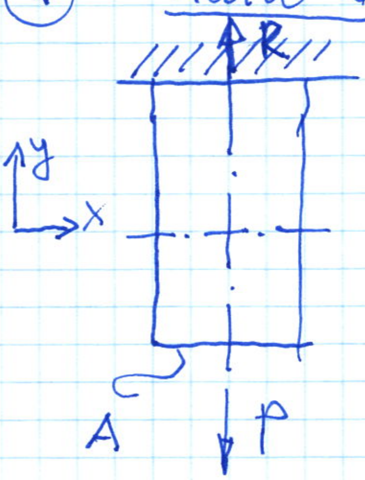
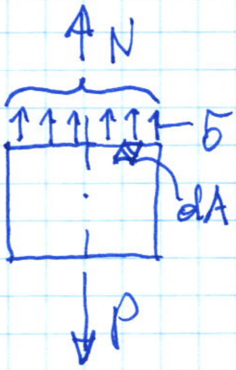
Tension, compression. Introduction

1. static conditions

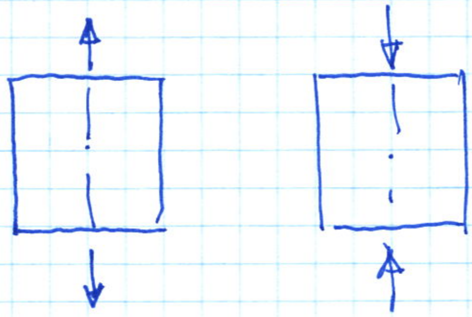




$$\Sigma P_{iy} = P - N = 0$$

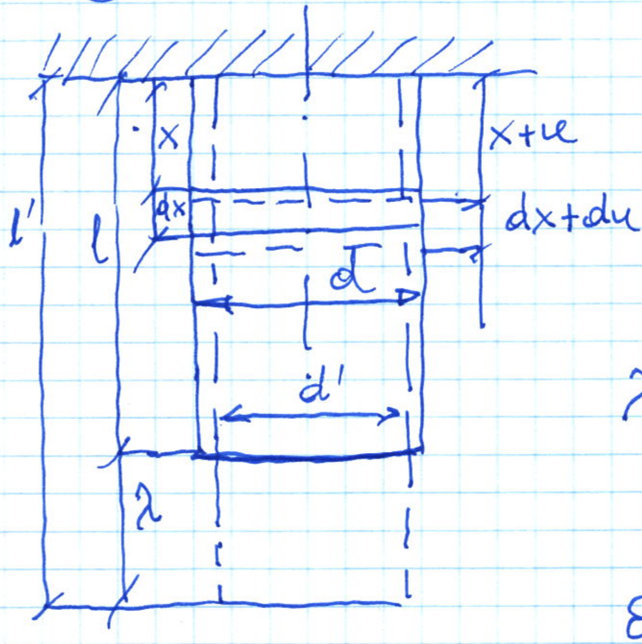
$$\int_A \sigma \cdot dA = N$$

Agreement as to the sign of force



tension (positive force) compression (negative force)

2. geometric conditions



$$\frac{du}{dx} = \varepsilon \quad (\text{def})$$

axial strain - longitudinal strain

$$\lambda = \Delta l = l' - l$$

elongation

$$\lambda = u_{x=l} = \int_0^l \varepsilon \cdot dx = \varepsilon \cdot l$$

$$\varepsilon = \frac{\lambda}{l} = \frac{\Delta l}{l}$$

$$\varepsilon' = \frac{d' - d}{d}$$

(for tension $d > d'$, $\varepsilon' < 0$

transverse strain (lateral strain)

3. Physical conditions, Hooke's law

$$\boxed{\varepsilon = \sigma \frac{1}{E}} \quad , \quad \Delta l = \frac{P \cdot l}{E \cdot A}$$

$$\boxed{\varepsilon' = -\nu \cdot \varepsilon} \qquad \frac{1}{6} < \nu < \frac{1}{2} \qquad \nu \cong 0{,}3$$

(for steel)

$$\sigma = \frac{N}{A}$$

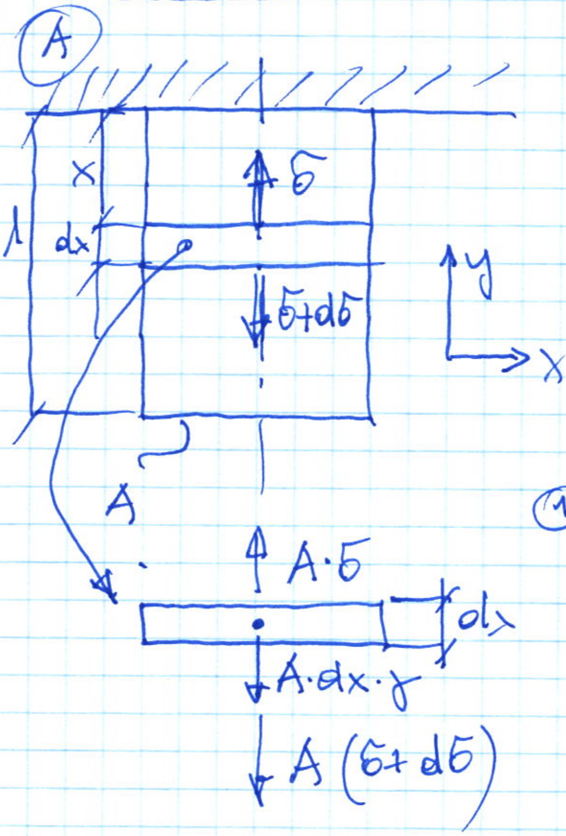
$$\lambda = \Delta l = \int_0^l \varepsilon \, dx = \int_0^l \frac{\sigma}{E} \cdot dx = \int_0^l \frac{N}{A \cdot E} dx = \frac{N \cdot l}{E \cdot A}$$

$$\boxed{\Delta l = \lambda = \frac{N \cdot l}{E \cdot A}}$$

$E \cdot A$ – stiffness for tension/compression

$\frac{EA}{l}$ – unit stiffness for tension/compression

Tension. Influence of weight on stresses.



!

Assumptions: P, $A = \text{const}$, l, γ

to designate: $\sigma(x), \varepsilon(x)$ – ?

γ – specific gravity/specific weight

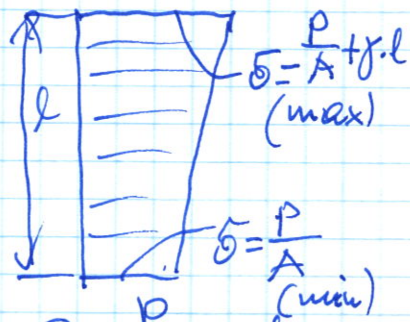
static equation

(1) $\sum P_{iy} = A(\sigma + d\sigma) + A \cdot \gamma \cdot dx - A \cdot \sigma = 0$

$\cancel{A \cdot \sigma} + A \cdot d\sigma + A \cdot \gamma \cdot dx - \cancel{A \cdot \sigma} = 0 \quad | : A$

$d\sigma = -\gamma \cdot dx$

$$\boxed{\sigma = -\gamma \cdot x + C}$$



but from static equation

$$\sigma_{x=l} = \frac{P}{A} \Rightarrow \frac{P}{A} = -\gamma \cdot l + C \Rightarrow C = \frac{P}{A} + \gamma \cdot l$$

$$\sigma = -\gamma \cdot x + \frac{P}{A} + \gamma \cdot l \Rightarrow \boxed{\sigma = \frac{P}{A} + \gamma (l - x)}$$

stress state

strain state

$$\varepsilon = \frac{\sigma}{E} = \frac{1}{E}\left[\frac{P}{A} + \gamma (l - x)\right]$$

$$\lambda = \Delta l = \int_0^l \varepsilon(x)\, dx = \frac{1}{E}\int_0^l \left[\frac{P}{A} + \gamma (l - x)\right] dx = \frac{1}{E}\left[\frac{P}{A} \cdot x + \gamma \cdot l x - \gamma \frac{x^2}{2}\right]_0^l =$$

$$\lambda = \Delta l = \frac{1}{E}\left[\frac{Pl}{A} + \gamma l^2 - \frac{\gamma l^2}{2}\right] = \frac{1}{E}\left[\frac{P \cdot l}{A} + \frac{\gamma l^2}{2}\right]$$

$$\lambda = \Delta l = \frac{P \cdot l}{EA} + \frac{\gamma l^2}{2E}$$

but assume that $Q = A \cdot l \cdot \gamma$

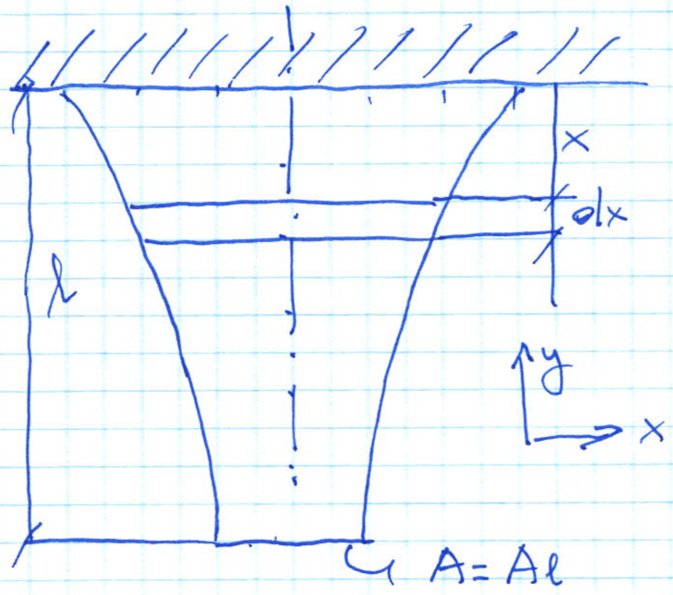
Q – weight

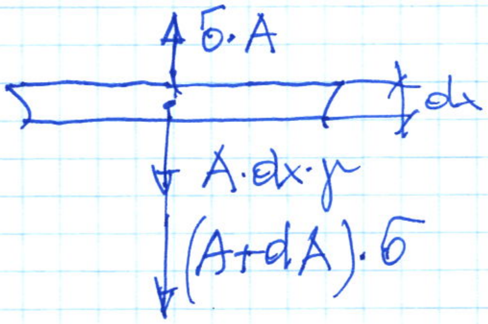
$$\boxed{\lambda = \frac{P \cdot l}{EA} + \frac{Q \cdot l}{2EA}}$$

(B)

<u>Assumptions</u>: $l, P, A_{x=l} = A$, $\sigma = \text{const}$!, γ

to designate: $A(x)$ − ?





static equation

(1) $$\Sigma P_{iy} = (A+dA)\sigma + A\cdot\gamma\cdot dx - \sigma\cdot A = 0$$

$$\cancel{A\cdot\sigma} + dA\cdot\sigma + A\cdot\gamma\cdot dx - \cancel{\sigma A} = 0 \quad \Big| : A\cdot\sigma$$

$$\frac{1}{A}\,dA = -\frac{\gamma}{\sigma}\,dx$$

$$\ln A + C = -\frac{\gamma}{\sigma}\cdot x$$

but $$A_{x=l} = A_l = \frac{P}{\sigma}$$

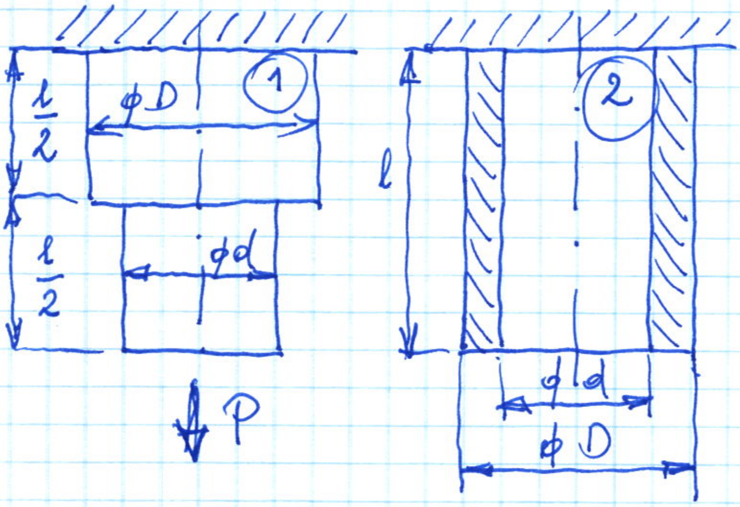
$$\ln A_l + C = -\frac{\gamma}{\sigma}\cdot l \implies C = -\ln A_l - \frac{\gamma}{\sigma}\cdot l$$

$$\ln A - \ln A_l = \frac{\gamma}{\sigma}\cdot l - \frac{\gamma}{\sigma}\cdot x$$

$$\ln \frac{A}{A_l} = \frac{\gamma}{\sigma}(l - x)$$

$$\boxed{A = A_l \cdot e^{\frac{\gamma}{\sigma}(l-x)}}$$

Ex. 1. Compare the stiffnesses of two bars



E, $\alpha = \frac{d}{D} = 0{,}8$, (P), D

$C = \frac{EA}{l}$ - ?

C_1, C_2 - ?

C - unit stiffness

$$C = \frac{P}{\Delta l} \qquad \Delta l = \frac{P \cdot l}{EA}$$

$$C = \frac{\cancel{P} \cdot EA}{\cancel{P} \cdot l} = \frac{EA}{l} \Rightarrow \text{unit stiffness}$$

(A) Elongation Δl_1 (for bar (1))

$$\Delta l = \frac{P \cdot \frac{l}{2}}{E \frac{\pi d^2}{4}} + \frac{P \frac{l}{2}}{E \cdot \frac{\pi D^2}{4}} = \frac{4 P \cdot l}{2 \pi E d^2} + \frac{4 P \cdot l}{2 \pi E D^2} = \frac{2 P l}{\pi E}\left(\frac{1}{d^2} + \frac{1}{D^2}\right)$$

$$\Delta l_1 = \frac{2 P l}{\pi E} \frac{D^2 + d^2}{d^2 D^2}$$

and $$C_1 = \frac{P}{\Delta l_1} = \frac{\pi E d^2 D^2}{2 l (d^2 + D^2)} = \frac{\pi E \alpha^2 D^2 \cdot D^2}{2 l D^2 (\alpha^2 + 1)} = \frac{\pi E \alpha^2 D^2}{2 l (1 + \alpha^2)}$$

(B) Elongation Δl_2 (for bar/tube (2))

$$\Delta l_2 = \frac{P \cdot l}{E \pi \frac{(D^2 - d^2)}{4}} = \frac{4 \cdot P \cdot l}{\pi E (D^2 - d^2)}$$

$$C_2 = \frac{P}{\Delta l_2} = \frac{\pi E (D^2 - \alpha^2 D^2)}{4 l} = \frac{\pi E D^2 (1 - \alpha^2)}{4 l}$$

(C) $$k = \frac{C_1}{C_2} = \frac{\cancel{\pi E} \alpha^2 \cancel{D^2}}{\cancel{2}^{} \cancel{l} (1 + \alpha^2)} \cdot \frac{\cancel{4}^{2} \cancel{l}}{\cancel{\pi E D^2} (1 - \alpha^2)} = \frac{2 \alpha^2}{1 - \alpha^4}$$

for $\alpha = 0{,}8$ $$k = \frac{2 \cdot 0{,}8^2}{1 - 0{,}8^4} = 2{,}17$$

(D) for which k value - the unit stiffnesses will be identical

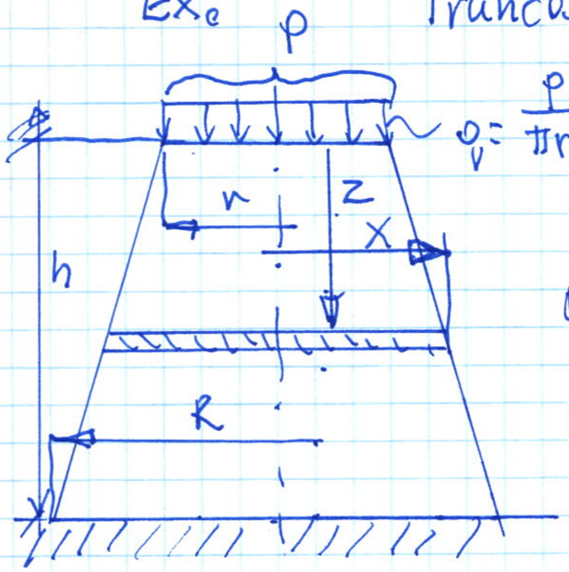
$$k = \frac{2\alpha^2}{1 - \alpha^4} = 1 \rightarrow \alpha^4 + 2\alpha^2 - 1 = 0 \qquad \alpha^2 = z$$

$$z^2 + 2z - 1 = 0$$

$$z_1 = \frac{-2 + 2\sqrt{2}}{2} = \sqrt{2} - 1 = 0{,}41 \qquad \Delta = 4 + 4 = 8 \quad \sqrt{\Delta} = 2\sqrt{2}$$

$$\hookrightarrow \alpha = 0{,}643 \,!$$

Ex. Truncated cone, weight effect



$$\frac{h, R, r, \gamma, P}{X_{\sigma min} - ?}$$

Note: for rising z, both - the weight and the surface area increase

(1) We need to find a relationships between Z and X

$$\frac{z}{h} = \frac{x-r}{R-r} \Rightarrow x = r + \frac{z(R-r)}{h}$$

then $dx = \frac{R-r}{h} dz \Rightarrow dz = \frac{h}{R-r} dx$

(2) The cross-sectional area of radius x is affected by the weight G(x)

$$G(x) = \pi\gamma \int_0^z x^2 \cdot dz = \pi \cdot \gamma \int_r^x x^2 \frac{h}{R-r} dx = \frac{\pi \gamma \cdot h}{R-r} \frac{x^3}{3}\Big|_r^x =$$

$$= \underbrace{\frac{\pi \gamma \cdot h}{3(R-r)}}_{a} (x^3 - r^3) = a(x^3 - r^3), \text{ where}$$

$$\boxed{a = \frac{\pi\gamma \cdot h}{3(R-r)} = \text{const}}$$

(3) The stress for radius x

$$\sigma(x) = \frac{P + G(x)}{\pi x^2} = \frac{P + a(x^3 - r^3)}{\pi x^2}$$

(4) Extreme of the function $\sigma(x)$

$$\boxed{\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}}$$

$$\frac{d\sigma(x)}{dx} = \frac{1}{\pi}\left[a - 2\left(\frac{P - ar^3}{x^3}\right)\right] = 0$$

$$a = \frac{2}{x^3}(P - ar^3) \Rightarrow$$

$$\Rightarrow x^3 = \frac{2}{a}(P - ar^3) \Rightarrow x = \sqrt[3]{2\left(\frac{P}{a} - r^3\right)} =$$

$$\sqrt[3]{\frac{6P(R-r)}{\pi \cdot \gamma \cdot h} - 2r^3}$$

but x must be:

— positive (negative doesn't make sense)

— $r \leq x \leq R$

It is easy to notice that x is positive when

$$p > ar^3$$

At the same time, for the extremum to have minimal value (minimum), the second derivative must be greater then zero !

$$\frac{d^2\sigma(x)}{dx^2} > 0 \quad !$$

$$\frac{d^2\sigma(x)}{dx} = \frac{6\left(p - ar^3\right)}{\pi \cdot x^4}$$

for $p > ar^3$, $\frac{d^2\sigma(x)}{dx^2} > 0$

It means, that $r \leq x \leq R$!