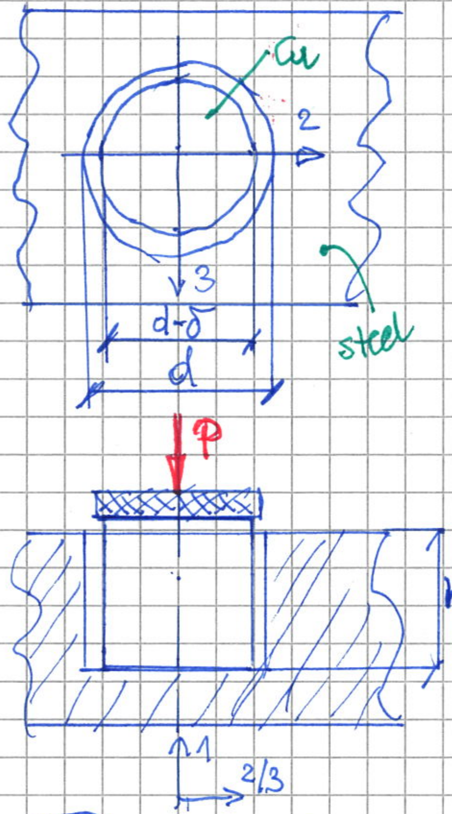
The generalized Hooke's law, examples of exercises

Ex. 1. / ćw. 1.



In the perfectly solid (rigid) plate, a hole of diameter d was made. A cylinder of diameter $d-\delta$ was inserted into the hole. The cylinder was loaded by the force P by a small rigid element.

Determine:

- pressures (stresses) between the cylinder and the wall of hole,
- cylinder volume change

$P = 981$ kN, $d = 0{,}06$ m

$\delta = 0{,}00005$ m, $h = 0{,}08$ m

$E = 1{,}01 \cdot 10^5$ MPa, $\nu = 0{,}33$

$\sigma_1, \sigma_1', \sigma_2', \sigma_{1c}, \Delta V$ – ?

(1.) $$P = P_1 + P_1'$$

P_1 – This force removes δ

P_1' – This force exerts pressure (stress) on the walls of hole

(2) $$\sigma_1 = \frac{P_1}{\frac{\pi d^2}{4}} \qquad \delta = \nu \cdot \varepsilon_1 \cdot d = \nu \frac{\sigma_1}{E} \cdot d \Rightarrow \sigma_1$$

$$\sigma_1 = \frac{\delta \cdot E}{\nu d} = \frac{P_1}{\frac{\pi d^2}{4}} = \frac{4P_1}{\pi d^2} \Rightarrow P_1$$

$$P_1 = \frac{\delta \cdot E \cdot \pi d^2}{4 \nu d} = \frac{\delta E \pi d}{4\nu}$$

$$P_1 = \frac{\pi E \delta d}{4\nu}$$

(3) $$P_1' = P - P_1 = P - \frac{\delta E \pi d}{4\nu}$$

(4) $$\sigma_1' = \frac{P_1'}{\frac{\pi d^2}{4}} = \frac{4P\nu - \delta\pi E d}{4\nu}$$

(5) Once there is contact between the cylinder and the walls of the hole, the force P_1' is responsible for the stresses σ_2' and σ_3' (but $\sigma_2' = \sigma_3'$ circular hole symmetry)

$$\varepsilon_2 = \varepsilon_3 = \frac{1}{E}\left[\sigma_2' - \nu\left(\sigma_3' + \sigma_1'\right)\right] = 0$$

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$$\sigma_2' - \nu\sigma_2' - \nu\sigma_1' = 0 \qquad \sigma_2'(1-\nu) = \nu\cdot\sigma_1'$$

$$\sigma_2' = \sigma_3' = \frac{\nu\cdot\sigma_1'}{1-\nu} = \frac{\cancel{\nu}}{1-\nu}\,\frac{4P\nu - \delta\pi E d}{4\cancel{\nu}}$$

$$\sigma_2' = \sigma_3' = \frac{4P\cdot\nu - \delta\pi E d}{4(1-\nu)}$$

(6) Total stress σ_{1C}

$$\sigma_{1C} = \sigma_1 + \sigma_1' = \frac{\delta\cdot E}{\nu d} + \frac{4P\cdot\nu - \delta\cdot\pi E d}{4\nu} =$$

$$\sigma_{1C} = \frac{4\delta E + 4P\cdot\nu - \delta\cdot\pi E d}{4\nu d}$$

$\sigma_2' = \sigma_3'$

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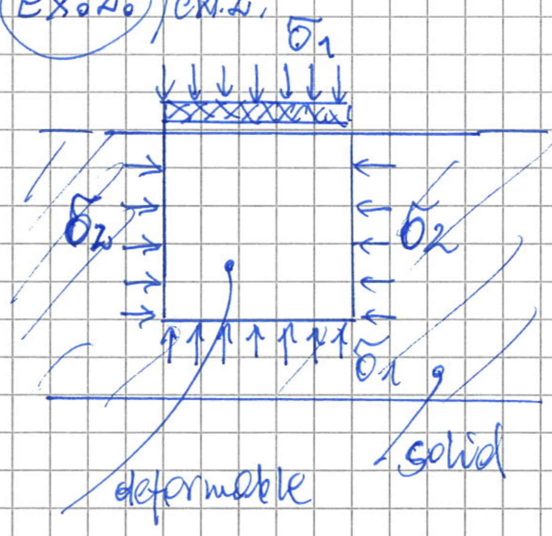
(7) Volume change ΔV

$$\frac{\Delta V}{V_0} = \vartheta \Rightarrow \Delta V = V_0\cdot\vartheta = V_0\cdot\frac{1-2\nu}{E}\left(\sigma_{1C} + \sigma_2' + \sigma_3'\right)$$

$$\Delta V = \frac{\pi d^2}{4}\cdot h\cdot\frac{1-2\nu}{E}\left(\frac{4\delta E + 4P\nu - \delta\pi E d}{4\nu d} + 2\,\frac{4P\nu - \delta\pi E d}{4(1-\nu)}\right)$$

Ex. 2. / cw. 2.



$\sigma_1 = 160\ MPa$, $\nu = 0{,}3$

$E = 1{,}1 \cdot 10^5\ MPa$ $\quad (\sigma_3 = 0)$

$\sigma_2 - ?$, $\varepsilon_1 - ?$, $\varepsilon_3 - ?$

(1) $\varepsilon_2 = 0$!

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu(\sigma_3 + \sigma_1)\right] = \frac{1}{E}(\sigma_2 - \nu\sigma_1) = 0$$

$$\sigma_2 = \nu \cdot \sigma_1$$

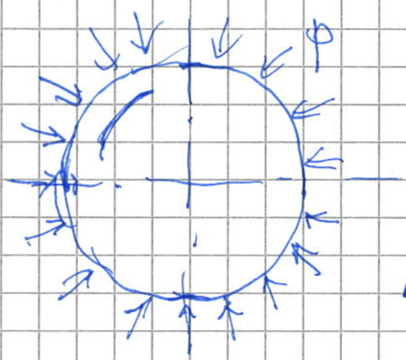
$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right] = \frac{1}{E}(\sigma_1 - \nu \cdot \nu \cdot \sigma_1) =$$

$$\varepsilon_1 = \frac{1-\nu^2}{E} \cdot \sigma_1$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_2 + \sigma_1)\right] = \frac{1}{E}\left[-\nu(\nu\sigma_1 + \sigma_1)\right]$$

$$\varepsilon_2 = \frac{1}{E}\left[-\nu\sigma_1(1+\nu)\right] = \frac{-\nu(1+\nu)}{E} \cdot \sigma_1$$

Ex. 3. / cw. 3



The ball of radius r is subjected to pressure.

Calculate the change in ball volume

$r = 15$, $p = 8\ MPa$, $E = 2 \cdot 10^5\ MPa$

$$\vartheta = \frac{-p}{K} = \frac{-p}{\frac{E}{3(1-2\nu)}}$$

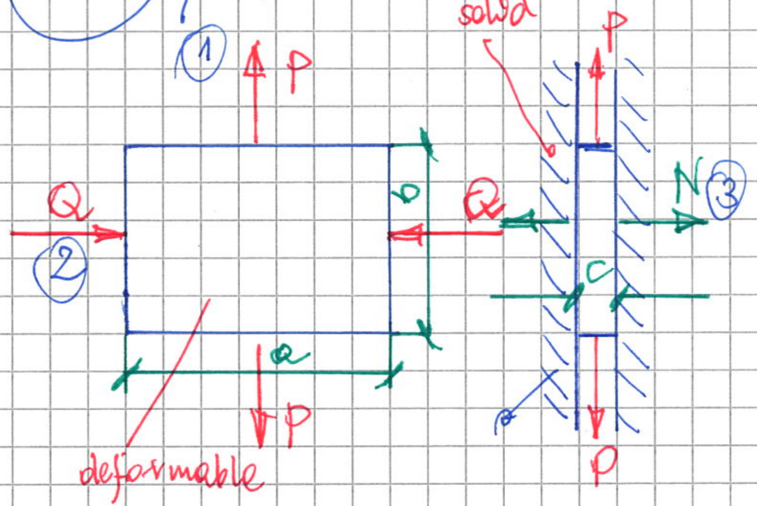
$$\Delta V = V_0 \cdot \vartheta = \frac{4}{\cancel{3}}\pi r^3 \cdot \frac{-p\,\cancel{3}(1-2\nu)}{E} = \frac{-4\pi r^3 \cdot p\,(1-2\nu)}{E}$$

Ex. 4. | ćw. 4

a, b, c, ν, E, P, Q

$Q > P$

$N - ?$, $\Delta V - ?$



$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_2 + \sigma_1)\right] = 0 \,!$$

$$\sigma_3 - \nu(\sigma_2 + \sigma_1) = 0 \qquad \sigma_3 = \frac{-N}{ab}, \quad \sigma_2 = \frac{-Q}{b \cdot c}, \quad \sigma_1 = \frac{P}{a \cdot c}$$

$$\sigma_3 = -\nu(\sigma_2 + \sigma_1)$$

$$-\frac{N}{a \cdot b} = -\nu\left(\frac{-Q}{b \cdot c} + \frac{P}{a \cdot c}\right) = -\nu\frac{(-Qa + P \cdot b)}{abc} \quad \Big| \cdot (-1)$$

$$\frac{N}{a \cdot b} = \nu\frac{(Pb - Qa)}{abc} \quad \Big| \cdot ab$$

$$N = \frac{\nu \cdot \cancel{ab}(Pb - Qa)}{\cancel{ab}c} = \frac{\nu}{c}(Pb - Qa)$$

$$\Delta V = V_0 \cdot \vartheta = a \cdot b \cdot c \cdot \frac{1-2\nu}{E}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\Delta V = a \cdot b \cdot c \cdot \frac{1-2\nu}{E}\left(\frac{P}{ac} - \frac{Q}{b \cdot c} - \frac{N}{ab}\right) =$$

$$\Delta V = \frac{(1-2\nu)\cancel{abc}}{E} \cdot \frac{(Pb - Qa - Nc)}{\cancel{ab \cdot c}}$$

$$\Delta V = \frac{1-2\nu}{E}(P \cdot b - Q \cdot a - N \cdot c)$$