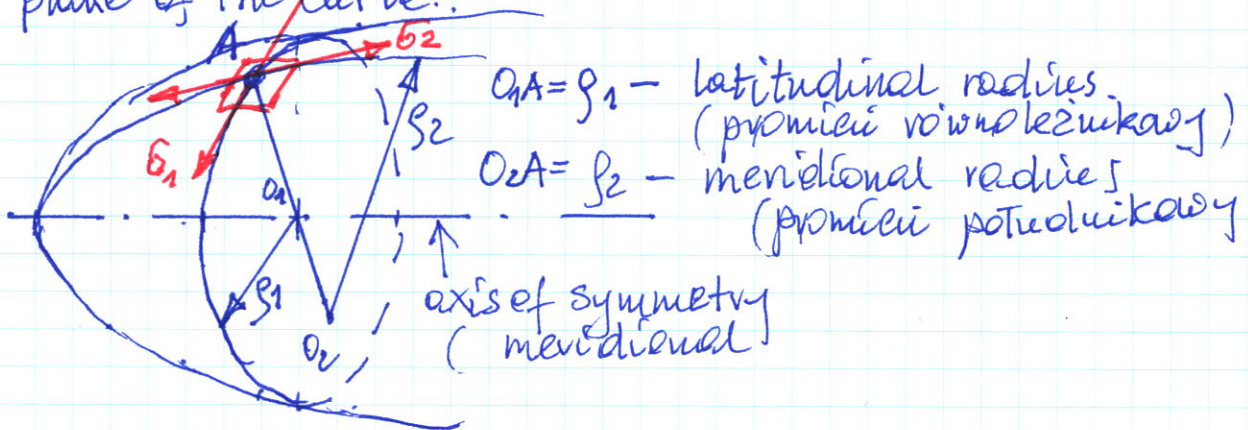


Thin-walled vessels

(zbiorniki cienkościenne)

Thin-walled pressure vessels such as cylindrical, spherical, conical, or toroidal shells subject to internal or external pressure from a gas or a liquid.

The shell of revolution is formed by rotating a plane curve ~~of~~ (the meridian) about an axis lying in the plane of the curve.



$\frac{1}{r_1}$ - latitudinal curvature

$\frac{1}{r_2}$ - meridional curvature

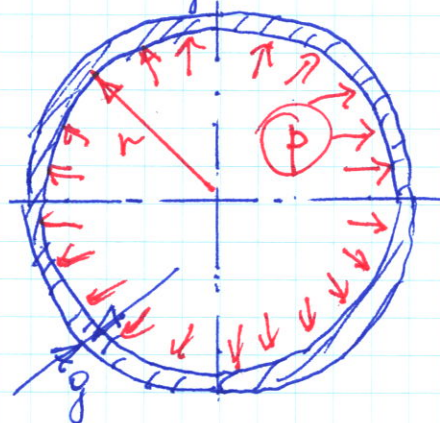
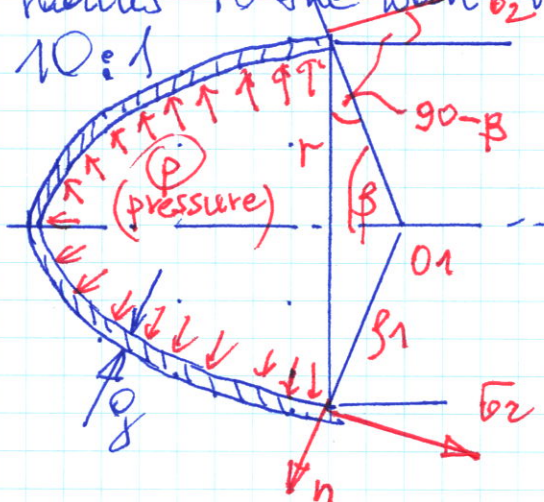
σ_1 - latitudinal stress (hoop or circumferential)

σ_2 - meridional stress

Examples of pressure vessels

- compressed tanks
- pipelines
- water tower
- balloon
- airplanes
- inflatable boats

Thin-walled structures if ratio of the inside radius to the wall ~~σ_2~~ thickness is greater than about 10:1



The pressure p acts on the cut-off part of the vessel, giving the force P , where

$$P = p \cdot F, \text{ where } F - \text{projection of the cut-off part of the vessel}$$

$$F = \pi \cdot r^2, \text{ where } r = \rho_1 \cdot \sin \beta$$

and substitute

$$P = p \cdot F = p \cdot \pi r^2 = p \cdot \pi \cdot \rho_1^2 \sin^2 \beta$$

The pressure force P is balanced by the force in the vessel wall (F_{wall}), i.e.

$$P = F_{\text{wall}}, \text{ where}$$

~~$$F_{\text{wall}} \approx 2\pi r \cdot g = 2\pi g \cdot \rho_1 \cdot \sin \beta$$~~

$$F_{\text{wall}} = A_{\text{wall}} \cdot \sigma_2 \cdot \sin \beta$$

$$\text{wall section} = A_{\text{wall}} = 2\pi r \cdot g = 2\pi g \cdot \rho_1 \cdot \sin \beta$$

and substitute

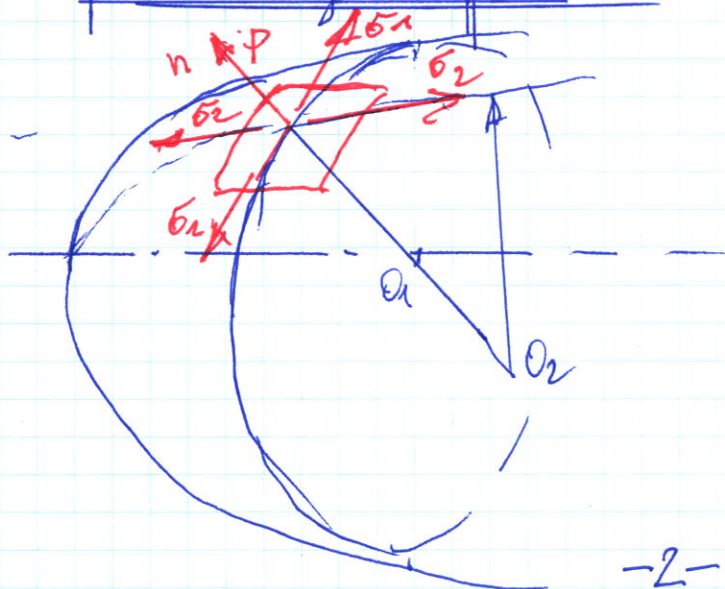
$$F_{\text{wall}} = (2\pi g \cdot \rho_1 \cdot \sin \beta) \cdot \sigma_2 \cdot \sin \beta = 2\pi g \rho_1 \sigma_2 \sin^2 \beta$$

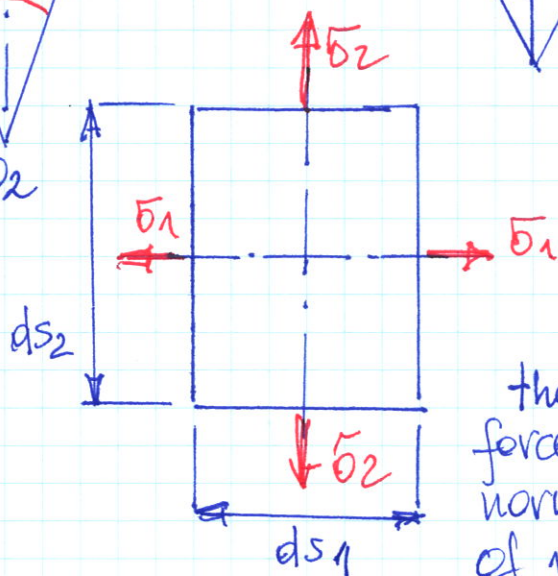
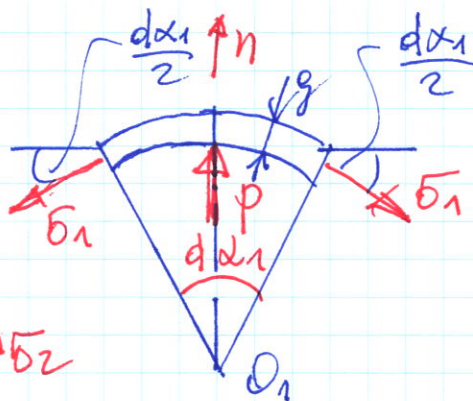
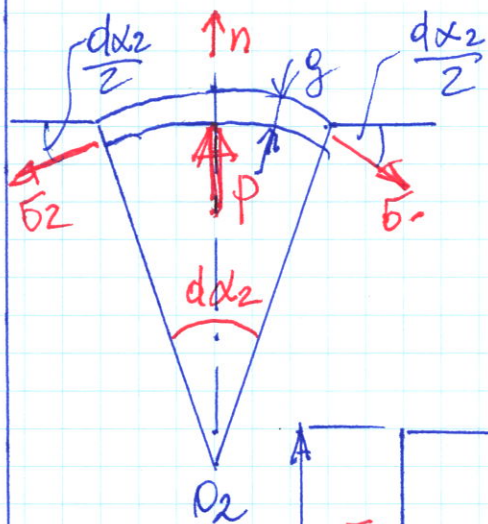
$$P = F_{\text{wall}}$$

~~$$p \cdot \pi \cdot \rho_1^2 \cdot \sin^2 \beta = 2\pi g \rho_1 \sigma_2 \sin^2 \beta$$~~

$$\sigma_2 = \frac{\rho_1}{2g} \cdot p$$

meridional stress





$$ds_1 = g_1 \cdot d\alpha_1$$

$$ds_2 = g_2 \cdot d\alpha_2$$

the sum of projections all forces on the direction of normal to the surface of vessel ($\uparrow n$)

$$\sum p_n = p \cdot ds_1 \cdot ds_2 - 2\sigma_1 \cdot g \cdot ds_2 \cdot \sin \frac{d\alpha_1}{2} - 2\sigma_2 \cdot g \cdot ds_1 \cdot \sin \frac{d\alpha_2}{2}$$

for small angles:

$$\sin \frac{d\alpha_1}{2} \approx \frac{d\alpha_1}{2} \quad \text{and} \quad \sin \frac{d\alpha_2}{2} \approx \frac{d\alpha_2}{2}$$

$$p \cdot ds_1 \cdot ds_2 - 2\sigma_1 \cdot g \cdot ds_2 \cdot d\alpha_1 - 2\sigma_2 \cdot g \cdot ds_1 \cdot \frac{d\alpha_2}{2} = 0$$

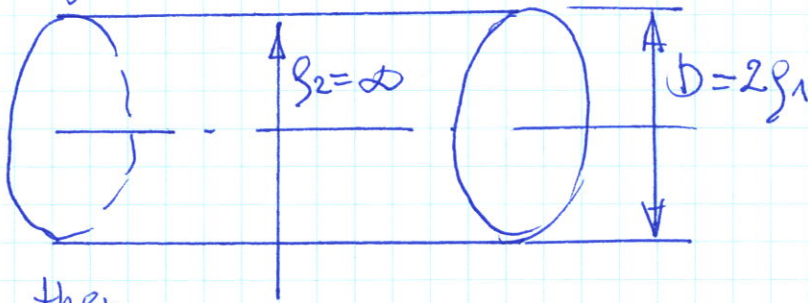
$$\frac{p}{g} - \sigma_1 \frac{d\alpha_1}{ds_1} - \sigma_2 \frac{d\alpha_2}{ds_2} = 0 \quad \Big| : g \cdot ds_1 \cdot ds_2$$

but $\frac{d\alpha_1}{ds_1} = \frac{1}{g_1}$ and $\frac{d\alpha_2}{ds_2} = \frac{1}{g_2}$ then finally

$$\frac{\sigma_1}{g_1} + \frac{\sigma_2}{g_2} = \frac{p}{g}$$

Laplace formula
(basic equation of the shell state)

Cylindrical shells



p, D, g

$$r_1 = \frac{D}{2}$$

$$r_2 = \infty$$

then

$$\sigma_2 = \frac{r_1}{2g} \cdot p = \frac{D}{2 \cdot 2g} \cdot p = \frac{D \cdot p}{4g}$$

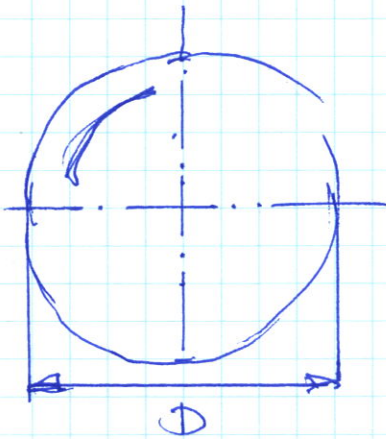
from Laplace's formulae

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{g} \Rightarrow \frac{\sigma_1}{\frac{D}{2}} + \frac{D \cdot p}{4g \cdot \infty} = \frac{p}{g}$$

$$\frac{2\sigma_1}{D} = \frac{p}{g} \Rightarrow \sigma_1 = \frac{Dp}{2g}$$

so-called
Poisson's formulae

Spherical shells (vessels)



p, D, g

$$r_1 = r_2 = \frac{D}{2}$$

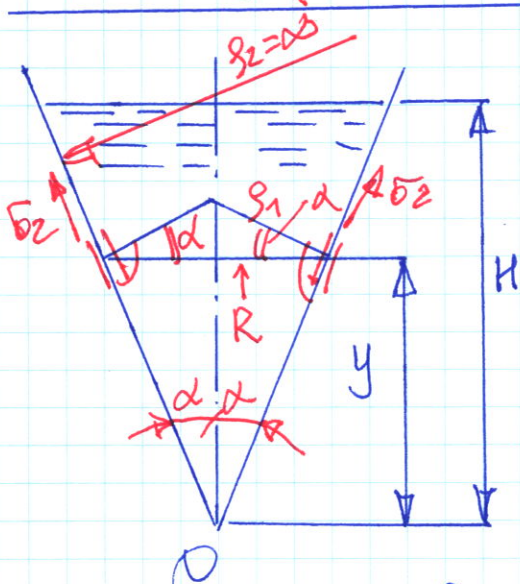
$$\sigma_1 = \sigma_2$$

$$\sigma_1 = \sigma_2 = \frac{Dp}{4g}$$

$$\sigma_2 (\text{cylindrical}) = \sigma_2 (\text{spherical})$$

$$\sigma_1 (\text{cylindrical}) = 2 \cdot \sigma_1 (\text{spherical})$$

Conical ~~fluid~~ shell



H, α, r, g

$$\frac{R}{y} = \tan \alpha \Rightarrow R = y \cdot \tan \alpha$$

$$\frac{R}{s_1} = \cos \alpha \Rightarrow s_1 = \frac{R}{\cos \alpha}$$

$$s_1 = \frac{R}{\cos \alpha} = \frac{y \cdot \tan \alpha}{\cos \alpha}$$

$$s_2 = \infty$$

At a distance y from the top O , the ^{hydrostatic} pressure is

$$p_h = \rho (H - y)$$

but

$$\frac{\sigma_1}{s_1} + \frac{\sigma_2}{\infty} = \frac{p_h}{g}$$

$$\frac{\sigma_1 \cdot \cos \alpha}{y \cdot \tan \alpha} + 0 = \frac{\rho (H - y)}{g} \Rightarrow \sigma_1 = \frac{\rho \cdot y (H - y) \tan \alpha}{g \cdot \cos \alpha}$$

the sum of projections of all forces on the direction to the vessel axis is

$$\sigma_2 \cdot 2\pi R \cdot g \cos \alpha - p_h \cdot \pi R^2 - \frac{1}{3} \pi R^2 \cdot \rho \cdot y = 0$$

force in the wall

force from the hydrostatic pressure

fluid weight for height = y

$$\sigma_2 \cdot 2\pi g \cos \alpha \cdot y \tan \alpha - \rho (H - y) \cdot \pi y^2 \tan^2 \alpha - \frac{1}{3} \pi \rho \cdot y \cdot y^2 \tan^2 \alpha = 0$$

$$\sigma_2 \cdot 2\pi g y \cos \alpha \tan \alpha - \rho (H - y) \pi y^2 \tan^2 \alpha - \frac{1}{3} \pi \rho y^3 \tan^2 \alpha = 0$$

$$\sigma_2 \cdot 2g \cos \alpha = \rho (H - y) \cdot y \tan \alpha + \frac{1}{3} \rho y^2 \tan \alpha$$

$$\sigma_2 \cdot 2g \cos \alpha = \rho y \tan \alpha \left(H - y + \frac{1}{3} y \right) = \rho \cdot y \cdot \tan \alpha \left(H - \frac{2}{3} y \right)$$

$$\sigma_2 = \frac{\rho \cdot y \left(H - \frac{2}{3} y \right) \cdot \tan \alpha}{2g \cdot \cos \alpha}$$

$$\bar{\sigma}_{2 \max} = ?$$

$$\frac{d\bar{\sigma}_2(y)}{dy} = 0 \Rightarrow H - \frac{4}{3}y = 0 \Rightarrow y = \frac{3}{4}H$$

$$\bar{\sigma}_{2 \max} = \frac{r \cdot \tan \alpha}{2 \cdot g \cdot \cos \alpha} \cdot \frac{3}{4}H \left(H - \frac{2}{3} \cdot \frac{3}{4}H \right) =$$

$$\frac{r \cdot \tan \alpha}{2g \cos \alpha} \cdot \frac{3}{4}H \cdot \frac{H}{2} = \frac{3}{16} \frac{H^2 \cdot r \cdot \tan \alpha}{g \cdot \cos \alpha} = \bar{\sigma}_{2 \max}$$

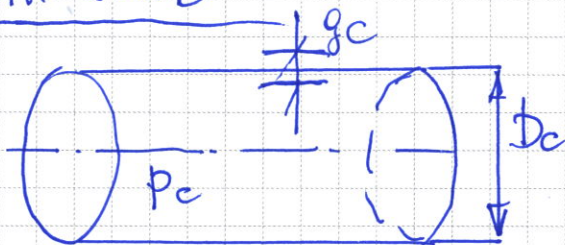
$$\bar{\sigma}_{1 \max} = ?$$

$$\frac{d\bar{\sigma}_1(y)}{dy} = 0 \Rightarrow H - 2y = 0 \Rightarrow y = \frac{H}{2}$$

$$\bar{\sigma}_{1 \max} = \frac{r \cdot \frac{H}{2} \left(H - \frac{H}{2} \right) \cdot \tan \alpha}{g \cdot \cos \alpha} = \frac{r \cdot \tan \alpha}{g \cdot \cos \alpha} \cdot \frac{H^2}{4}$$

$$\bar{\sigma}_{1 \max} = \frac{H^2 \cdot r \cdot \tan \alpha}{4 \cdot g \cdot \cos \alpha}$$

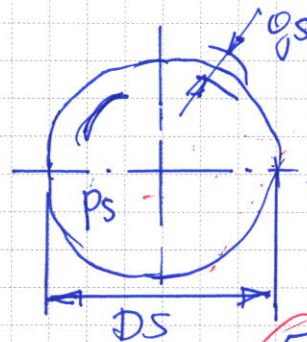
Homework



c - cylindrical
s - spherical

$$D_c = D, D_s = 1,5D, q_c = q, q_s = 1,2q$$

$$p_c = 2p, p_s = 3p$$



$$\frac{\bar{\sigma}_{1c}}{\bar{\sigma}_{1s}} = ?$$

$$\frac{\bar{\sigma}_{2c}}{\bar{\sigma}_{2s}} = ?$$

