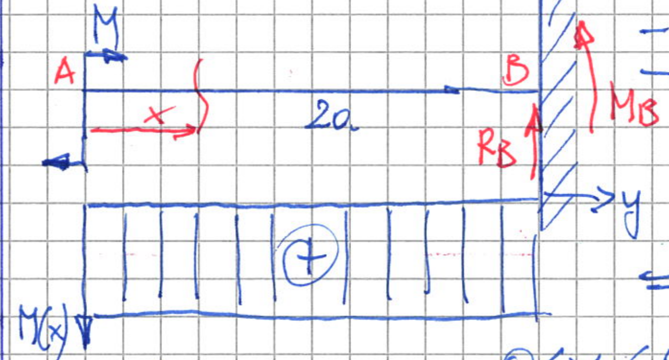
Bending / Zginanie

- Pure bending / czyste zginanie
- Bending with shear / zginanie ze ścinaniem



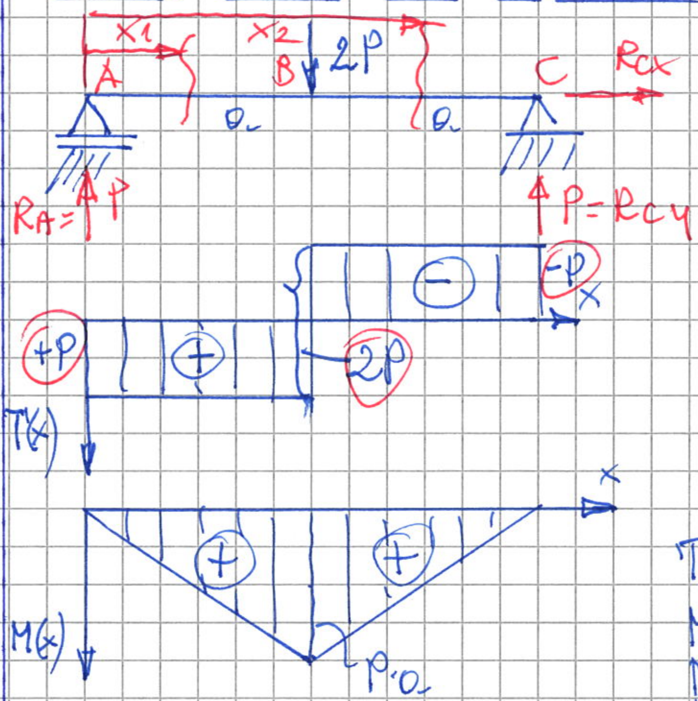
① $\sum P_{iy} = R_B = 0$

② $\sum M_{iB} = M - M_B = 0 \Rightarrow M_B = M = const$

$\Longrightarrow$ pure bending

$M(x) = M = const$ $\quad 0 \le x \le 2a$

$$T(x) = \frac{dM(x)}{dx} = 0$$



Bending with shear

① $\sum P_{ix} = R_{Cx} = 0$

② $\sum P_{iy} = R_A - 2P + R_{Cy} = 0$

③ $\sum M_i^C = R_A \cdot 2a - 2P \cdot a = 0$

$\hookrightarrow$ $R_A = P$

$\rightarrow$ $P - 2P + R_{Cy} = 0 \Rightarrow R_{Cy} = P$

$0 \le x_1 \le a$	$a \le x_2 \le 2a$
$T_1(x) = +P$	$T_2(x) = +P - 2P = -P$
$M_1(x) = P \cdot x$	$M_2(x) = R_A \cdot x - 2P(x-a) =$
$M_1(0) = 0$	$= P \cdot x - 2P \cdot x + 2P \cdot a =$
$M_1(a) = P \cdot a$	$= -P \cdot x + 2P \cdot a$
	$M_2(a) = P \cdot a$
	$M_2(2a) = 0$

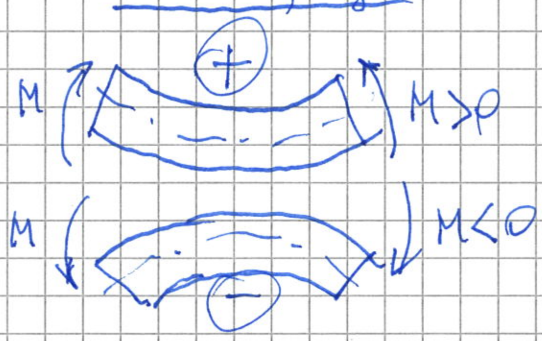
$M_1(x) = P \cdot x$

$$T_1(x) = \frac{dM_1(x)}{dx} = P$$

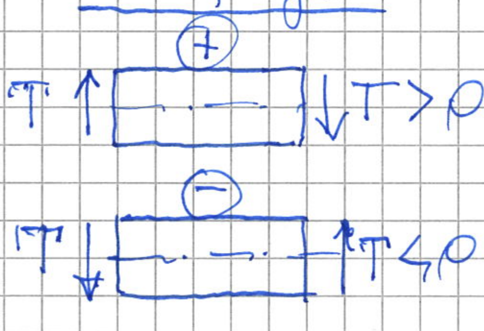
$M_2(x) = -P \cdot x + 2Pa$

$$T_2(x) = \frac{dM_2(x)}{x} = -P$$

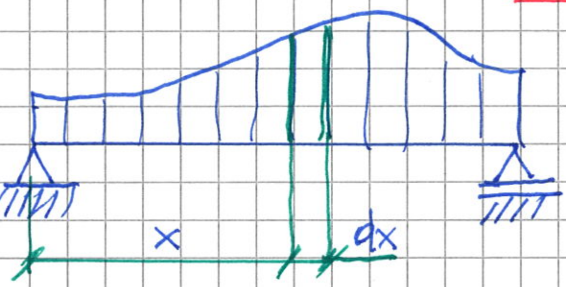
Moment, sign

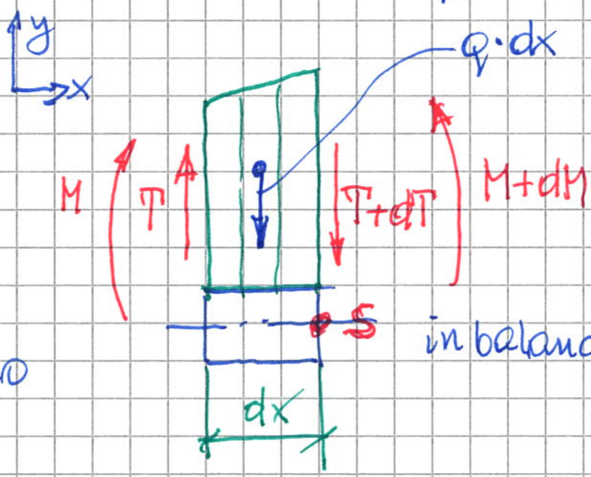


Shear, sign



Schwedler - Żurawski equations





in balance

① $\Sigma P_{iy} = T - q \cdot dx - (T + dT) = 0$

$\cancel{T} - q \cdot dx - \cancel{T} - dT = 0$

$$\boxed{\frac{dT(x)}{dx} = -q(x)}$$

② $\Sigma M_i^S = M + T \cdot dx - q \cdot dx \cdot \frac{dx}{2} - (M + dM) = 0$

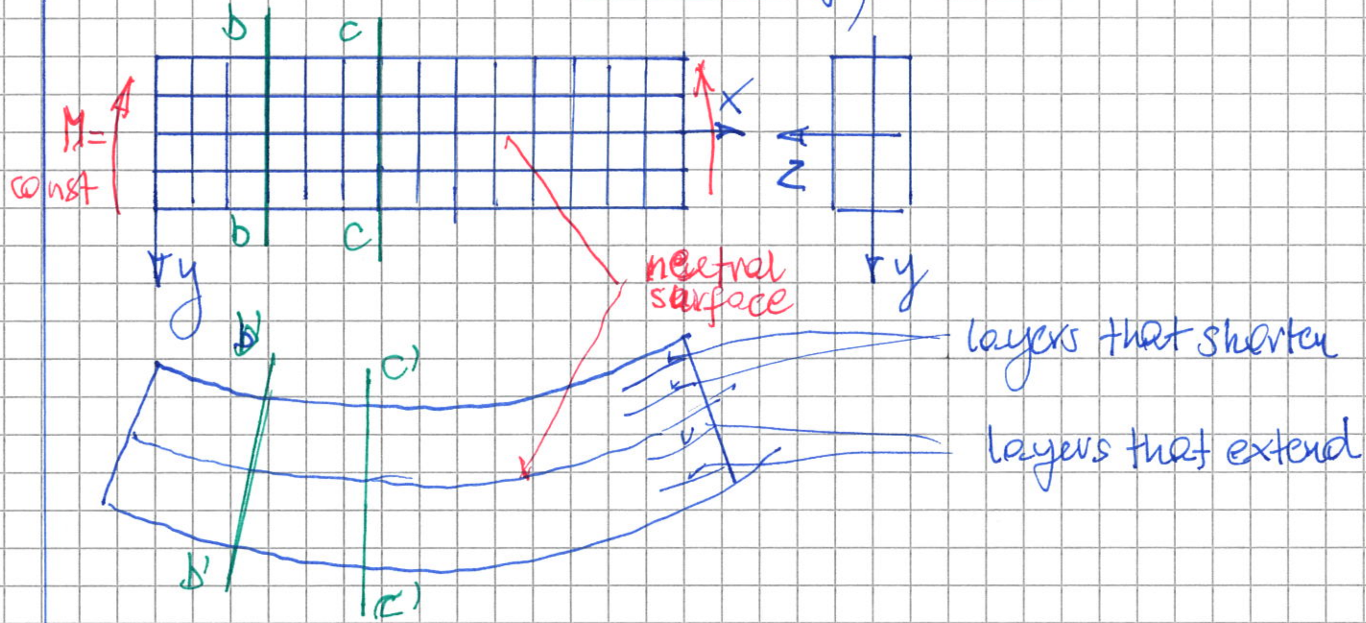
$\cancel{M} + T \cdot dx - q \cdot dx \cdot \frac{dx}{2} - \cancel{M} - dM = 0$

$q \cdot dx \cdot \frac{dx}{2} \approx 0$

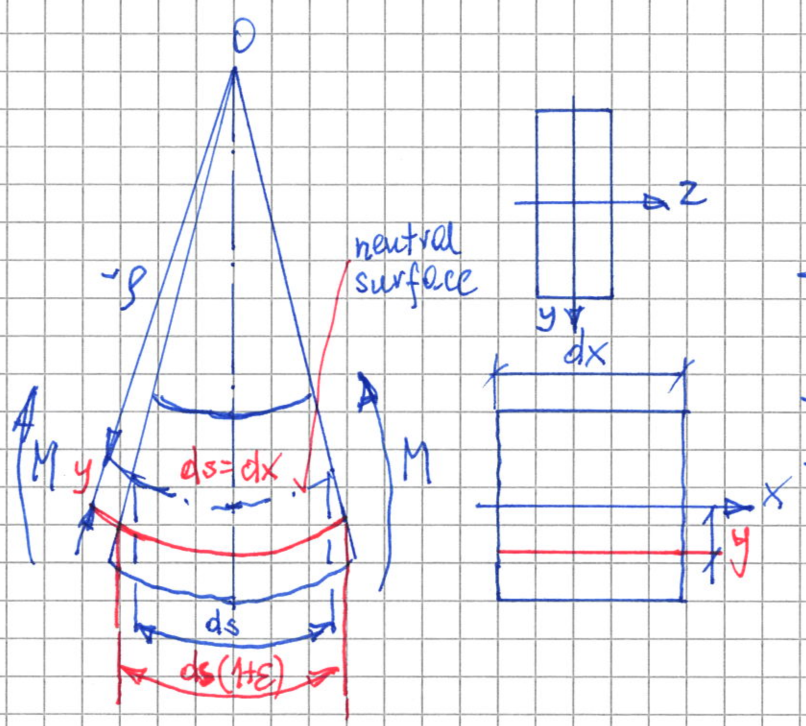
$$\boxed{\frac{dM(x)}{dx} = T(x)}$$

Sch-Ż. equations

Pure bending, strains and stresses



only normal stresses,
because we have only
tension and compression

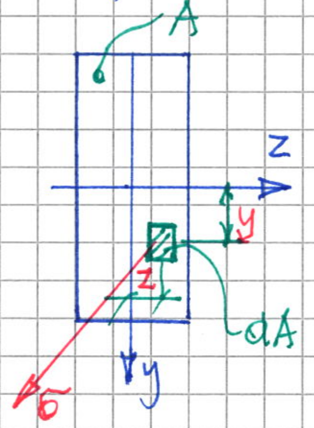


Assumptions:

- the beam section is flat before and after deformation
- exists neutral surface
- only normal stresses

$ds = dx$ = element before deformation

$ds(1+\varepsilon)$ – element after deformation



for the layer distant by y from the neutral surface

$$\frac{ds(1+\varepsilon)}{-\rho+y} = \frac{ds}{-\rho} \Rightarrow \frac{1+\varepsilon}{-\rho+y} = \frac{1}{-\rho}$$

$$-\rho(1+\varepsilon) = -\rho + y$$

$$\cancel{-\rho} - \rho\cdot\varepsilon = \cancel{-\rho} + y$$

$$\boxed{\varepsilon = \frac{-y}{\rho}}$$

(1) $\sum P_{ix} = 0 \qquad \int_A \sigma\cdot dA = 0$

(2) $\sum M_{iy} = 0 \qquad \int_A \sigma\cdot dA\cdot z = 0$

(3) $\sum M_{iz} = 0 \qquad \int_A \sigma\cdot dA\cdot y - M = 0$

for elastic region ⇒ Hooke's law

$$\varepsilon = \frac{\sigma}{E} \Rightarrow \sigma = \varepsilon\cdot E = E\cdot\frac{-y}{\rho} = -\frac{E}{\rho}\cdot y \Rightarrow (3)$$

$$\int_A -\frac{E}{\rho}\cdot y\cdot y\cdot dA - M = 0 \Rightarrow -\frac{E}{\rho}\int_A y^2\cdot dA = M$$

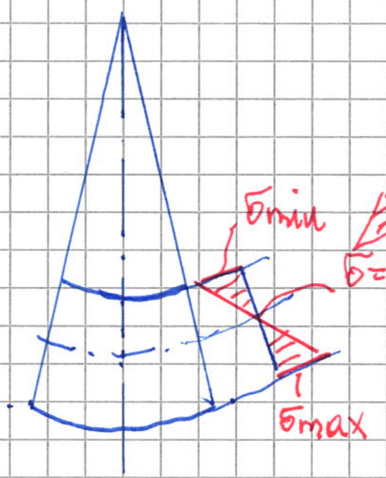
$$\int_A y^2\, dA = I_z$$

$$-\frac{E}{\rho} I_z = M \Rightarrow \frac{1}{\rho} = -\frac{M}{E\cdot I_z}$$

$$\sigma = \varepsilon \cdot E = -\frac{E}{\rho}\cdot y \Rightarrow -\frac{1}{\rho} = \frac{\sigma}{E\cdot y}$$

$$\frac{\sigma}{E\cdot y} = \frac{M}{E\cdot I_z} \Rightarrow \boxed{\sigma = \frac{M\cdot y}{I_z}}$$

$$\boxed{\frac{I_z}{y_{max}} = W_z}$$



$$I_z = \frac{bh^3}{12}$$

$$W_z = \frac{bh^2}{6}$$

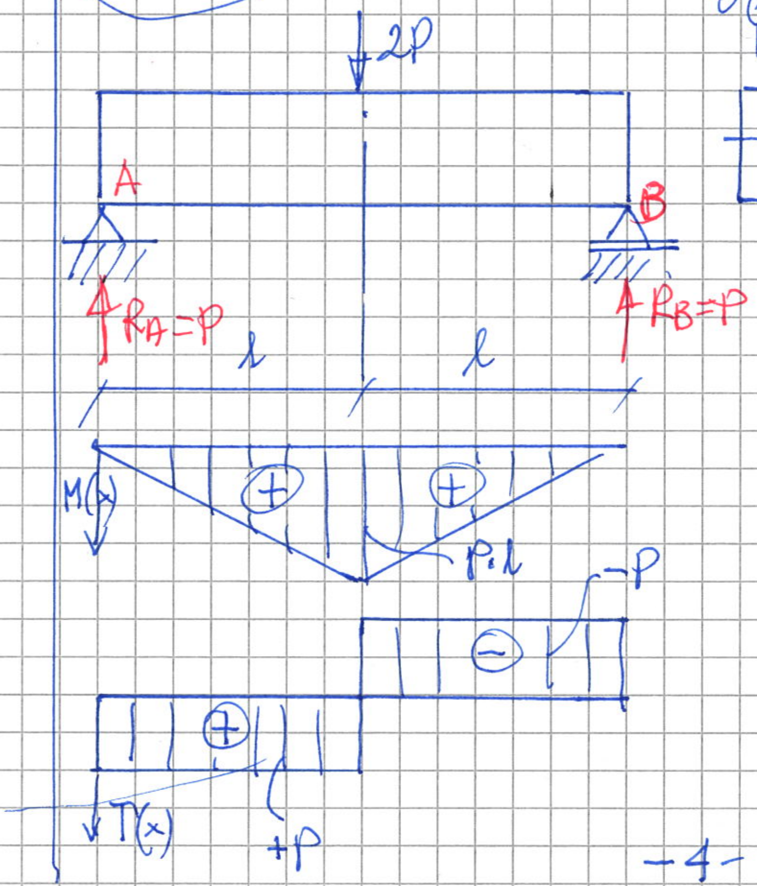
Constant stress beam

$$\frac{M(x)}{W_z(x)} = \sigma(x) = \text{const} \;!$$

Ex. A.

$\sigma(x) = \sigma_{cr} = \text{const}$, $2P$, $h = \text{const}$, l

$b(x) - ?$



① $\sum P_{iy} = R_A + R_B - 2P = 0$

② $\sum M_i^A = 2P\cdot l - R_B \cdot 2l = 0$

$R_B = P \qquad R_A = P$

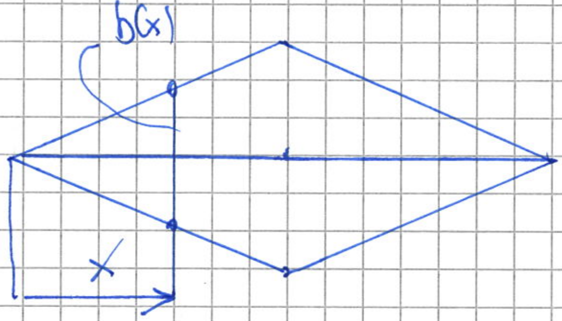
$$W_z(x) = \frac{b(x)\cdot h^2}{6}$$

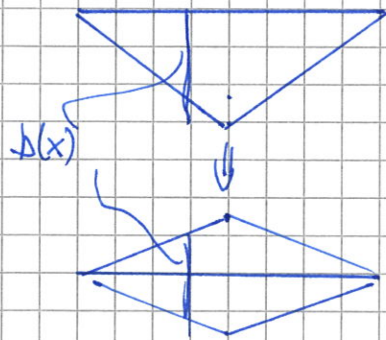
$$W_z(x) = \frac{M(x)}{\sigma_{cr}} = \frac{P\cdot x}{\sigma_{cr}}$$

$$\frac{b(x)\cdot h^2}{6} = \frac{P\cdot x}{\sigma_{cr}}$$

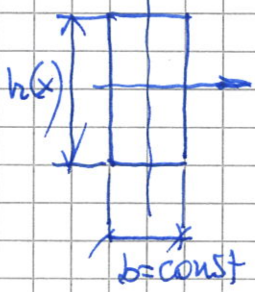
$$b(x) = \frac{6\cdot P\cdot x}{\sigma_{cr}\cdot h^2} = \frac{6P}{\sigma_{cr}\cdot h^2}\cdot x$$

linear function





Ex. B



$$\frac{b=\text{const},\ P,\ l,\ \sigma=\text{const}=\sigma_{cr}}{h(x)-?}$$

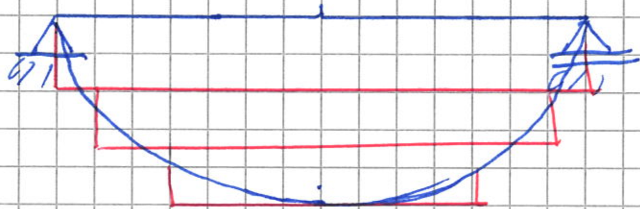
$$M = P\cdot x, \quad W_z(x) = \frac{M(x)}{\sigma_{cr}}$$

$$W_z(x) = \frac{b\cdot h^2(x)}{6}$$

$$W_z(x) = \frac{P\cdot x}{\sigma_{cr}}$$

$$\frac{b\cdot h^2(x)}{6} = \frac{P\cdot x}{\sigma_{cr}} \Rightarrow h^2(x) = \frac{6P}{\sigma_{cr}\cdot b}\cdot x$$

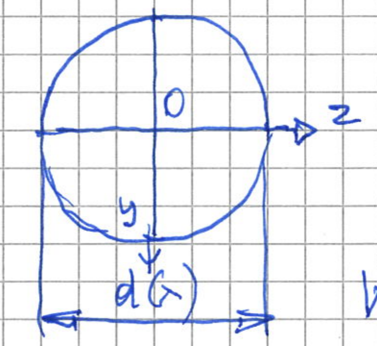
$$h(x) = \sqrt{\frac{6P}{\sigma_{cr}\cdot b}}\cdot\sqrt{x}$$



Ex. C

$$P,\ l,\ \sigma = const = \sigma_{cr}$$

$$d(x) - ?$$



$$I_0 = \frac{\pi d^4}{32}$$

$$I_z = I_y = \frac{\pi d^4}{64}$$

$$W_z = W_y = \frac{\pi d^3}{32}$$

$$W_z(x) = \frac{\pi d^3(x)}{32} \ ; \quad W_z(x) = \frac{M(x)}{\sigma_{cr}} = \frac{P \cdot x}{\sigma_{cr}}$$

$$\frac{\pi d^3(x)}{32} = \frac{P \cdot x}{\sigma_{cr}} \Rightarrow d^3(x) = \frac{32P}{\pi \sigma_{cr}} \cdot x$$

$$d(x) = \sqrt[3]{\frac{32P}{\pi \sigma_{cr}}} \cdot \sqrt[3]{x}$$