdot 2l}{EA} + \frac{R_B \cdot l}{EA} =$$

$$= \frac{3 R_B \cdot l}{EA}$$

III $\curvearrowright$ II

(2) $\Delta l_{\Sigma P} - \Delta l_{RB} = \frac{P \cdot l}{EA} - \frac{3 R_B \cdot l}{EA} = \delta \quad \Big| \cdot \frac{EA}{l}$

$$P - 3R_B = \frac{\delta EA}{l}$$

$$3R_B = P - \frac{\delta EA}{l} = \frac{P \cdot l - \delta EA}{l}$$

$$R_B = \frac{P \cdot l - \delta EA}{3 \cdot l} \Longrightarrow \text{I} \quad (\text{static eq})$$

$$R_A - 10P + 2P + R_B = 0 \qquad R_A - 8P + R_B = 0$$

$$R_A = 8P - R_B = 8P - \frac{Pl - \delta EA}{3l} =$$

$$= \frac{24P \cdot l - Pl + \delta EA}{3l} = \frac{23P \cdot l + \delta EA}{3l} = R_A$$

σ_{1}, σ_{2}, σ_{3}

$$\sigma_{1} = \frac{R_A}{2A} \qquad\qquad \sigma_{1} = \frac{-R_B - 2P + 10P}{2A}$$

$$\sigma_{2} = \frac{R_A - 10P}{2A} \qquad\qquad \sigma_{2} = \frac{-R_B - 2P}{2A}$$

$$\sigma_{3} = \frac{R_A - 10P}{A} \qquad\qquad \sigma_{3} = \frac{-R_B - 2P}{A}$$

$$\sigma_{4} = \frac{R_A - 10P + 2P}{A} \qquad\qquad \sigma_{4} = -\frac{R_B}{A}$$

2nd method

I static eq. (1) $\sum P_{iy} = P$

II Geometric eq:

$$\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4 = \delta \quad !$$

III ~~Stat~~ Physical conditions (Hooke's law)

$$\Delta l_1 = \frac{R_A \cdot l}{E \cdot 2A}, \quad \Delta l_2 = \frac{(R_A - 10P)l}{E \cdot 2A},$$

$$\Delta l_3 = \frac{(R_A - 10P)l}{EA}, \quad \Delta l_4 = \frac{(R_A - 10P + 2P)l}{EA}$$

III $\rightarrow$ II

$$\frac{R_A \cdot l}{2EA} + \frac{(R_A - 10P) \cdot l}{2EA} + \frac{(R_A - 10P)l}{EA} + \frac{(R_A - 10P + 2P) \cdot l}{EA} = \delta$$

$$\frac{R_A \cdot l}{2EA} + \frac{R_A \cdot l}{2EA} - \frac{10P \cdot l}{2EA} + \frac{R_A \cdot l}{EA} - \frac{10P \cdot l}{EA} + \frac{R_A \cdot l}{EA} - \frac{10P \cdot l}{EA} + \frac{2Pl}{EA} = \frac{3R_A l}{EA} - \frac{23Pl}{EA} = \delta \quad \Big| \cdot \frac{EA}{l}$$

$$3R_A - 23P = \frac{\delta EA}{l} \Rightarrow$$

$$3R_A = 23P + \frac{\delta EA}{l} \Rightarrow \frac{23P \cdot l + \delta EA}{3l}$$

$$R_A = \frac{23Pl + \delta EA}{3l}$$