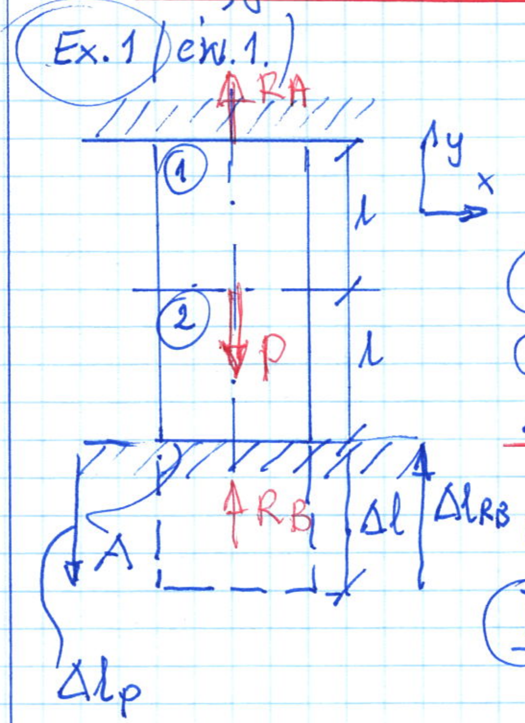
# Tension-compression, hyperstatic objects /1
# Rozciąganie-ściskanie, przypadki hiperstatyczne

## Ex. 1 / ćw. 1.




$$\frac{A, l, E, P}{R_A, R_B, \sigma_1, \sigma_2 \ -?}$$

I. statics eqs.

(1) $\sum P_{iy} = R_A - P + R_B = 0$ ?, ?

2 reactions - 1 st. eq $\Rightarrow$ 1× hyperstatic object

We need add. equation/condition!

II. Geometric condition

(2) $\Delta l_p - \Delta l_{RB} = 0$

III. Physical conditions (Hooke's law)

$$\Delta l_p = \frac{P \cdot l}{E \cdot A} \qquad \Delta l_{RB} = \frac{R_B \cdot 2l}{E \cdot A}$$

$$\sigma_1 = \frac{R_A}{A} = \frac{P}{2A}$$

$$\sigma_2 = \frac{R_A - P}{A} = \frac{\frac{P}{2} - P}{A} = -\frac{P}{2}$$

$\sigma_1$ - tension

$\sigma_2$ - compression

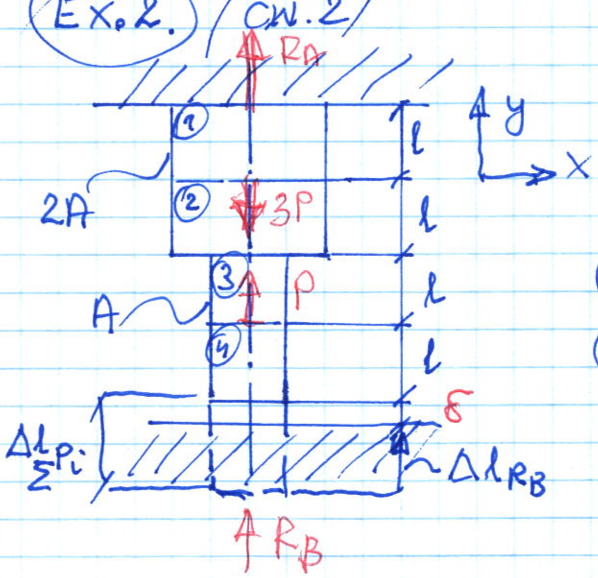
III $\rightarrow$ II

$$\frac{P \cdot l}{EA} - \frac{R_B \cdot 2l}{EA} = 0 \quad \Big| \cdot \frac{EA}{l}$$

$$P - 2R_B = 0 \Rightarrow R_B = \frac{P}{2} \Rightarrow (1)$$

$$R_A - P + R_B = R_A - P + \frac{P}{2} = 0 \Rightarrow R_A = \frac{P}{2}$$

## Ex. 2. / ćw. 2 /




$$\frac{A, l, E, P, \delta}{R_A, R_B, \sigma_1, \sigma_2, \sigma_3, \sigma_4 \ -?}$$

I. Statics equations

(1) $\sum P_{iy} = R_A - 3P + P + R_B = 0$ ?, ?

2 reactions - 1 static eq $\Rightarrow$ 1× hyperstatic

II Geometric condition

$$② \quad \Delta l_{\Sigma P_i} - \Delta l_{R_B} = \delta$$

III Physical conditions (Hooke's law)

$$\Delta l_{\Sigma P_i} = \frac{3P \cdot l}{E \cdot 2A} - \frac{P \cdot l}{EA} - \frac{P \cdot 2l}{E \cdot 2A}$$

$$\Delta l_{R_B} = \frac{R_B \cdot 2l}{EA} + \frac{R_B \cdot 2l}{E \cdot 2A}$$

III $\rightarrow$ II

$$\frac{3P \cdot l}{E \cdot 2A} - \frac{P \cdot l}{EA} - \frac{P \cdot \cancel{2}l}{E \cdot \cancel{2}A} - \frac{R_B \cdot 2l}{EA} - \frac{R_B \cdot \cancel{2}l}{E \cdot \cancel{2}A} = \delta \quad \Big| \; \frac{E \cdot 2A}{l}$$

$$3P - 2P - 2P - 4R_B - 2R_B = \frac{\delta E \cdot 2A}{l}$$

$$-P - 6R_B = \frac{\delta E \cdot 2A}{l}$$

$$6R_B = -P - \frac{\delta E \cdot 2A}{l} = \frac{-P \cdot l - \delta E \cdot 2A}{l}$$

$$R_B = \frac{-(Pl + \delta \cdot E \cdot 2A)}{6l} = -\frac{P}{6} - \frac{\delta EA}{3l}$$

from ① $R_A = 2P - R_B = 2P + \frac{P}{6} + \frac{\delta EA}{3l} = \frac{13}{6}P + \frac{\delta EA}{3l}$

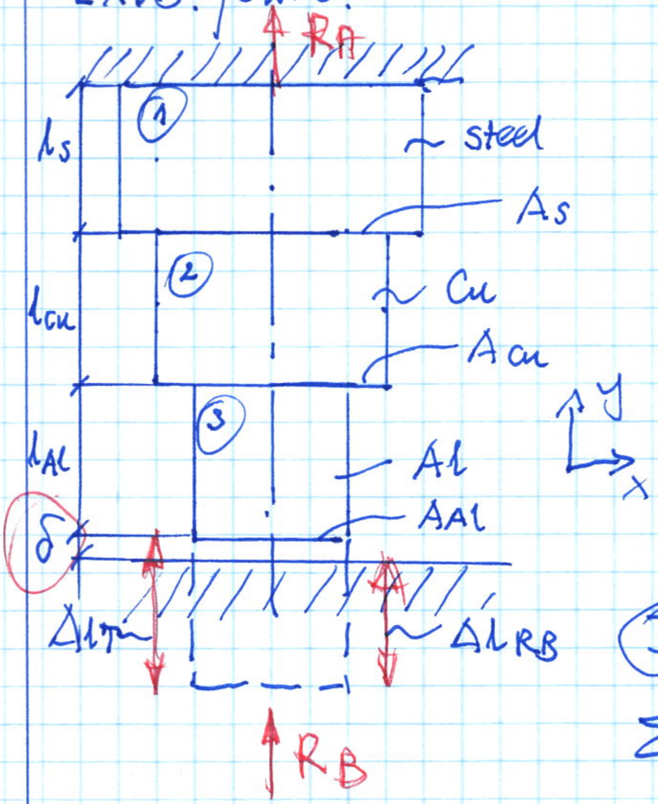
$$\sigma_{①} = \frac{R_A}{2A} \qquad \sigma_{①} = \frac{-R_B - P + 3P}{2A}$$

$$\sigma_{②} = \frac{R_A - 3P}{2A} \qquad \sigma_{②} = \frac{-R_B - P}{2A}$$

$$\sigma_{③} = \frac{R_A - 3P}{A} \qquad \sigma_{③} = \frac{-R_B - P}{A}$$

$$\sigma_{④} = \frac{R_A - 3P + P}{A} \qquad \sigma_{④} = \frac{-R_B}{A}$$

Ex. 3. / ćw. 3.




$l_s = l_{cu} = l_{Al} = l$, $\delta$

$A_s = 3A$, $A_{cu} = 2A$, $A_{Al} = A$

$E_s$, $E_{cu}$, $E_{Al}$, $\Delta T$

$\alpha_s$, $\alpha_{cu}$, $\alpha_{Al}$

$\alpha$ - coeff. of linear thermal expansion

$\sigma_{①}$, $\sigma_{②}$, $\sigma_{③}$ - ?

(I) statics equation

$$\Sigma P_{iy} = R_A + R_B = 0 \Rightarrow$$
$$R_A = -R_B$$

2 reactions - 1 st. eq $\Rightarrow$ 1x hyperstatic

(II) Geometric condition

$$(2) \quad \Delta l_T - \Delta l_{RB} = \delta$$

(III) Physical conditions: Hooke's law + linear thermal expansion

$$\Delta l_T = l_s \cdot \alpha_s \cdot \Delta T + l_{cu} \cdot \alpha_{cu} \cdot \Delta T + l_{Al} \cdot \alpha_{Al} \cdot \Delta T =$$
$$= l \cdot \alpha_s \cdot \Delta T + l \cdot \alpha_{cu} \cdot \Delta T + l \cdot \alpha_{Al} \cdot \Delta T =$$
$$= l \cdot \Delta T \left( \alpha_s + \alpha_{cu} + \alpha_{Al} \right)$$

$$\Delta l_{RB} = \frac{R_B \cdot l_{Al}}{A_{Al} \cdot E_{Al}} + \frac{R_B \cdot l_{cu}}{A_{cu} \cdot E_{cu}} + \frac{R_B \cdot l_s}{A_s \cdot E_s} =$$
$$= \frac{R_B \cdot l}{A \cdot E_{Al}} + \frac{R_B \cdot l}{2A \cdot E_{cu}} + \frac{R_B \cdot l}{3A \cdot E_s} =$$
$$= \frac{R_B \cdot l}{A} \left( \frac{1}{E_{Al}} + \frac{1}{2E_{cu}} + \frac{1}{3E_s} \right) =$$
$$= \frac{R_B \cdot l}{6A} \, \frac{6 E_{cu} E_s + 3 E_{Al} E_s + 2 E_{Al} \cdot E_{cu}}{E_{Al} E_{cu} E_s}$$

III → II

?

$$1 \cdot \Delta T (\alpha_s + \alpha_{cu} + \alpha_{Al}) - \frac{R_B \cdot l}{6A} \cdot \frac{6E_{cu}E_s + 3E_{Al} \cdot E_s + 2E_{Al} \cdot E_{cu}}{E_{Al} \cdot E_{cu} \cdot E_s} = 0$$

$$\Downarrow$$

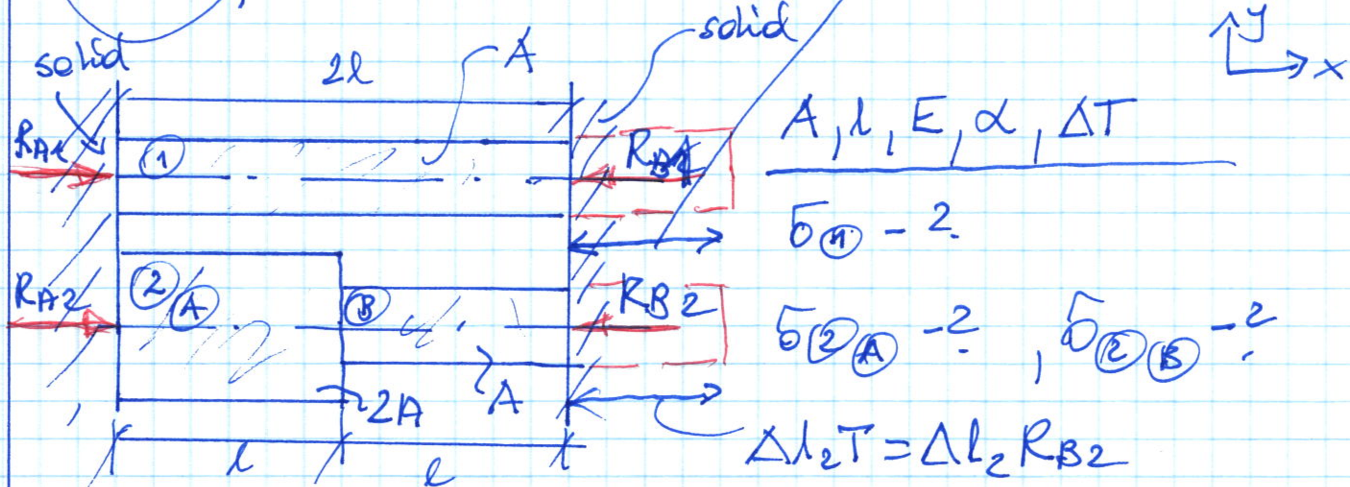
$$R_B \Longrightarrow R_A = -R_B$$

$$\sigma_1 = \frac{R_A}{3A}, \quad \sigma_2 = \frac{R_A}{2A}, \quad \sigma_3 = \frac{R_A}{A}$$

or

$$\sigma_1 = \frac{-R_B}{3A}, \quad \sigma_2 = \frac{-R_B}{2A}, \quad \sigma_3 = \frac{-R_B}{A}$$

Ex. 4 / ćw. 4




$A, l, E, \alpha, \Delta T$

$\sigma_1$ - ?

$\sigma_{2A}$ - ?, $\sigma_{2B}$ - ?

Bar 1

I static eq.

(1) $\Sigma P_{ix} = R_{A1} - R_{B1} = 0 \Rightarrow R_{A1} = R_{B1}$

1× hyperstatic

II Geometric condition

(2) $\Delta l_1 T - \Delta l_1 R_{B1} = 0$

III Physical conditions

$\Delta l_1 T = \alpha \cdot 2l \cdot \Delta T$

$\Delta l_1 R_{B1} = \frac{R_{B1} \cdot 2l}{E \cdot A}$

III $\rightarrow$ II

$$\alpha \cdot 2l \cdot \Delta T - \frac{R_{B1} \cdot 2l}{EA} = 0 \quad \Big| \cdot \frac{EA}{2l}$$

$$R_{B_1} = R_{A1} = \frac{\alpha \cdot \cancel{2l} \cdot \Delta T \cdot EA}{\cancel{2l}} = \alpha \cdot \Delta T \cdot EA$$

$$R_{B1} = R_{A1} = \alpha \cdot \Delta T \cdot EA$$

$$\sigma_{①} = -\frac{R_{A1}}{A} = \frac{\alpha \cdot \Delta T \cdot E \cdot \cancel{A}}{\cancel{A}} = \alpha \Delta T \cdot E$$

Bar 2

I static eq.

① $\sum F_{ix} = R_{A2} - R_{B2} = 0 \Rightarrow \quad R_{A2} = R_{B2}$

1x hyperstatic

II Geometric condition

② $\Delta l_2 T - \Delta l_{2 R_{B2}} = 0$

III Physical conditions

$$\Delta l_2 T = \alpha \cdot l \cdot \Delta T + \alpha \cdot l \cdot \Delta T = 2 \cdot \alpha \cdot l \cdot \Delta T$$

$$\Delta l_{2 R_{B2}} = \frac{R_{B2} \cdot l}{EA} + \frac{R_{B2} \cdot l}{E \cdot 2A} = \frac{R_{B2} l}{EA}\left(1 + \frac{1}{2}\right) = \frac{3 R_{B2} \cdot l}{2EA}$$

III $\rightarrow$ II

$$2\alpha \cdot l \cdot \Delta T - \frac{3R_{B2} \cdot l}{2EA} = 0 \quad \Big| \cdot \frac{2EA}{3l}$$

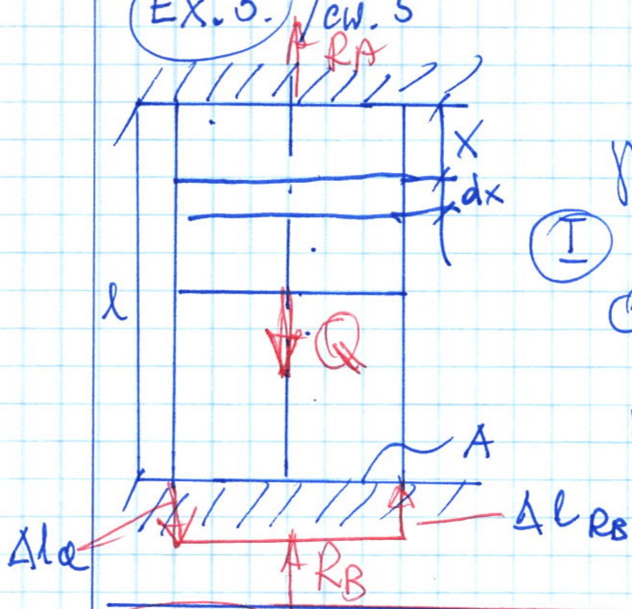
$$R_{B_2} = 2\alpha \cdot \cancel{l} \cdot \Delta T \cdot \frac{2EA}{3\cancel{l}} = \frac{4}{3} \alpha \cdot \cancel{l} \cdot \Delta T \cdot E \cdot A = R_{A2}$$

$$\sigma_{②A} = \frac{R_{A2}}{2A} = \frac{4}{6} \frac{\alpha \cancel{l} \cdot \Delta T \cdot E \cdot \cancel{A}}{\cancel{A}} = \frac{2}{3} \alpha \cancel{l} \cdot \Delta T \cdot E =$$

$$= \frac{2}{3} \alpha \cdot \Delta T \cdot E$$

$$\sigma_{②B} = \frac{R_{A2}}{A} = \frac{4}{3} \alpha \cdot \Delta T \cdot E$$

Ex. 5. / ćw. 5




$$\frac{A,\ l,\ \gamma,\ E}{\sigma(x) - ?}$$

$\gamma$ - specific gravity / specific weight

(I) static eq. ? ?

$$① \ \Sigma P_{iy} = R_A - Q + R_B = 0$$

where $Q = A \cdot l \cdot \gamma$

(II) Geometric condition

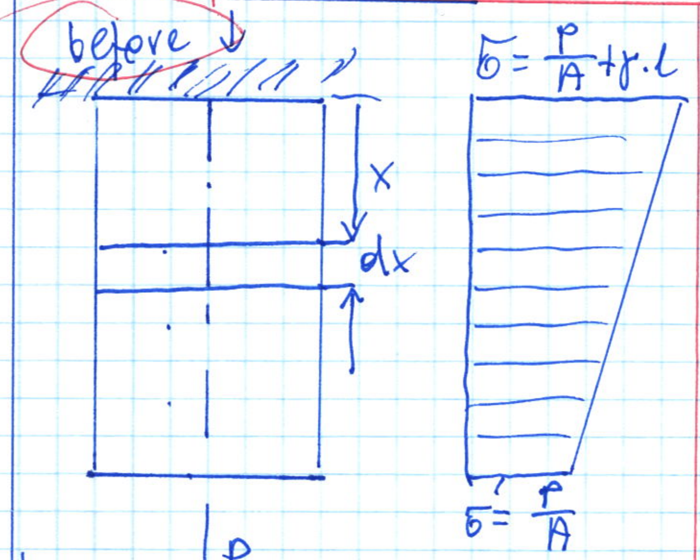
$$② \ \Delta l_Q - \Delta l_{R_B} = 0$$

(III) Physical conditions

$$\Delta l_Q = \frac{Q \cdot l}{2EA} = \frac{\cancel{A} l \cdot \gamma \cdot l}{2E\cancel{A}} = \frac{\gamma l^2}{2E}$$

$$\Delta l_{R_B} = \frac{R_B \cdot l}{EA}$$

(III) → (II)




$$\boxed{\Delta l = \frac{P \cdot l}{EA} + \frac{Q\, l}{2EA}}$$

$$\frac{\gamma l^2}{2E} - \frac{R_B \cdot l}{EA} = 0 \implies R_B = \frac{\gamma \cdot l \cdot A}{2}$$

$$R_A - Q + R_B = 0 = R_A - Al\gamma + \frac{\gamma l A}{2} = 0$$

$$R_A = R_B = \frac{\gamma l A}{2}$$

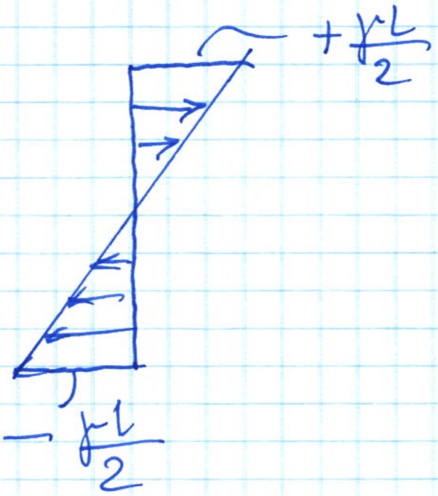
Weight for $x \Rightarrow \int^{x} A \cdot \gamma \cdot dx = A \cdot \gamma x + C = Q(x)$

for $x = 0 \Rightarrow Q(0) = 0 \Rightarrow C = 0$

$$Q(x) = A \cdot \gamma \cdot x$$

$$\sigma(x) = \frac{R_A}{A} - \frac{\cancel{A} \cdot \gamma \cdot x}{\cancel{A}} = \frac{\gamma \cdot l \cdot \cancel{A}}{2\cancel{A}} - \frac{\cancel{A} \cdot \gamma \cdot x}{\cancel{A}}$$

$$\boxed{\sigma(x) = \frac{\gamma \cdot l}{2} - \gamma \cdot x}$$




$$\sigma(x=0) = \frac{\gamma l}{2}$$

$$\sigma(x=l) = -\frac{\gamma l}{2}$$