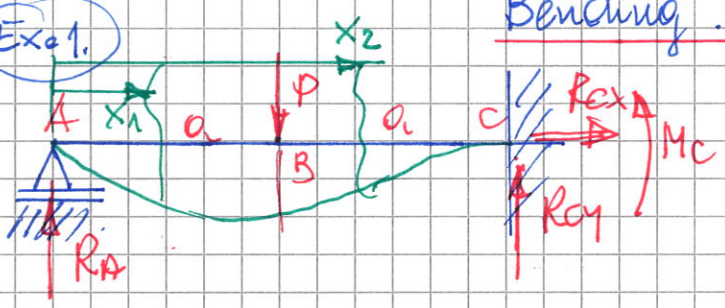
Bending. Hyperstatic beams.

Ex. 1.



P, a, EI = const

R_A, R_{Cx}, R_{Cy}, M_C – ?

(4 reactions)

(I) static eqs.

(1) $\sum P_{ix} = R_{Cx} = 0$

(2) $\sum P_{iy} = R_A - P + R_{Cy} = 0$

(3) $\sum M_i^C = R_A \cdot 2a - P \cdot a - M_C = 0$

4 reactions – 3 static equations $\Rightarrow$ 1× hyperstatic beam

(II) Differential equation

$0 \le x_1 \le a$

$M_1(x) = R_A \cdot x$

$EI \frac{d^2y}{dx^2} = -M_1(x) = -R_A x$

(**) $EI \frac{dy(x)}{dx} = -R_A \frac{x^2}{2} + C_1$

(*) $EI\, y(x) = -R_A \frac{x^3}{6} + C_1 x + D_1$

C_1, D_1 – ?

$a \le x_2 \le 2a$

$M_2(x) = R_A \cdot x - P(x-a) = R_A \cdot x - P \cdot x + P \cdot a$

$M_2(x) = (R_A - P) \cdot x + P \cdot a$

$EI \frac{d^2y(x)}{dx^2} = -(R_A - P) \cdot x - P \cdot a$

$EI \frac{dy(x)}{dx} = -(R_A - P) \frac{x^2}{2} - P \cdot a \cdot x + C_2$ (**)(*)(*)

$EI\, y(x) = -(R_A - P) \frac{x^3}{6} - P \cdot a \frac{x^2}{2} + C_2 x + D_2$ (4 (*))

C_2, D_2 – ?

C_1, D_1, C_2, D_2 – 4 unknown constants

4 unknown reactions + 4 unknown constants = 8 !

(III) Geometrical conditions (boundary conditions)

(1) $x = 0$, $y = 0$

(2) $x = a$, $v_L = v_R$

(3) $x = a$, $y_L = y_R$

(4) $x = 2a$, $v = 0$

(5) $x = 2a$, $y = 0$

L = R

8 = 8

4 reactions + 4 unknown constants =

= 3 static eqs + 5 boundary cond.

than a solution is possible!

① $x=0$, $y=0$ (*)

$EI\,y(0)=0=-R_A\frac{0^3}{6}+C_1\cdot 0+D_1 \Rightarrow D_1=0$

② $x=a$, $v_L=v_R$ (**) = (**)(*)

$-R_A\frac{a^2}{2}+C_1=-(R_A-P)\cdot\frac{a^2}{2}-P\cdot a\cdot a+C_2$

③ $x=a$, $y_L=y_R$ (*) = 4(*)

$-R_A\frac{a^3}{6}+C_1\cdot a+D_1=-(R_A-P)\frac{a^3}{6}-P\cdot a\frac{a^2}{2}+C_2\cdot a+D_2$

④ $x=2a$, $v=0$ (***)

$-(R_A-P)\frac{(2a)^2}{2}-P\cdot a\cdot 2a+C_2=0$

⑤ $x=2a$, $y=0$ 4(*)

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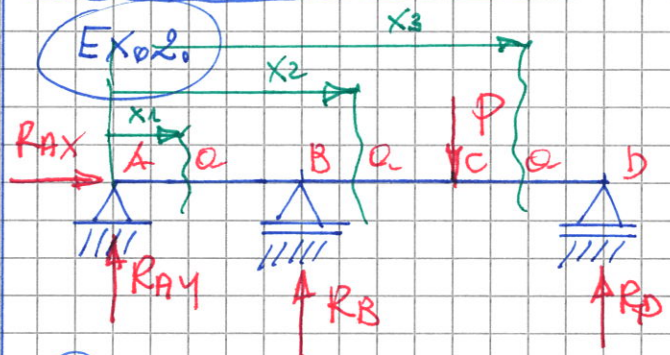
① ② ③ → static eqs. + ① ② ③ ④ ⑤ → b. conditions $\Rightarrow$

R_A, R_{Cx}, R_{Cy}, M_C
+
C_1, D_1, C_2, D_2

8

finish this ex. as homework



$P, a, EI=\text{const}$

R_{AX}, R_{AY}, R_B, R_D — ?

4 reactions

I Static eqs.

① $\sum P_{ix}=R_{AX}=0$

② $\sum P_{iy}=R_{AY}+R_B-P+R_D=0$

③ $\sum M_{iA}=-R_B\cdot a+P\cdot 2a-R_D\cdot 3a=0$

3 static eqs.

4 reactions − 3 st. eqs =
= 1x hyperstatic object !!

c

II Diff. eqs.

$0 \le x_1 \le a$

$M_1(x) = R_{Ay} \cdot x$

$EI \frac{d^2y(x)}{dx^2} = -R_A \cdot x$

$EI \frac{dy(x)}{dx} = -R_A \frac{x^2}{2} + C_1$ (**)

$EI\, y(x) = -R_A \frac{x^3}{6} + C_1 x + D_1$ (*)

$a \le x_2 \le 2a$

$M_2(x) = R_{Ay} \cdot x + R_B(x-a) = R_{Ay} \cdot x + R_B \cdot x - R_B \cdot a$

$EI \frac{d^2y(x)}{dx^2} = -R_{Ay} \cdot x - R_B x + R_B \cdot a$

$EI \frac{dy(x)}{dx} = -R_{Ay} \frac{x^2}{2} - R_B \frac{x^2}{2} + R_B \cdot a \cdot x + C_2$ (3*)

$EI\, y(x) = -R_{Ay} \frac{x^3}{6} - R_B \frac{x^3}{6} + R_B \cdot a \frac{x^2}{2} + C_2 x + D_2$ (4*)

$2a \le x_3 \le 3a$

$M_3(x) = R_A \cdot x + R_B(x-a) - P(x-2a)$

$M_3(x) = R_A \cdot x + R_B \cdot x - R_B \cdot a - P \cdot x + 2Pa$

$EI \frac{d^2y(x)}{dx^2} = -R_A \cdot x - R_B \cdot x + R_B \cdot a + P \cdot x - 2Pa =$

$(P - R_A - R_B) x + (R_B - 2P) \cdot a$

$EI \frac{dy(x)}{dx} = (P - R_A - R_B) \frac{x^2}{2} + (R_B - 2P) a \cdot x + C_3$ (5*)

$EI\, y(x) = (P - R_A - R_B) \frac{x^3}{6} + (R_B - 2P) a \frac{x^2}{2} + C_3 x + D_3$ (6*)

$C_1, D_1, C_2, D_2, C_3, D_3 - ?$

6

III Boundary conditions

(1) $x = 0$, $y = 0$ (*)

(2) $x = a$, $v_L = v_R$ (**) = 3(*)

(3) $x = a$, $y_L \Rightarrow 0$ (*)

(4) $x = a$, $y_R \Rightarrow 0$ 4(*)

(5) $x = 2a$, $v_L = v_R$ 3(*) = 5(*)

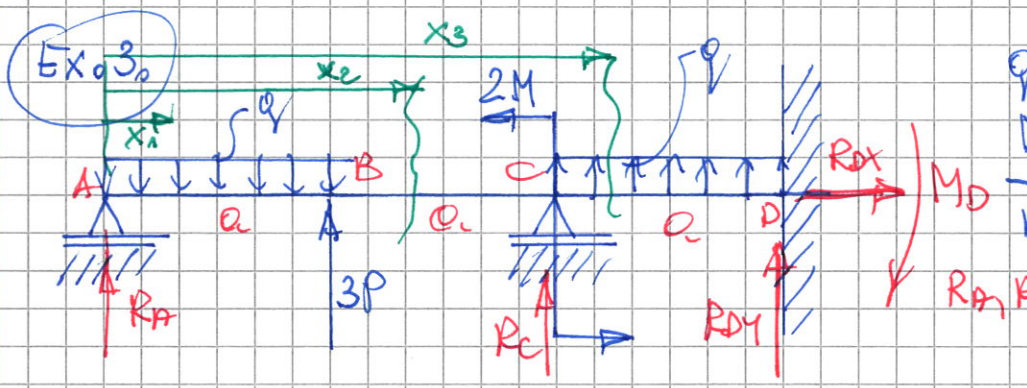
(6) $x = 2a$, $y_L = y_R$ 4(*) = 6(*)

(7) $x = 3a$, $y = 0$ (*)

L = 3 static eqs + 7 boundary cond = 10 !

R = 4 reactions + 6 constants = 10 !

$L = R$



$q, a, P = qa$

$M = qa^2$, $EI = const$

reactions - 2.

$R_A, R_C, R_{Dx}, R_{Dy}, M_D$ - ?

(I) Static eqs.

(1) $\sum R_{ix} = R_{Dx} = 0$

(2) $\sum P_{iy} = R_A - qa + 3P + R_C + qa + R_{Dy} = 0$

(3) $\sum M_i^D = R_A \cdot 3a - qa \cdot \frac{5}{2}a + 3P \cdot 2a - 2M + R_C \cdot a + \frac{qa^2}{2} + M_D = 0$

5 reactions - 3 static eqs. $\Rightarrow$ 2x hyperstatic beam !

(II) Diff. eqs.

$0 \le x_1 \le a$

$M_1(x) = R_A \cdot x - \frac{qx^2}{2}$

$EI \frac{d^2y(x)}{dx^2} = -R_A x + \frac{qx^2}{2}$

(*) $EI \frac{dy(x)}{dx} = -R_A \frac{x^2}{2} + \frac{qx^3}{6} + C_1$

$EI\, y(x) = -R_A \frac{x^3}{6} + \frac{qx^4}{24} + C_1 \cdot x + D_1$ (*)

$a \le x_2 \le 2a$

$M_2(x) = R_A \cdot x - qa\left(x - \frac{a}{2}\right) + 3P(x-a)$

$= R_A \cdot x - qa \cdot x + \frac{qa^2}{2} + 3P \cdot x - 3P \cdot a$

$= (R_A - qa + 3P)x + \frac{qa^2}{2} - 3P \cdot a$

$EI \frac{d^2y(x)}{dx^2} = -(R_A - qa + 3P) \cdot x - \frac{qa^2}{2} + 3P \cdot a$

$EI \frac{dy(x)}{dx} = -(R_A - qa + 3P)\frac{x^2}{2} - \frac{qa^2 x}{2} + 3P \cdot a \cdot x + C_2$ 3(*)

$EI\, y(x) = -(R_A - qa + 3P)\frac{x^3}{6} - \frac{qa^2 x^2}{4} + 3Pa \cdot \frac{x^2}{2} + C_2 \cdot x + D_2$ 4(*)

$2a \le x_3 \le 3a$

$M_3(x) = R_A \cdot x - qa\left(x - \frac{a}{2}\right) + 3P(x-a) - 2M + R_C(x-2a) + \frac{q(x-2a)^2}{2}$

$M_3(x) = R_A \cdot x - qa \cdot x + \frac{qa^2}{2} + 3P \cdot x - 3Pa - 2M + R_C \cdot x - 2R_C \cdot a + \frac{q}{2}(x^2 - 4ax + 4a^2)$

$$M_3(x) = (R_A - qa + 3P - R_C)\cdot x + \frac{qa^2}{2} - 3P\cdot a - 2R_C\cdot a + \frac{q}{2}(x^2 - 4ax + 4a^2)$$

$$EI\frac{d^2y(x)}{dx^2} = -M_3(x) = -(R_A - qa + 3P - R_C)x - \frac{qa^2}{2} + 3Pa + 2R_C a - \frac{q}{2}(x^2 - 4ax + 4a^2)$$

$$EI\frac{dy(x)}{dx} = -(R_A - qa + 3P - R_C)\frac{x^2}{2} - \frac{qa^2}{2}x + 3Pax + 2R_C ax - \frac{q}{2}\left(\frac{x^3}{6} - 4a\frac{x^2}{2} + 4a^2 x\right) + C_3 \quad 5\text{(*)}$$

$$EIy(x) = -(R_A - qa + 3P - R_C)\frac{x^3}{6} - \frac{qa^2}{2}\frac{x^2}{2} + 3P\frac{x^2}{2} + 2R_C a\frac{x^2}{2} - \frac{q}{a}\left(\frac{x^4}{24} - 4a\frac{x^3}{6} + 4a^2\frac{x^2}{2}\right) + C_3 x + D_3 \quad 6\text{(*)}$$

$C_1, D_1, C_2, D_2, C_3, D_3 - ?$

6

III Boundary conditions

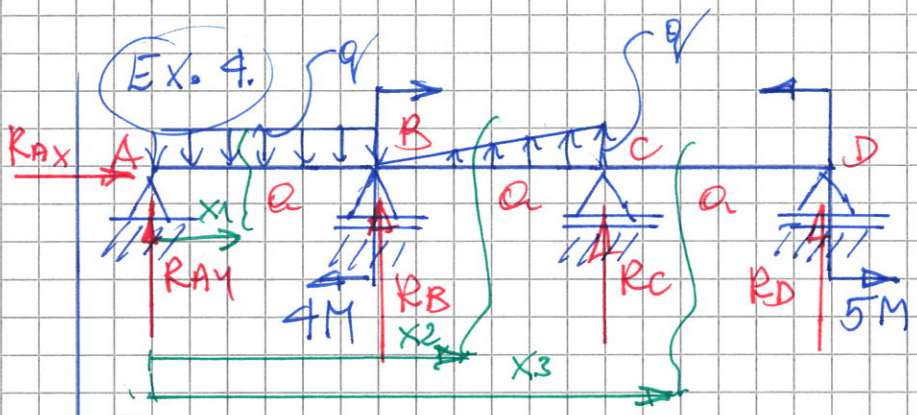
1. $x = 0$, $y = 0$ (*)
2. $x = a$, $\vartheta_L = \vartheta_R$ (*)(*) = 3(*)
3. $x = a$, $y_L = y_R$ (*) = 4(*)
4. $x = 2a$, $\vartheta_L = \vartheta_R$ 3(*) = 5(*)
5. $x = 2a$, $y_L \Rightarrow 0$ 4(*)
6. $x = 2a$, $y_R \Rightarrow 0$ 6(*)
7. $x = 3a$, $\nu = 0$ 5(*)
8. $x = 3a$, $y = 0$ 6(*)

L = 3 static eqs + 8 b. conditions = 11 !

R = 5 reactions + 6 constants = 11 !

$L = R$

Ex. 4.



$q, a, M = qa^2, EI = const$

reactions - ?

$R_{AX}, R_{AY}, R_B, R_C, R_D$

(2x hyperstatic beam)

(I) Static eqs.

(1) $\sum P_{ix} = R_{AX} = 0$

(2) $\sum P_{iy} = R_{AY} - qa + R_B + \frac{1}{2}qa + R_C + R_D = 0$

(3) $\sum M_{iA} = \frac{qa^2}{2} + 4M - R_B \cdot a - \frac{1}{2}qa \cdot \frac{5}{3}a - R_C \cdot 2a - R_D \cdot 3a - 5M = 0$

5 reactions - 3 static eqs. $\Rightarrow$ 2x hyperstatic

(II) Diff. eqs.

$0 \le x_1 \le a$

$a \le x_2 \le 2a$

$2a \le x_3 \le 3a$

$\Rightarrow$ please complete as homework!

$\Downarrow$

$C_1, D_1, C_2, D_2, C_3, D_3 \Rightarrow 6$

(III) Boundary conditions

(1) $x = 0, \; y = 0$

(2) $x = a, \; \vartheta_L = \vartheta_R$

(3) $x = a, \; y_L = y_R$

(4) $x = a, \; y_R = 0$

(5) $x = 2a, \; \vartheta_L = \vartheta_R$

(6) $x = 2a, \; y_L = 0$

(7) $x = 2a, \; y_R = 0$

(8) $x = 3a, \; y = 0$

$\Rightarrow$ please connect the boundary conditions to the differential equations

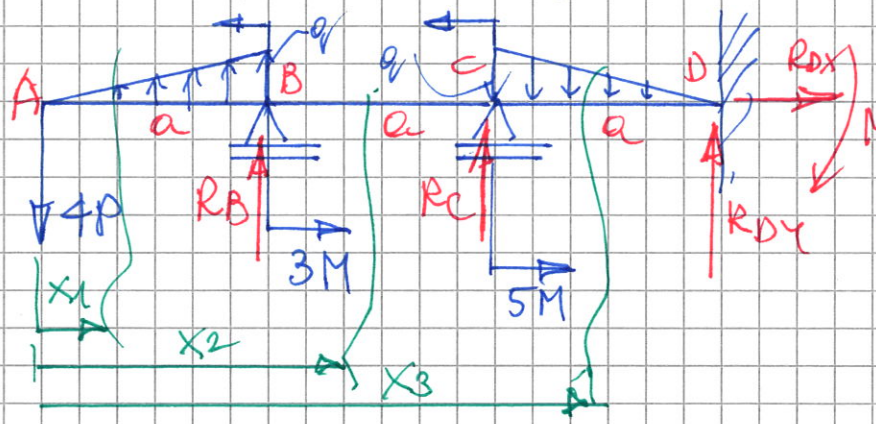
-6-

L = 3 static eqs + 8 b. conditions $\Rightarrow$ M !

R = 5 reactions + 6 constants $\Rightarrow$ M !

$L = R$

Ex. 5. Homework !



$q, a, P = qa$,

$M = qa^2$

$EI = \text{const}$

$R_B, R_C, R_{DX}, R_{DY}, M_D$ - ?

y_A - ?

5-3 $\Rightarrow$ 2x hyperstatic beam !