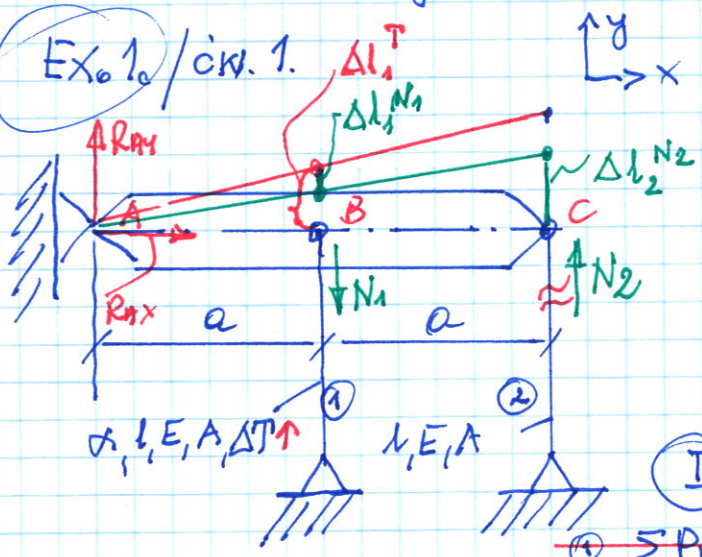
Tension-compression, hyperstatic objects / 4
Rozciąganie - ściskanie, przypadki hiperstek.

Ex. 1 / str. 1.



$a, l, E, A, \Delta T\uparrow, \alpha$ (heating / grzanie)

$\sigma_{(1)}, \sigma_{(2)}$ - ?

$\Downarrow \quad \Downarrow$

$\frac{N_1}{A}, \frac{N_2}{A}$

Reactions: ~~R_{Ax}, R_{Ay}~~, N_1, N_2

(I) static eqs.

(1) ~~$\sum P_{ix} = R_{Ax} = 0$~~

(2) ~~$\sum P_{iy} = R_{Ay} - N_2 + N_2 = 0$~~

(3) $\sum M_i^A = N_1 \cdot a - N_2 \cdot 2a = 0 \quad | : a$

$N_1 - 2N_2 = 0 \Rightarrow N_2 = \frac{N_1}{2}$

~~4~~ reactions - 3 st. eqs $\Rightarrow$ 1x hyperstatic

2 reactions - 1 st. eq. $\Rightarrow$ 1x hyperstatic

(II) Geometric eqs.

$$\frac{\Delta l_1^T - \Delta l_1^{N_1}}{a} = \frac{\Delta l_2^{N_2}}{2a} \quad | \cdot 2a$$

$$2(\Delta l_1^T - \Delta l_1^{N_1}) = \Delta l_2^{N_2}$$

$$2\Delta l_1^T - 2\Delta l_1^{N_1} - \Delta l_2^{N_2} = 0 \Rightarrow 2\Delta l_1^T = 2\Delta l_1^{N_1} + \Delta l_2^{N_2}$$

(III) Physical conditions

$$\Delta l_1^T = \alpha \cdot l \cdot \Delta T$$

$$\Delta l_1^{N_1} = \frac{N_1 \cdot l}{EA} \quad ; \quad \Delta l_2^{N_2} = \frac{N_2 \cdot l}{EA}$$

(III) $\rightarrow$ (II)

$$2\alpha \cdot l \cdot \Delta T = \frac{2N_1 \cdot l}{EA} + \frac{N_2 \cdot l}{EA} \quad | \cdot \frac{EA}{l}$$

$$2\alpha \cdot \Delta T \cdot \frac{E \cdot A}{\cancel{l}} = 2N_1 + N_2 \qquad \text{but } N_2 = \frac{N_1}{2}$$

$$2\alpha \cdot \Delta T \cdot EA = 2N_1 + \frac{N_1}{2} = \frac{5}{2} N_1 \Rightarrow N_1$$

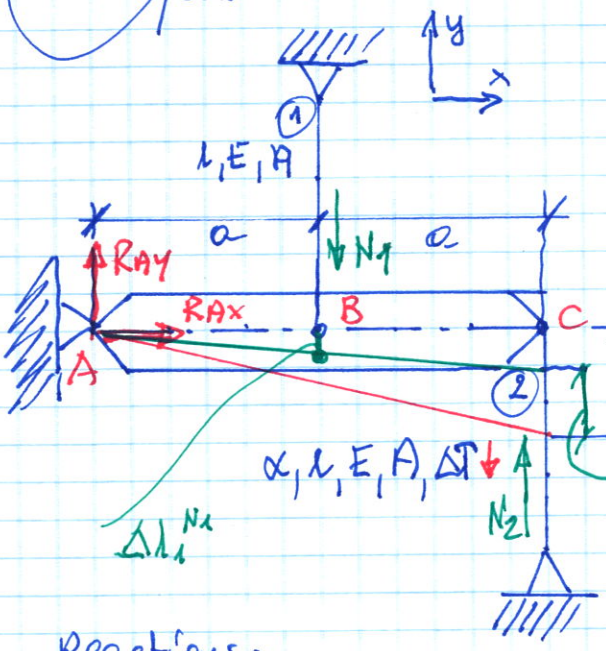
$$N_1 = \frac{4}{5} \alpha \cdot \Delta T \cdot E \cdot A \quad , \quad N_2 = \frac{4}{10} \alpha \cdot \Delta T \cdot E \cdot A$$

$$\sigma_1 = \frac{4}{5} \alpha \cdot \Delta T \cdot E \qquad \sigma_2 = \frac{4}{10} \alpha \cdot E \cdot \Delta T$$

Ex. 2. / Ćw. 2

$a, l, E, A, \Delta T\downarrow, \alpha$

(cooling / ochłodzenie)



$\sigma_{(1)}$, $\sigma_{(2)}$ - ?

$\sigma_{(1)} \Rightarrow \frac{N_1}{A}$, $\sigma_{(2)} \Rightarrow \frac{N_2}{A}$

Reactions:

~~R_{Ax}, R_{Ay}~~, N_1, N_2

(I) Static eqs.

(1) ~~$\sum P_{ix} = R_{Ax} = 0$~~

(2) ~~$\sum P_{iy} = R_{Ay} - N_1 + N_2 = 0$~~

(3) $\sum M_i^A = N_1 \cdot a - N_2 \cdot 2a = 0 \Rightarrow$

$\Rightarrow N_1 = 2N_2 \Rightarrow N_2 = \frac{N_1}{2}$

4 reactions - 3 st. eqs $\Rightarrow$ 1x hyperstatic

2 reactions - 1 st. eq. $\Rightarrow$ 1x hyperstatic

(II) Geometric eqs.

$$\frac{\Delta l_1^{N_1}}{a} = \frac{\Delta l_2^T - \Delta l_2^{N_2}}{2a} \quad | \cdot 2a$$

$$2\Delta l_1^{N_1} = \Delta l_2^T - \Delta l_2^{N_2}$$

$$\Delta l_2^T = 2\Delta l_1^{N_1} + \Delta l_2^{N_2}$$

(III) Physical conditions

$$\Delta l_1^{N_1} = \frac{N_1 \cdot l}{EA} \; ; \; \Delta l_2^{N_2} = \frac{N_2 \cdot l}{EA} \; ; \; \Delta l_2^T = \alpha \cdot l \cdot \Delta T$$

(III) $\rightsquigarrow$ (II)

$$\alpha \cdot l \cdot \Delta T = \frac{2N_1 \cdot l}{EA} + \frac{N_2 \cdot l}{EA} \quad | \cdot \frac{AE}{l}$$

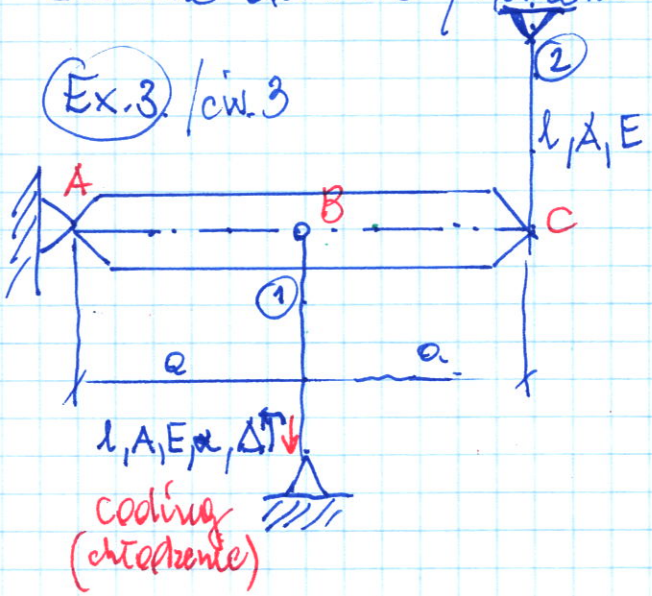
$$\frac{\alpha \cdot \cancel{l} \cdot \Delta T \cdot EA}{\cancel{l}} = 2N_1 + N_2 = 2N_1 + \frac{N_1}{2} = \frac{5}{2} N_1$$

$$\alpha \cdot \Delta T \cdot E \cdot A = \frac{5}{2} N_1 \Rightarrow N_1 = \frac{2}{5} \alpha \cdot \Delta T \cdot E \cdot A \; , \; N_2 = \frac{1}{5} \alpha \cdot \Delta T \cdot EA$$

$$\sigma_1 = \frac{2}{5} \alpha \cdot \Delta T \cdot E \; , \; \sigma_2 = \frac{1}{5} \alpha \cdot \Delta T \cdot E$$

Zadanie domowe / homework

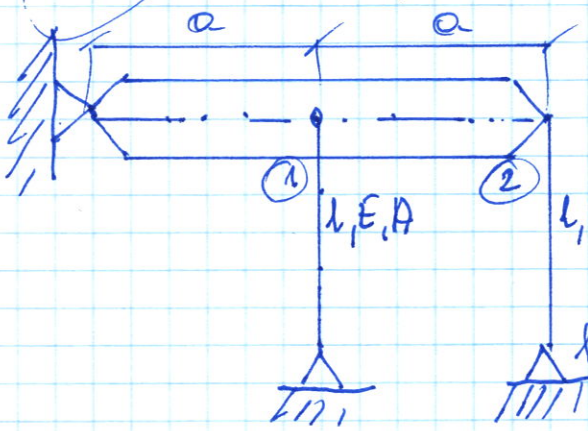
Ex. 3 / ćw. 3



$a, l, E, A, \alpha, \Delta T\downarrow$

$\sigma_{①}, \sigma_{②}$ - ?

Ex. 4. / ćw. 4. a) i b)

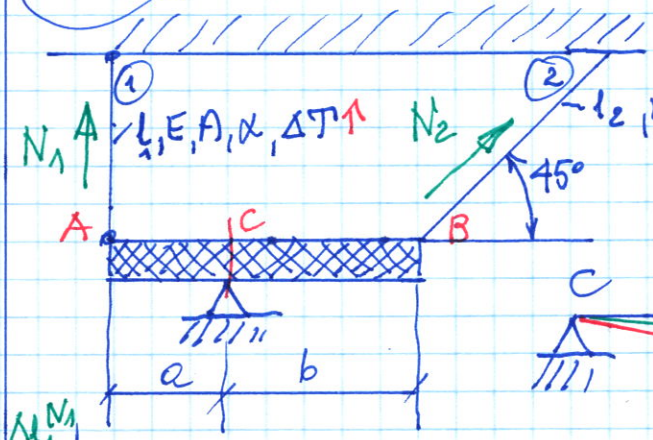


$a, l, E, A, \alpha, \Delta T\downarrow / \Delta T\uparrow$

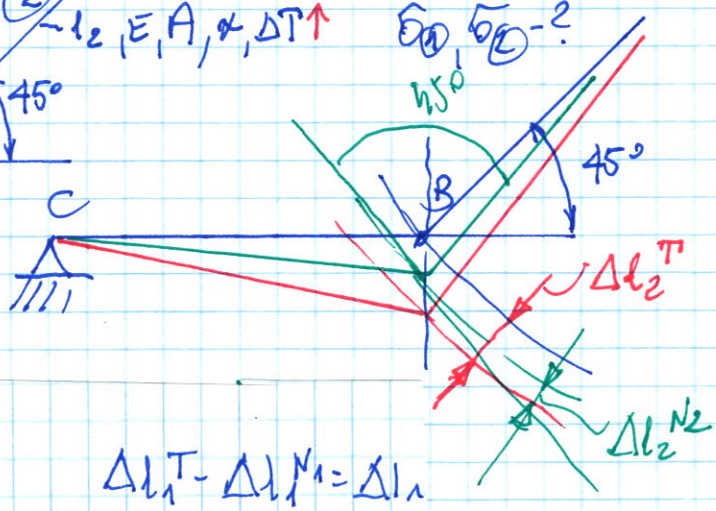
Ex. 5. / ćw. 5

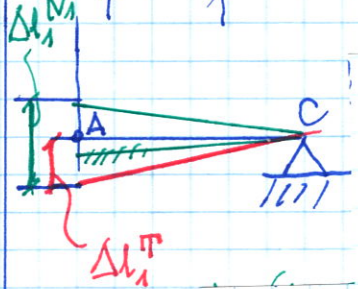
$l_1 = l$, $l_2 = \sqrt{2}\cdot l$, $E, A, \alpha, \Delta T\uparrow$

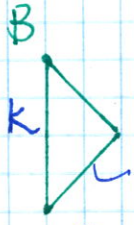
(heating / ogrzanie)



$\sigma_{①}, \sigma_{②}$ - ?







$\Delta l_1^T - \Delta l_1^{N_1} = \Delta l_1$

$\Delta l_2^T - \Delta l_2^{N_2} = \Delta l_2$

(I) static eq.

(1) $\sum M_i^C = N_1 \cdot a - N_2 \cdot b \frac{\sqrt{2}}{2} = 0 \Rightarrow N_2 = \sqrt{2} \cdot N_1 \cdot \frac{a}{b}$

$N_1 - ?$, $N_2 - ?$

1x hyperstatic object

(II) Geometric eq.

$$\frac{\Delta l_1}{a} = \frac{k}{b}$$

$$\frac{\Delta l_2}{k} = \frac{\sqrt{2}}{2} \Rightarrow \sin 45^\circ$$

$$k = \frac{\Delta l_2}{\sin 45^\circ}$$

$$\frac{\Delta l_1}{a} = \frac{\Delta l_2}{b \cdot \sin 45^\circ} \quad | \cdot a \cdot \sin 45^\circ$$

$$\frac{\Delta l_1 \cdot \sin 45^\circ}{\Delta l_2} = \frac{a}{b}$$

(III) Physical conditions

$$\Delta l_1 = \Delta l_1^T - \Delta l_1^{N_1} = \alpha \cdot l \cdot \Delta T - \frac{N_1 \cdot l}{EA}$$

$$\Delta l_2 = \Delta l_2^T - \Delta l_2^{N_2} = \alpha \cdot l\sqrt{2} \cdot \Delta T - \frac{N_2 \cdot l \cdot \sqrt{2}}{E \cdot A}$$

(III) ∨ (II)

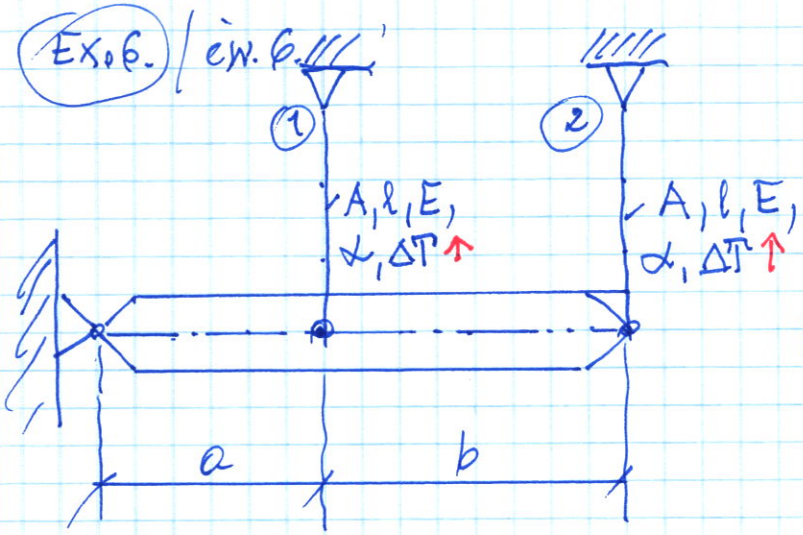
$$\frac{\left(\alpha \cdot l \cdot \Delta T - \frac{N_1 \cdot l}{EA}\right) \cdot \sin 45^\circ}{\alpha \cdot l\sqrt{2} \cdot \Delta T - \frac{N_2 \cdot l \cdot \sqrt{2}}{EA}} = \frac{a}{b}$$

$$\frac{\left(\alpha \cdot l \cdot \Delta T - \frac{N_1 \cdot l}{EA}\right) \cdot \sin 45^\circ}{l \cdot \sqrt{2}\left(\alpha \cdot \Delta T - \frac{N_2}{EA}\right)} = \frac{a}{b}$$

$$\frac{\left(\alpha \cdot l \cdot \Delta T - \frac{N_1 \cdot l}{EA}\right) \frac{\sqrt{2}}{2}}{l \cdot \sqrt{2}\left(\alpha \cdot \Delta T - \frac{\sqrt{2} \cdot N_1 \cdot a}{b \cdot EA}\right)} = \frac{a}{b} \Rightarrow N_1 \Rightarrow N_2$$

$N_{(1)} \Rightarrow \sigma_{(1)}$, $N_{(2)} \Rightarrow \sigma_{(2)}$

Ex. 6. / ew. 6.



$a, b, A, l, E, \alpha, \Delta T\uparrow$

$\overline{5}_{①}$, $\overline{5}_{②}$ - ?

Homework