BIEM modeling and numerical simulation of 3D problems for an composite structures with crack-like damages

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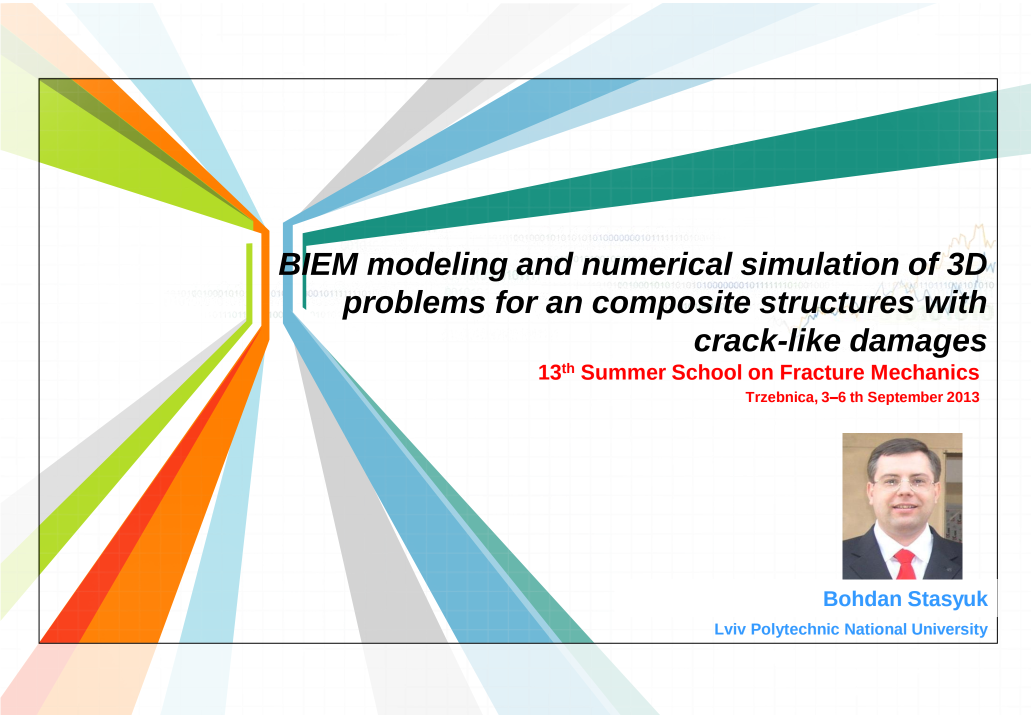

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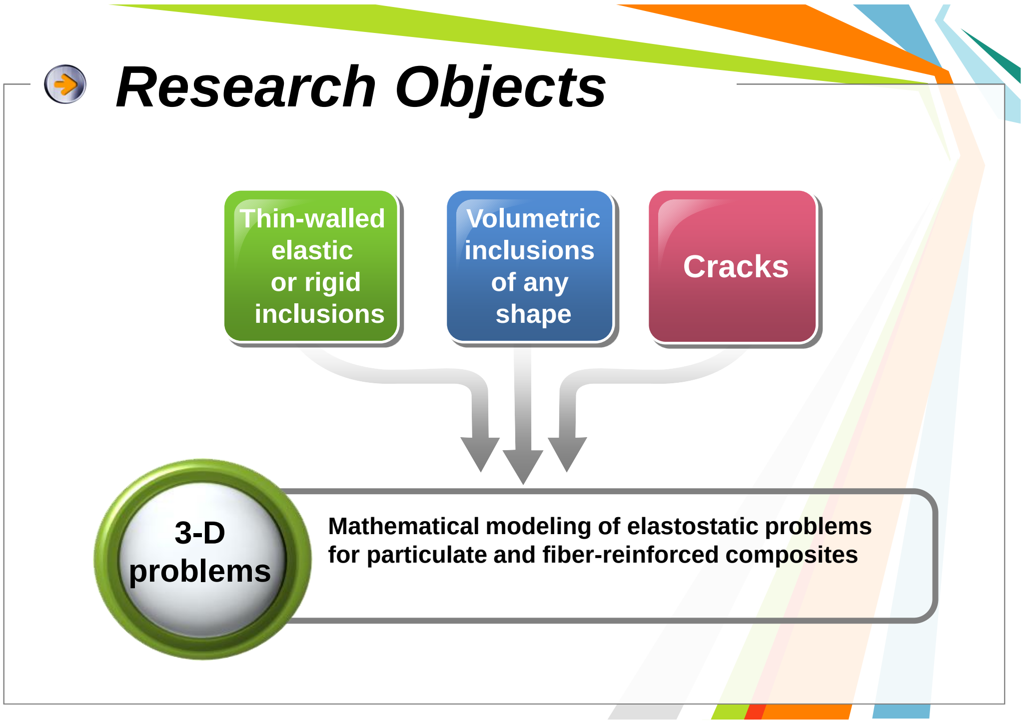
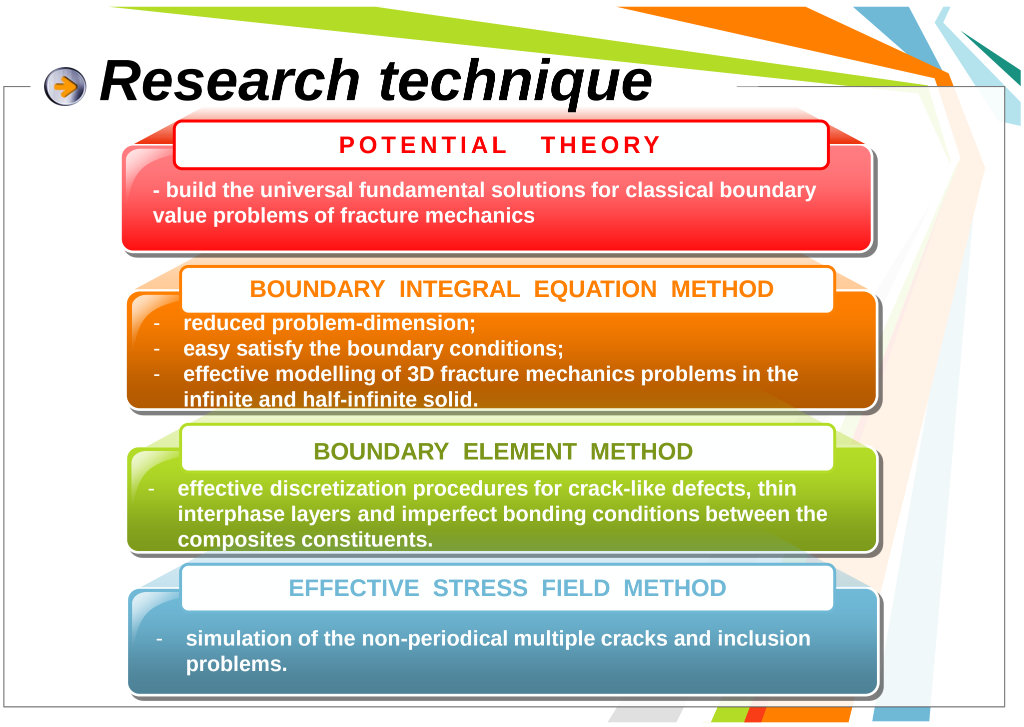
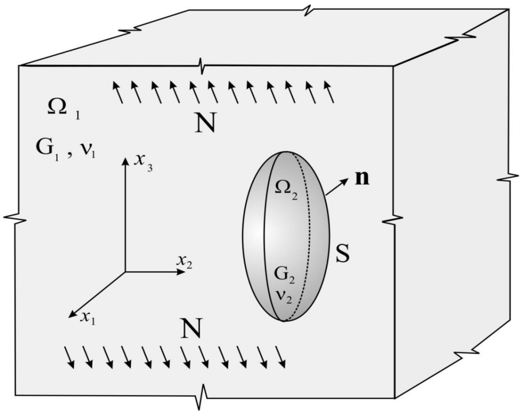

Research Objects
Thin-walled elastic or rigid inclusions
Volumetric inclusions of any shape
Cracks
3-D problems
Mathematical modeling of elastostatic problems for particulate and fiber-reinforced composites


Research technique
POTENTIAL THEORY
- build the universal fundamental solutions for classical boundary value problems of fracture mechanics
BOUNDARY INTEGRAL EQUATION METHOD
- reduced problem-dimension;
- easy satisfy the boundary conditions;
- effective modelling of 3D fracture mechanics problems in the infinite and half-infinite solid.
BOUNDARY ELEMENT METHOD
- effective discretization procedures for crack-like defects, thin interphase layers and imperfect bonding conditions between the composites constituents.
EFFECTIVE STRESS FIELD METHOD
- simulation of the non-periodical multiple cracks and inclusion problems.

FEM vs BEM

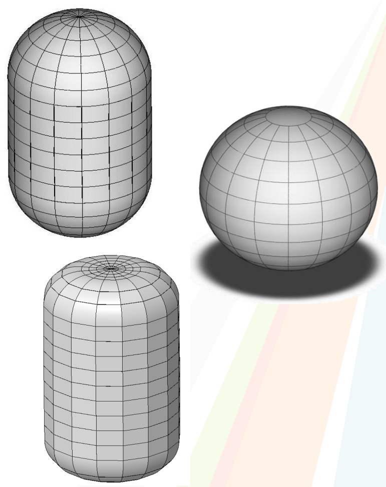
Integral equations + Boundary element method	Differential equations + Finite element method
• Discretization of the 2D domain. • Effective modelling of the imperfect boundary conditions.	• Discretization of the 3D domain.
• Regularization of the singular and hypersingular integrals. • Linear problem only.	• Nonlinear problem • Easy numerical procedures and large finite element library

Microanalysis of particulate and fiber-reinforced composites



Problem statement

Topological shapes of the inclusion



Microanalysis of particulate and fiber-reinforced composites

1 Integral representation of displacements in the inclusion

$$u_i^I(\mathbf{x}) = \iint_S U_{ij}^I(\mathbf{x},\xi)\, t_j^I(\xi)\, dS_\xi - \iint_S T_{ij}^I(\mathbf{x},\xi)\, u_j^I(\xi)\, dS_\xi, \qquad \mathbf{x} \in \Omega_I, \;\; i = 1,2,3$$

2 Integral representation of displacements in the matrix

$$u_i^{M-I}(\mathbf{x}) = \iint_S U_{ij}^M(\mathbf{x},\xi)\, t_j^M(\xi)\, dS_\xi - \iint_S T_{ij}^M(\mathbf{x},\xi)\, u_j^M(\xi)\, dS_\xi, \qquad \mathbf{x} \in \Omega_M,$$

$$U_{ij}(\mathbf{x},\xi) = \frac{1}{16\pi G(1-\nu)}\left[(3-4\nu)\frac{\delta_{ij}}{|\mathbf{x}-\xi|} + \frac{(x_i-\xi_i)(x_j-\xi_j)}{|\mathbf{x}-\xi|^3}\right] \qquad i, j = \overline{1,3}$$

$$T_{ij}(\mathbf{x},\xi) = \frac{\left((1-2\nu)\delta_{ij} + 3\frac{(x_i-\xi_i)(x_j-\xi_j)}{|\mathbf{x}-\xi|^2}\right)\sum_{k=1}^{3}(x_k-\xi_k)n_k(\xi) - (1-2\nu)\left((x_i-\xi_i)n_j(\xi) - (x_j-\xi_j)n_i(\xi)\right)}{8\pi(1-\nu)|\mathbf{x}-\xi|^3}$$

Microanalysis of particulate and fiber-reinforced composites

1 Integral representation of stresses in the inclusion

$$\sigma_{ij}^I(\mathbf{x}) = \frac{1}{1-\nu_I}\sum_{k=1}^{3}\iint_S D_{ijk}^I(\mathbf{x},\xi)\, t_k(\xi)\, dS_\xi - \frac{G_I}{1-\nu_I}\sum_{k=1}^{3}\iint_S S_{ijk}^I(\mathbf{x},\xi)\, u_k(\xi)\, dS_\xi, \; \mathbf{x} \in \Omega_I, \; i = 1,2,3$$

2 Integral representation of stresses in the matrix

$$\sigma_{ij}^{M-I}(\mathbf{x}) = -\frac{1}{1-\nu_M}\sum_{k=1}^{3}\iint_S D_{ijk}^M(\mathbf{x},\xi)\, t_k(\xi)\, dS_\xi + \frac{G_M}{1-\nu_M}\sum_{k=1}^{3}\iint_S S_{ijk}^M(\mathbf{x},\xi)\, u_k(\xi)\, dS_\xi, \; \mathbf{x} \in \Omega_M,$$

$$D_{ijk}(\mathbf{x},\xi) = -\frac{1}{8\pi|\mathbf{x}-\xi|^2}\left\{(1-2\nu)\left(\frac{(x_i-\xi_i)}{|\mathbf{x}-\xi|}\delta_{jk} + \frac{(x_j-\xi_j)}{|\mathbf{x}-\xi|}\delta_{ki} - \frac{(x_k-\xi_k)}{|\mathbf{x}-\xi|}\delta_{ij}\right) + 3\frac{(x_i-\xi_i)(x_j-\xi_j)(x_k-\xi_k)}{|\mathbf{x}-\xi|^3}\right\}$$

$$S_{ijk}(\mathbf{x},\xi) = -\frac{1}{4\pi|\mathbf{x}-\xi|^3}\left\{3\frac{x_l-\xi_l}{|\mathbf{x}-\xi|}n_l(\xi)\left[(1-2\nu)\delta_{ij}\frac{x_k-\xi_k}{|\mathbf{x}-\xi|} + \nu\left(\delta_{jk}\frac{x_i-\xi_i}{|\mathbf{x}-\xi|} + \delta_{ik}\frac{x_j-\xi_j}{|\mathbf{x}-\xi|}\right) - 5\frac{(x_i-\xi_i)(x_j-\xi_j)(x_k-\xi_k)}{|\mathbf{x}-\xi|^3}\right] + \right.$$

$$\left. +3\left[\nu n_i(\xi)\frac{(x_j-\xi_j)(x_k-\xi_k)}{|\mathbf{x}-\xi|^2} + \nu n_j(\xi)\frac{(x_i-\xi_i)(x_k-\xi_k)}{|\mathbf{x}-\xi|^2} + (1-2\nu)n_k(\xi)\frac{(x_i-\xi_i)(x_j-\xi_j)}{|\mathbf{x}-\xi|^2}\right] + (1-2\nu)\left[\delta_{jk}n_i(\xi) + \delta_{ik}n_j(\xi)\right] - (1-4\nu)\delta_{ij}n_k(\xi)\right\}$$

$$i, j, k = \overline{1,3}$$

Boundary condition

Perfect contact

$$\mathbf{u}^{(1)}(\mathbf{x})=\mathbf{u}^{(1)}(\mathbf{x})=\mathbf{u}(\mathbf{x}) \qquad P_j^{(2)}(\mathbf{x})=-P_j^{(1)}(\mathbf{x})=P_j(\mathbf{x}) \qquad j=\overline{1,3} \qquad \mathbf{x}\in S$$

Thin interphase layer

$$\Delta\mathbf{u}(\mathbf{x})=\mathbf{u}^{(1)}(\mathbf{x})-\mathbf{u}^{(2)}(\mathbf{x}) \qquad \mathbf{P}^{(2)}(\mathbf{x})=-\mathbf{P}^{(1)}(\mathbf{x})=\mathbf{P}(\mathbf{x}) \qquad \mathbf{x}\in S$$

$$\mathbf{P}_n(\mathbf{x})=f\,\Delta\mathbf{u}_n(\mathbf{x}) \qquad f=\frac{2G_0}{h(1-\nu_0)} \qquad g=\frac{G_0}{h}$$

$$\mathbf{P}_\tau(\mathbf{x})=g\,\Delta\mathbf{u}_\tau(\mathbf{x})$$

Sliding contact

$$t_n^{(1)}(\mathbf{x})=-t_n^{(1)}(\mathbf{x})=t_n(\mathbf{x}) \qquad t_\tau^{(1)}(\mathbf{x})=t_\tau^{(2)}(\mathbf{x})=0$$

$$u_n^{(1)}(\mathbf{x})=u_n^{(2)}(\mathbf{x})=u_n(\mathbf{x}) \qquad t_r^{(1)}(\mathbf{x})=t_r^{(2)}(\mathbf{x})=0$$

Boundary integral equation

Boundary properties of potentials

$$\lim_{\substack{\mathbf{x}\to S\\ \mathbf{x}\in\Omega_I}}\iint_S T_{ij}^I(\mathbf{x},\boldsymbol{\eta})u_j^I(\boldsymbol{\eta})\,dS_{\boldsymbol{\eta}}=-\frac{1}{2}u_i^I(\mathbf{x})+\iint_S T_{ij}^I(\mathbf{x},\boldsymbol{\eta})u_j^I(\boldsymbol{\eta})\,dS_{\boldsymbol{\eta}}$$

$$\lim_{\substack{\mathbf{x}\to S\\ \mathbf{x}\in\Omega_M}}\iint_S T_{ij}^M(\mathbf{x},\boldsymbol{\eta})u_j^M(\boldsymbol{\eta})\,dS_{\boldsymbol{\eta}}=\frac{1}{2}u_i^M(\mathbf{x})+\iint_S T_{ij}^M(\mathbf{x},\boldsymbol{\eta})u_j^M(\boldsymbol{\eta})\,dS_{\boldsymbol{\eta}}$$

$$\iint_S T_{ij}(\mathbf{x},\boldsymbol{\eta})\,dS_{\boldsymbol{\eta}}=-\frac{1}{2}\delta_{ij}$$

First step of regularization

$$\iint_S T_{ij}^I(\mathbf{x},\boldsymbol{\xi})u_j^I(\boldsymbol{\xi})\,dS_{\xi}=u_j^I(\mathbf{x})\iint_S T_{ij}^I(\mathbf{x},\boldsymbol{\xi})\,dS_{\xi}+\iint_S T_{ij}^I(\mathbf{x},\boldsymbol{\xi})\left[u_j^I(\boldsymbol{\xi})-u_j^I(\mathbf{x})\right]dS_{\xi}=$$

$$=-\frac{1}{2}u_j^I(\mathbf{x})+\iint_S T_{ij}^I(\mathbf{x},\boldsymbol{\xi})\left[u_j^I(\boldsymbol{\xi})-u_j^I(\mathbf{x})\right]dS_{\xi}$$

Perfect contact

$$\sum_{j=1}^{3}\iint_S T_{ij}^I(\mathbf{x},\boldsymbol{\xi})\left[u_j(\boldsymbol{\xi})-u_j(\mathbf{x})\right]dS_{\xi}-\sum_{j=1}^{3}\iint_S U_{ij}^I(\mathbf{x},\boldsymbol{\xi})t_j(\boldsymbol{\xi})\,dS_{\xi}=0, \qquad i,j=\overline{1,3}$$

$$u_i(\mathbf{x})-\sum_{j=1}^{3}\iint_S T_{ij}^M(\mathbf{x},\boldsymbol{\xi})\left[u_j(\boldsymbol{\xi})-u_j(\mathbf{x})\right]d_{\xi}S+\sum_{j=1}^{3}\iint_S U_{ij}^M(\mathbf{x},\boldsymbol{\xi})t_j(\boldsymbol{\xi})\,dS_{\xi}=u_{0i}(\mathbf{x})$$

Boundary integral equation

Thin interphase layer

$$\sum_{j=1}^{3}\iint_S T_{ij}^I(\mathbf{x},\xi)\left[u_j^I(\xi)-u_j^I(\mathbf{x})\right]dS_\xi-\sum_{j=1}^{3}\iint_S U_{ij}^I(\mathbf{x},\xi)t_j(\xi)dS_\xi=0,$$

$$u_i^I(\mathbf{x})-\sum_{j=1}^{3}\iint_S T_{ij}^M(\mathbf{x},\xi)\left[u_j^I(\xi)-u_j^I(\mathbf{x})\right]d_\xi S+\sum_{j=1}^{3}\iint_S U_{ij}^M(\mathbf{x},\xi)t_j(\xi)dS_\xi+ \qquad i,j=\overline{1,3}$$

$$+\sum_{j=1}^{3}R_{ij}^*(\mathbf{x})t_j(\xi)-\sum_{j=1}^{3}\iint_S R_{ij}(\mathbf{x},\xi)t_j(\xi)dS_\xi=u_{0i}(\mathbf{x})$$

$$R_{ij}^*(\mathbf{x})=g\,\delta_{ij}+(f-g)n_i n_j+\left(g+(f-g)n_j^2\right)\iint_S T_{ij}^M(\mathbf{x},\xi)dS_\xi+(f-g)\sum_{k=1}^{3}n_j n_k\iint_S T_{ik}^M(\mathbf{x},\xi)dS_\xi$$

$$R_{ij}(\mathbf{x},\xi)=\left(g+(f-g)n_j^2\right)T_{ij}^M(\mathbf{x},\xi)+(f-g)\sum_{k=1}^{3}n_j n_k T_{ik}^M(\mathbf{x},\xi)$$

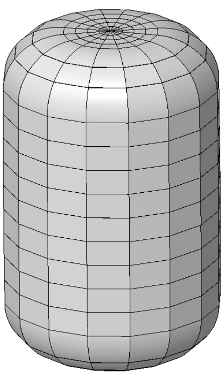
Sliding contact

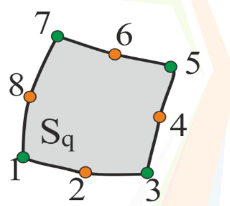
$$\sum_{j=1}^{3}\left[\iint_S T_{ij}^I(\mathbf{x},\xi)\left[u_j^I(\xi)-u_i^I(\mathbf{x})\right]dS_\xi-\iint_S U_{ij}^I(\mathbf{x},\xi)t_j(\xi)dS_\xi\right]=0 \qquad i,j=\overline{1,3}$$

$$u_i^I(\mathbf{x})+r_i(\mathbf{x})\Delta u_r(\mathbf{x})+\tau_i(\mathbf{x})\Delta u_\tau(\mathbf{x})-\iint_S T_{ij}^M(\mathbf{x},\xi)\left[u_j^I(\xi)-u_j^I(\mathbf{x})\right]dS_\xi+\left[r_j(\mathbf{x})\Delta u_r(\mathbf{x})+\tau_j(\mathbf{x})\Delta u_\tau(\mathbf{x})\right]\times$$

$$\times\iint_S T_{ij}^M(\mathbf{x},\xi)dS_\xi-\iint_S T_{ij}^M(\mathbf{x},\xi)\left[r_j(\xi)\Delta u_r(\xi)+\tau_j(\xi)\Delta u_\tau(\xi)\right]dS_\xi+\iint_S U_{ij}^M(\mathbf{x},\xi)n_j(\xi)t_n(\xi)dS_\xi=u_{0i}(\mathbf{x})$$

Boundary element formulation





$$x_{iq}\big|_{S_q}=\sum_{n=1}^{8}x_{iqn}N^n(\xi_1,\xi_2)$$

$$N^1(\xi_1,\xi_2)=\frac{1}{4}(1-\xi_1)(1-\xi_2)(-\xi_1-\xi_2-1)$$

$$N^2(\xi_1,\xi_2)=\frac{1}{2}(1-\xi_1^2)(1-\xi_2)$$

$$N^3(\xi_1,\xi_2)=\frac{1}{4}(1+\xi_1)(1-\xi_2)(\xi_1-\xi_2-1)$$

$$N^4(\xi_1,\xi_2)=\frac{1}{2}(1-\xi_2^2)(1+\xi_1)$$

$$N^5(\xi_1,\xi_2)=\frac{1}{4}(1+\xi_1)(1+\xi_2)(\xi_1+\xi_2-1)$$

$$N^6(\xi_1,\xi_2)=\frac{1}{2}(1-\xi_1^2)(1+\xi_2)$$

$$N^7(\xi_1,\xi_2)=\frac{1}{4}(1-\xi_1)(1+\xi_2)(-\xi_1+\xi_2-1)$$

$$N^8(\xi_1,\xi_2)=\frac{1}{2}(1-\xi_2^2)(1-\xi_1)$$

$$u_{iq}(\xi)\big|_{S_q}=\sum_{n=1}^{4}u_{iq2n-1}^{(2)}M^n(\xi_1,\xi_2)$$

$$t_{iq}(\xi)\big|_{S_q}=\sum_{n=1}^{4}t_{iq,2n-1}M^n(\xi_1,\xi_2)$$

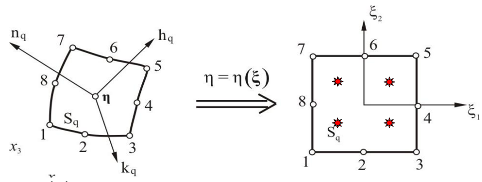
$$M^1(\xi_1,\xi_2)=\frac{1}{4}(1-\xi_1)(1-\xi_2)$$

$$M^2(\xi_1,\xi_2)=\frac{1}{4}(1+\xi_1)(1-\xi_2)$$

$$M^3(\xi_1,\xi_2)=\frac{1}{4}(1+\xi_1)(1+\xi_2)$$

$$M^4(\xi_1,\xi_2)=\frac{1}{4}(1-\xi_1)(1+\xi_2)$$

Regular integration



$x_i^* = \pm 0{,}57735\ 02691\ 89626$

$w_i = 1$

$$\iint_{S_q} T_{ij}^I(x_k,\eta)u_j(\eta)\,dS_\eta = \int_{-1}^{1}\int_{-1}^{1} T_{ij}^I(x_k,\eta(\xi))u_j(\eta(\xi))J_q(\xi)\,d\xi_1\,d\xi_2 = \sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}^I(x_k,\eta(\xi_l;\xi_m))u_j(\eta(\xi_l;\xi_m))J_q(\xi_l;\xi_m) =$$

$$= \sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}^I\left(x1_k, x2_k, x3_k, \sum_{n=1}^{8} x1_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x2_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x3_{qn}N^n(\xi_l,\xi_m)\right)\left[\sum_{s=1}^{4} uj_{q,2s-1}M^s(\xi_l,\xi_m)\right]J_q(\xi_l;\xi_m) =$$

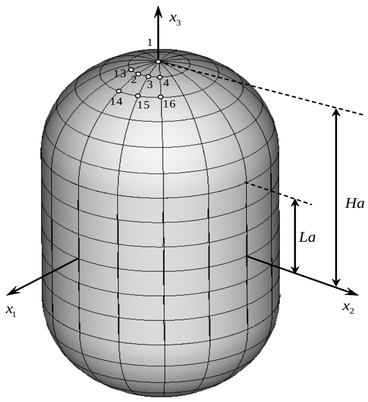
$$= uj_{q,1}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}^I\left(x1_k, x2_k, x3_k, \sum_{n=1}^{8} x1_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x2_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x3_{qn}N^n(\xi_l,\xi_m)\right)J_q(\xi_l;\xi_m)M^1(\xi_l,\xi_m) +$$

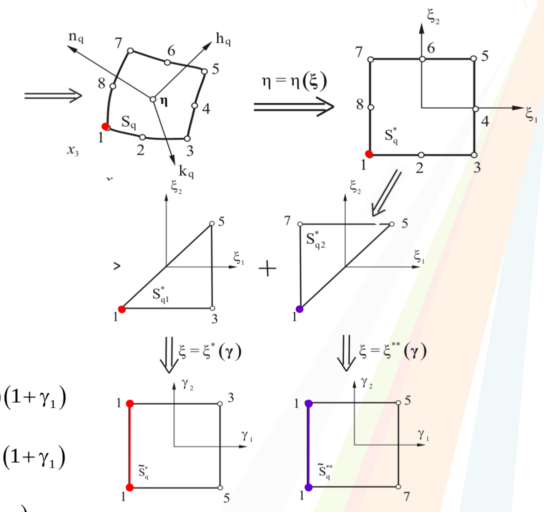
$$+ uj_{q,3}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}^I\left(x1_k, x2_k, x3_k, \sum_{n=1}^{8} x1_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x2_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x3_{qn}N^n(\xi_l,\xi_m)\right)J_q(\xi_l;\xi_m)M^2(\xi_l,\xi_m) +$$

$$+ uj_{q,5}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}^I\left(x1_k, x2_k, x3_k, \sum_{n=1}^{8} x1_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x2_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x3_{qn}N^n(\xi_l,\xi_m)\right)J_q(\xi_l;\xi_m)M^3(\xi_l,\xi_m) +$$

$$+ uj_{q,7}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}^I\left(x1_k, x2_k, x3_k, \sum_{n=1}^{8} x1_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x2_{qn}N^n(\xi_l,\xi_m), \sum_{n=1}^{8} x3_{qn}N^n(\xi_l,\xi_m)\right)J_q(\xi_l;\xi_m)M^4(\xi_l,\xi_m)$$

Regularization of a weakly singular integrals





$$\xi_1^* = -\frac{1}{2}(1-\gamma_1) + \frac{1}{4}(1-\gamma_2)(1+\gamma_1) + \frac{1}{4}(1+\gamma_2)(1+\gamma_1)$$

$$\xi_2^* = -\frac{1}{2}(1-\gamma_1) + \frac{1}{4}(1-\gamma_2)(1+\gamma_1) - \frac{1}{4}(1+\gamma_2)(1+\gamma_1)$$

$$K^*(\gamma_1;\gamma_2) = \frac{\partial\xi_1}{\partial\gamma_1}\frac{\partial\xi_2}{\partial\gamma_2} - \frac{\partial\xi_1}{\partial\gamma_2}\frac{\partial\xi_2}{\partial\gamma_1} = -\frac{1}{2}(1+\gamma_1)$$

Singular integration

$$\iint_{S_q} T_{ij}\left(x_{q1},\eta\right)u_j\left(\eta\right)dS_\eta = \int_{-1}^{1}\int_{-1}^{1} T_{ij}\left(x_{q1},\eta(\xi)\right)u_j\left(\eta(\xi)\right)J_q(\xi)\,d\xi_1\,d\xi_2 = \int_{-1}^{1}\int_{-1}^{\xi_1} T_{ij}\left(x_{q1},\eta(\xi)\right)u_j\left(\eta(\xi)\right)J_q(\xi)\,d\xi_1\,d\xi_2 + \int_{-1}^{1}\int_{\xi_1}^{1} T_{ij}\left(x_{q1},\eta(\xi)\right)u_j\left(\eta(\xi)\right)J_q(\xi)\,d\xi_1\,d\xi_2 =$$

$$= \int_{-1}^{1}\int_{-1}^{1} T_{ij}\left(x_{q1},\eta\left(\xi^*(\gamma)\right)\right)u_j\left(\eta\left(\xi^*(\gamma)\right)\right)J_q\left(\xi^*(\gamma)\right)\left(-\frac{1}{2}\right)(1+\gamma_1)\,d\gamma_1\,d\gamma_2 + \int_{-1}^{1}\int_{-1}^{1} T_{ij}\left(x_{q1},\eta\left(\xi^{**}(\gamma)\right)\right)u_j\left(\eta\left(\xi^{**}(\gamma)\right)\right)J_q\left(\xi^{**}(\gamma)\right)\left(-\frac{1}{2}\right)(1+\gamma_1)\,d\gamma_1\,d\gamma_2 =$$

$$= -\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x_{q1},\eta\left(\xi^*(\gamma_l;\gamma_m)\right)\right)u_j\left(\eta\left(\xi^*(\gamma_l;\gamma_m)\right)\right)J_q\left(\xi^*(\gamma_l;\gamma_m)\right)(1+\gamma_l) - \frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x_{q1},\eta\left(\xi^{**}(\gamma_l;\gamma_m)\right)\right)u_j\left(\eta\left(\xi^{**}(\gamma_l;\gamma_m)\right)\right)J_q\left(\xi^{**}(\gamma_l;\gamma_m)\right)(1+\gamma_l) =$$

$$-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right)\right)\left[\sum_{s=1}^{4}uj_{q,2s-1}M^s\left(\xi^*(\gamma_l;\gamma_m)\right)\right]J_q\left(\xi^*(\gamma_l;\gamma_m)\right)(1+\gamma_l) -$$

$$-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right)\right)\left[\sum_{s=1}^{4}uj_{q,2s-1}M^s\left(\xi^{**}(\gamma_l;\gamma_m)\right)\right]J_q\left(\xi^{**}(\gamma_l;\gamma_m)\right)(1+\gamma_l) =$$

$$= uj_{q,1}\left[-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right)\right)M^1\left(\xi^*(\gamma_l;\gamma_m)\right)J_q\left(\xi^*(\gamma_l;\gamma_m)\right)(1+\gamma_l) -\right.$$

$$\left.-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right)\right)M^1\left(\xi^{**}(\gamma_l;\gamma_m)\right)J_q\left(\xi^{**}(\gamma_l;\gamma_m)\right)(1+\gamma_l)\right] +$$

$$+ uj_{q,3}\left[-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right)\right)M^2\left(\xi^*(\gamma_l;\gamma_m)\right)J_q\left(\xi^*(\gamma_l;\gamma_m)\right)(1+\gamma_l) -\right.$$

$$\left.-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right)\right)M^2\left(\xi^{**}(\gamma_l;\gamma_m)\right)J_q\left(\xi^{**}(\gamma_l;\gamma_m)\right)(1+\gamma_l)\right] +$$

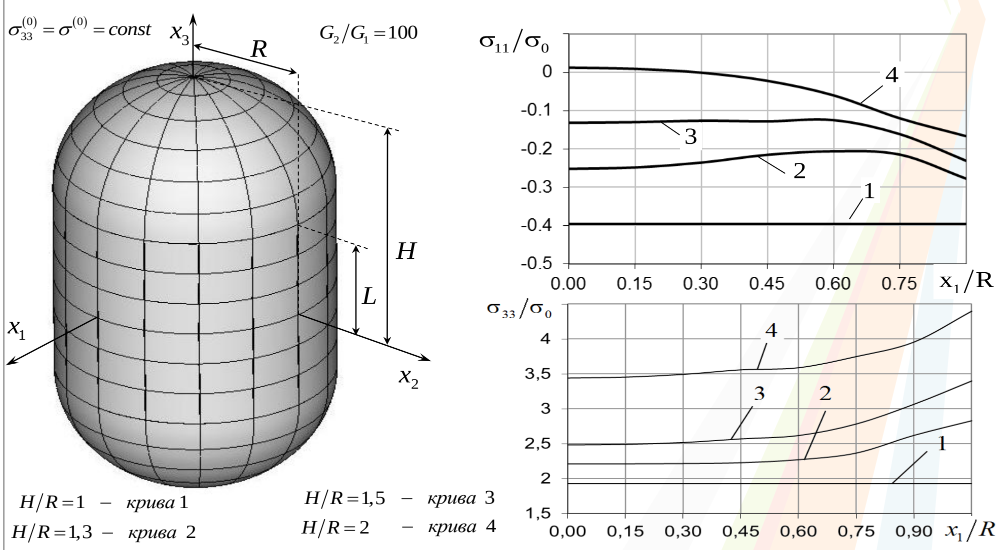
$$+ uj_{q,5}\left[-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right)\right)M^3\left(\xi^*(\gamma_l;\gamma_m)\right)J_q\left(\xi^*(\gamma_l;\gamma_m)\right)(1+\gamma_l) -\right.$$

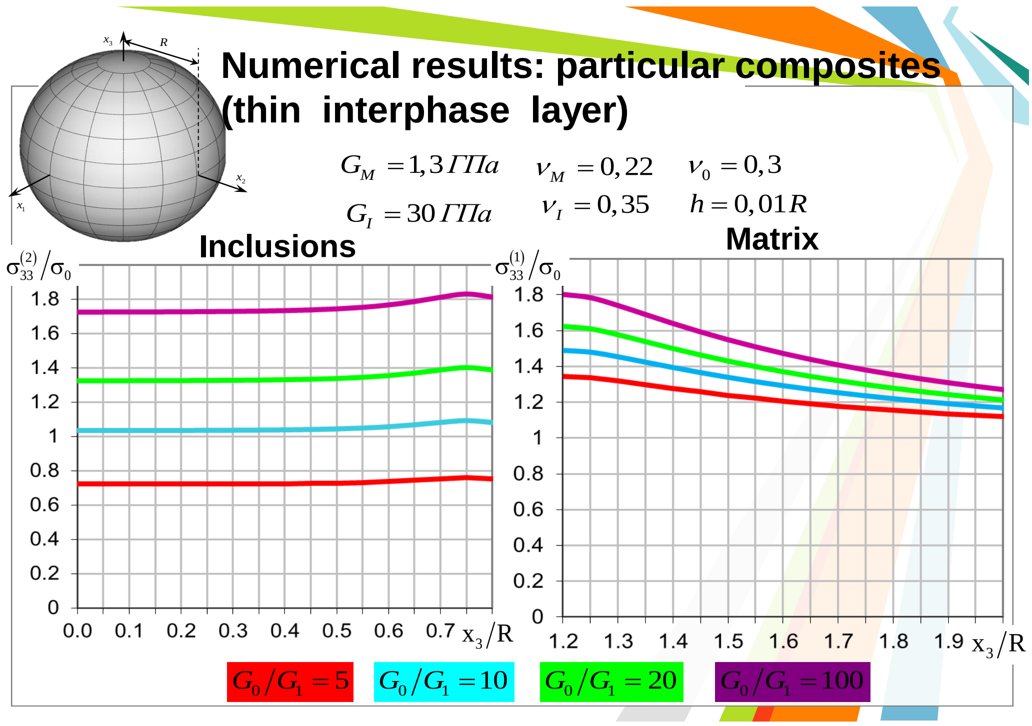
$$\left.-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right)\right)M^3\left(\xi^{**}(\gamma_l;\gamma_m)\right)J_q\left(\xi^{**}(\gamma_l;\gamma_m)\right)(1+\gamma_l)\right] +$$

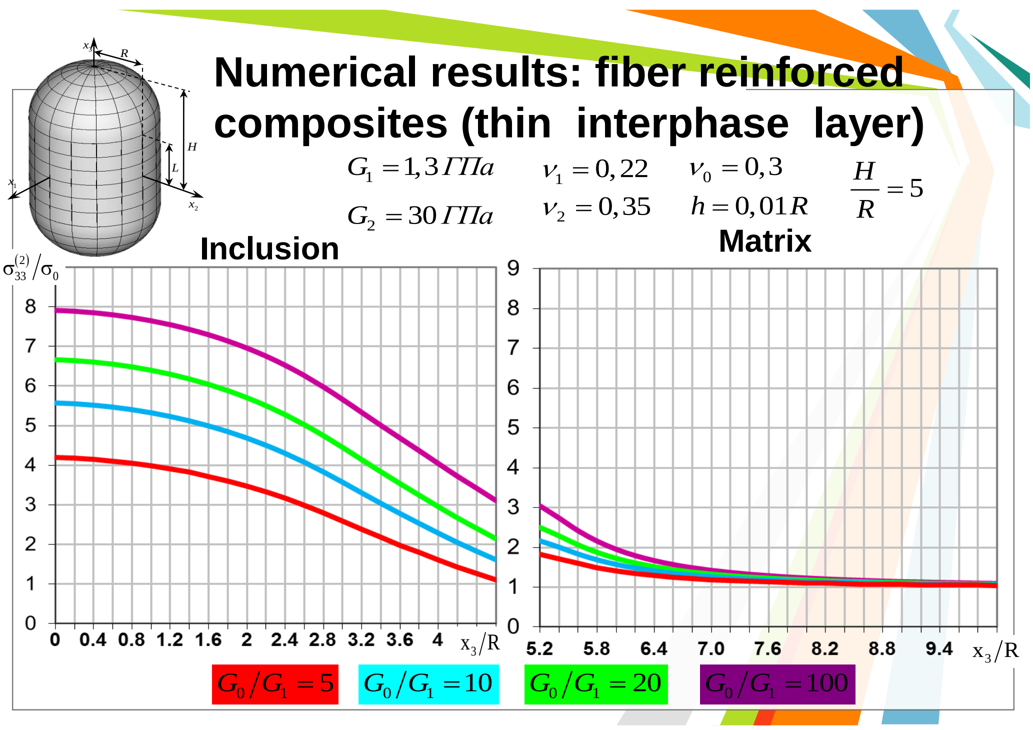
$$+ uj_{q,7}\left[-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^*(\gamma_l;\gamma_m)\right)\right)M^4\left(\xi^*(\gamma_l;\gamma_m)\right)J_q\left(\xi^*(\gamma_l;\gamma_m)\right)(1+\gamma_l) -\right.$$

$$\left.-\frac{1}{2}\sum_{l=1}^{2}\sum_{m=1}^{2} T_{ij}\left(x1_{q1},x2_{q2},x3_{q3},\sum_{n=1}^{8}x1_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x2_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right),\sum_{n=1}^{8}x3_{qn}N^n\left(\xi^{**}(\gamma_l;\gamma_m)\right)\right)M^4\left(\xi^{**}(\gamma_l;\gamma_m)\right)J_q\left(\xi^{**}(\gamma_l;\gamma_m)\right)(1+\gamma_l)\right]$$

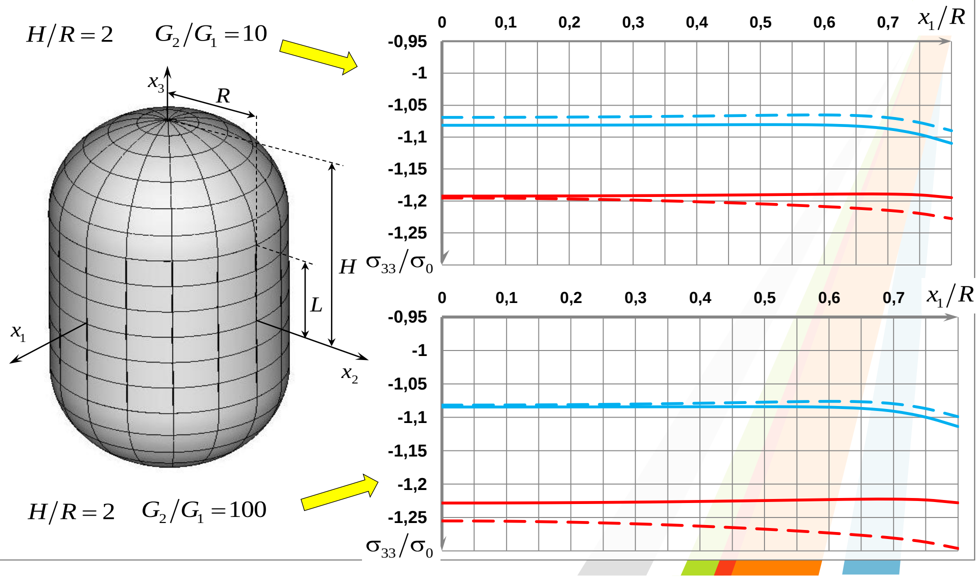
Numerical results: fiber reinforced composites (perfect contact)



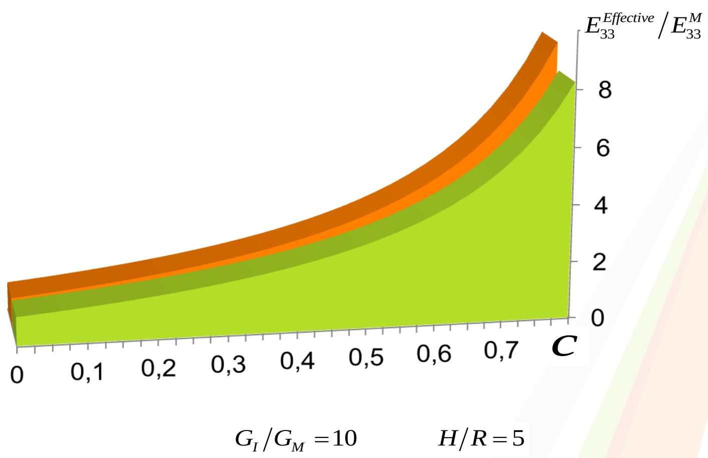

Numerical results: particular composites (thin interphase layer)
$G_M = 1,3\,ГПа$ $\nu_M = 0,22$ $\nu_0 = 0,3$
$G_I = 30\,ГПа$ $\nu_I = 0,35$ $h = 0,01R$
Inclusions
Matrix
$\sigma_{33}^{(2)}/\sigma_0$
$\sigma_{33}^{(1)}/\sigma_0$
x_3/R
$G_0/G_1 = 5$ $G_0/G_1 = 10$ $G_0/G_1 = 20$ $G_0/G_1 = 100$


Numerical results: fiber reinforced composites (thin interphase layer)
$G_1 = 1,3\,ГПа$ $\nu_1 = 0,22$ $\nu_0 = 0,3$ $\frac{H}{R} = 5$
$G_2 = 30\,ГПа$ $\nu_2 = 0,35$ $h = 0,01R$
Inclusion
Matrix
$\sigma_{33}^{(2)}/\sigma_0$
x_3/R
$G_0/G_1 = 5$ $G_0/G_1 = 10$ $G_0/G_1 = 20$ $G_0/G_1 = 100$

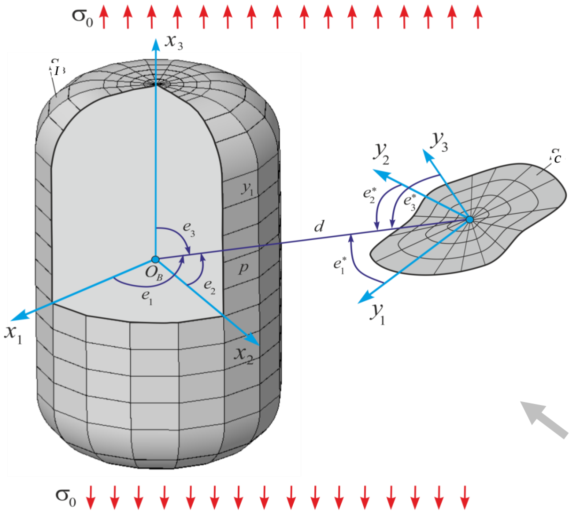
Numerical results: fiber reinforced composites (sliding and perfect contact)



Effective elastic moduli: fiber reinforced composites (sliding and perfect contact)



Inclusion- crack interaction: problem statement



$$d$$

$$l_{ij} = \cos\left(O_B\, x_i, O_T\, y_j\right)$$

$$e_i = \cos\left(O_B\, x_i\, ,\, d\right)$$

$$e_i^* = \cos\left(O_T\, y_i\, ,\, d\right)$$

Integral representation

Superposition principle

$$u^M(\mathbf{x}) = u_0(\mathbf{x}) + u^C(\mathbf{x}) + u^{M-I}(\mathbf{x})$$

$$\sigma^M(\mathbf{x}) = \sigma_0(\mathbf{x}) + \sigma^C(\mathbf{x}) + \sigma^{M-I}(\mathbf{x})$$

$$\mathbf{x} \in \Omega_M,$$

$$u_i^C(\mathbf{x}) = \sum_{j=1}^{3} \iint_{S_C} \Phi_{ij}(\mathbf{x},\xi)\, \alpha_j(\xi)\, d\, S_\xi$$

$$\sigma_{j3}^C(\mathbf{x}) = \frac{G_M}{1-\nu_M} \sum_{i=1}^{3} \iint_{S_C} \alpha_i(\xi)\, K_{ij}(\xi,\mathbf{x})\, d\, S_\xi$$

$$\Phi_{ii}(\mathrm{x},\xi) = -\frac{x_3}{2(1-\nu)|\mathbf{x}-\xi|^3}\left(1-2\nu+\frac{3(x_i-\xi_i)^2}{|\mathbf{x}-\xi|^2}\right)$$

$$\Phi_{ij}(\mathrm{x},\xi) = -\frac{(x_i-\xi_i)^{1-\delta_{i3}}(x_i-\xi_i)^{1-\delta_{j3}}}{2(1-\nu)|\mathbf{x}-\xi|^3}\left((1-2\nu)(\delta_{j3}-\delta_{i3})+\frac{3x_3^2}{|\mathbf{x}-\xi|^2}\right)$$

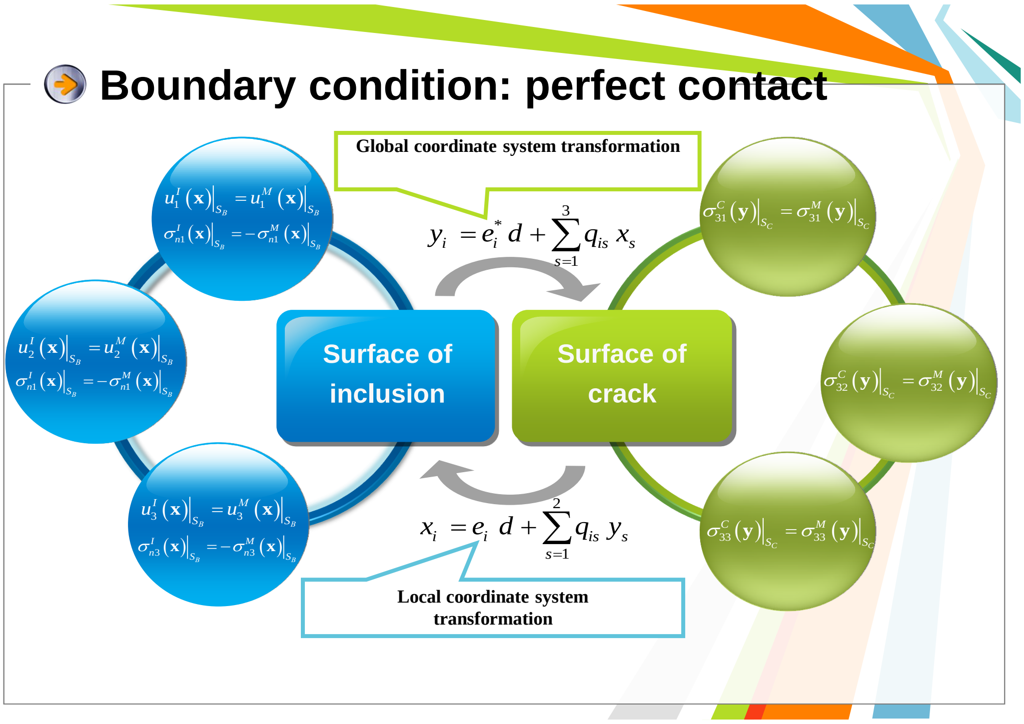
$$K_{ijm}(\mathbf{x},\xi) = -\frac{a_{ijm}}{|\mathbf{x}-\xi|^3} - \frac{\mathrm{b}_{ijm}(\mathbf{x},\xi)}{|\mathbf{x}-\xi|^5} + \frac{c_{ijm}(\mathbf{x},\xi)}{|\mathbf{x}-\xi|^7}$$

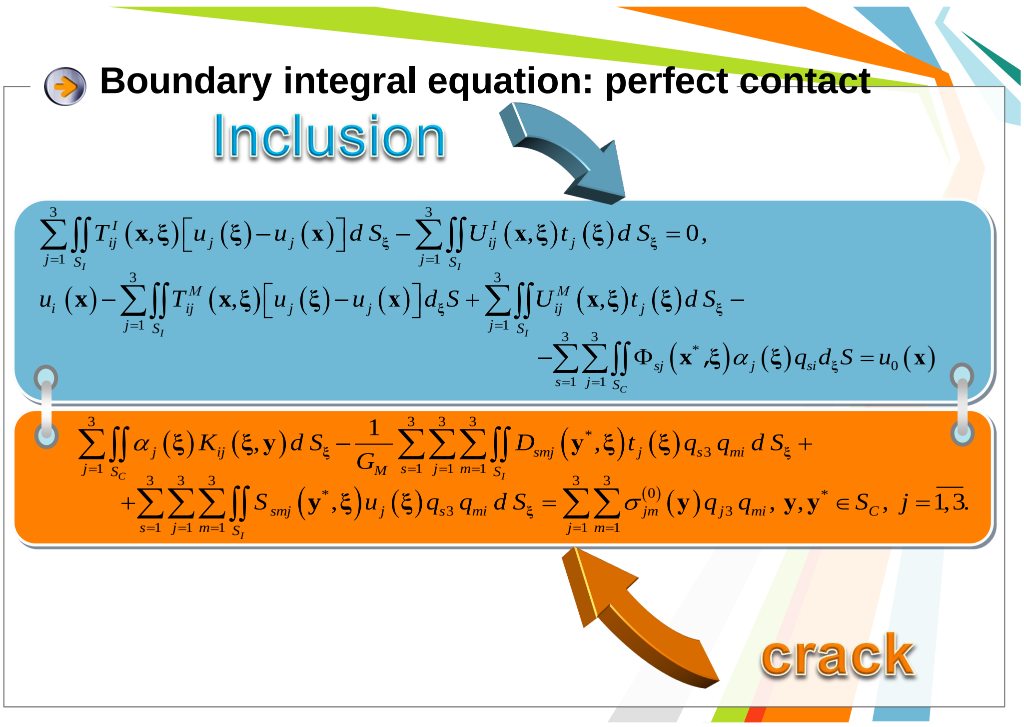
$$|\mathbf{x}-\xi| = \sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2+x_3^2}$$

$$a_{ijm} = Q_{1ijm} + Q_{2ijm} + Q_{3ijm}$$

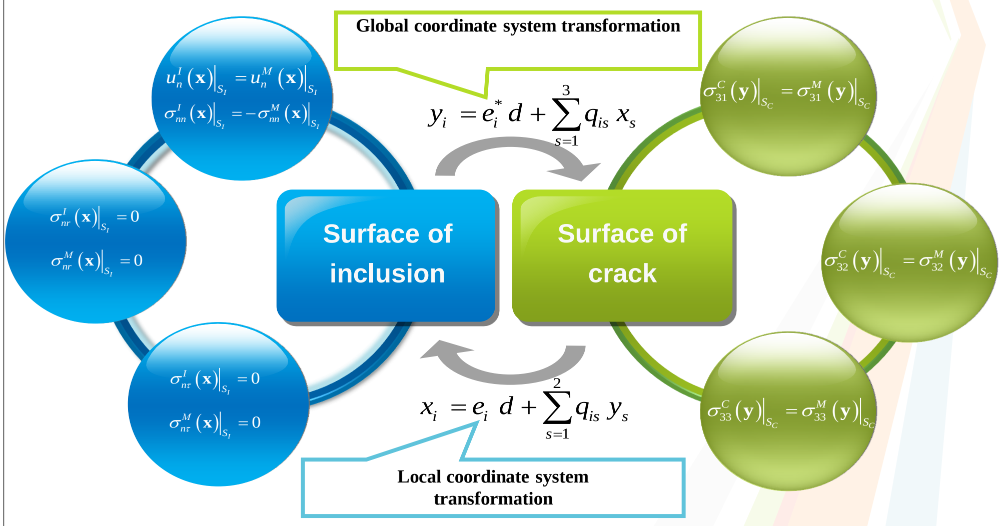
$$\begin{aligned} b_{ijm}(\mathbf{x},\xi) = {} & 3\left(x_m-(1-\delta_{3m})\xi_m\right)\left\{\delta_{im}\left(x_j-(1-\delta_{3j})\xi_j\right)+\delta_{1j}\left[2q_{1im}(x_1-\xi_1)+q_{1ij}^*(x_2-\xi_2)+q_{3ij}^*\,x_3\right]+\right. \\ & \left. +\delta_{2j}\left[2q_{2in}(x_2-\xi_2)+q_{1ij}^*(x_1-\xi_1)+q_{2ij}^*\,x_3\right]+\delta_{3j}\left[2q_{3im}\,x_3+q_{2ij}^*(x_2-\xi_2)+q_{3ij}^*(x_1-\xi_1)\right]\right\}- \\ & -3Q_{1ijm}(x_1-\xi_1)^2-3Q_{2ijm}(x_2-\xi_2)^2-3Q_{3ijm}\,x_3^2-3Q_{1ijm}^*(x_1-\xi_1)(x_2-\xi_2)-3Q_{2ijm}^*(x_2-\xi_2)x_3-3Q_{3ijm}^*(x_1-\xi_1)x_3 \end{aligned}$$

$$\begin{aligned} c_{ijm}(\mathbf{x},\xi) = {} & 15\left(x_m-(1-\delta_{3m})\xi_m\right)\left(x_j-(1-\delta_{3j})\xi_j\right)\left\{q_{1jm}(x_1-\xi_1)^2+q_{2jm}(x_2-\xi_2)^2+q_{3jm}\,x_3^2+q_{1jm}^*(x_1-\xi_1)(x_2-\xi_2)+\right. \\ & \left. +q_{2jm}^*(x_2-\xi_2)x_3+q_{3jm}^*(x_1-\xi_1)x_3\right\} \end{aligned}$$


Boundary condition: perfect contact
Global coordinate system transformation
$y_i = e_i^* d + \sum_{s=1}^{3} q_{is} x_s$
$u_1^I(\mathbf{x})|_{S_B} = u_1^M(\mathbf{x})|_{S_B}$, $\sigma_{n1}^I(\mathbf{x})|_{S_B} = -\sigma_{n1}^M(\mathbf{x})|_{S_B}$
$u_2^I(\mathbf{x})|_{S_B} = u_2^M(\mathbf{x})|_{S_B}$, $\sigma_{n1}^I(\mathbf{x})|_{S_B} = -\sigma_{n1}^M(\mathbf{x})|_{S_B}$
$u_3^I(\mathbf{x})|_{S_B} = u_3^M(\mathbf{x})|_{S_B}$, $\sigma_{n3}^I(\mathbf{x})|_{S_B} = -\sigma_{n3}^M(\mathbf{x})|_{S_B}$
Surface of inclusion
Surface of crack
$\sigma_{31}^C(\mathbf{y})|_{S_C} = \sigma_{31}^M(\mathbf{y})|_{S_C}$
$\sigma_{32}^C(\mathbf{y})|_{S_C} = \sigma_{32}^M(\mathbf{y})|_{S_C}$
$\sigma_{33}^C(\mathbf{y})|_{S_C} = \sigma_{33}^M(\mathbf{y})|_{S_C}$
$x_i = e_i d + \sum_{s=1}^{2} q_{is} y_s$
Local coordinate system transformation


Boundary integral equation: perfect contact
Inclusion
$\sum_{j=1}^{3}\iint_{S_I} T_{ij}^I(\mathbf{x},\xi)\left[u_j(\xi)-u_j(\mathbf{x})\right] dS_\xi - \sum_{j=1}^{3}\iint_{S_I} U_{ij}^I(\mathbf{x},\xi) t_j(\xi) dS_\xi = 0,$
$u_i(\mathbf{x}) - \sum_{j=1}^{3}\iint_{S_I} T_{ij}^M(\mathbf{x},\xi)\left[u_j(\xi)-u_j(\mathbf{x})\right] d_\xi S + \sum_{j=1}^{3}\iint_{S_I} U_{ij}^M(\mathbf{x},\xi) t_j(\xi) dS_\xi -$
$-\sum_{s=1}^{3}\sum_{j=1}^{3}\iint_{S_C} \Phi_{sj}(\mathbf{x}^*,\xi)\alpha_j(\xi) q_{si} d_\xi S = u_0(\mathbf{x})$
$\sum_{j=1}^{3}\iint_{S_C} \alpha_j(\xi) K_{ij}(\xi,\mathbf{y}) dS_\xi - \frac{1}{G_M}\sum_{s=1}^{3}\sum_{j=1}^{3}\sum_{m=1}^{3}\iint_{S_I} D_{smj}(\mathbf{y}^*,\xi) t_j(\xi) q_{s3} q_{mi} dS_\xi +$
$+\sum_{s=1}^{3}\sum_{j=1}^{3}\sum_{m=1}^{3}\iint_{S_I} S_{smj}(\mathbf{y}^*,\xi) u_j(\xi) q_{s3} q_{mi} dS_\xi = \sum_{j=1}^{3}\sum_{m=1}^{3}\sigma_{jm}^{(0)}(\mathbf{y}) q_{j3} q_{mi},\ \mathbf{y},\mathbf{y}^* \in S_C,\ j=\overline{1,3}.$
crack

Boundary condition: sliding contact



Boundary integral equation: sliding contact

Inclusion

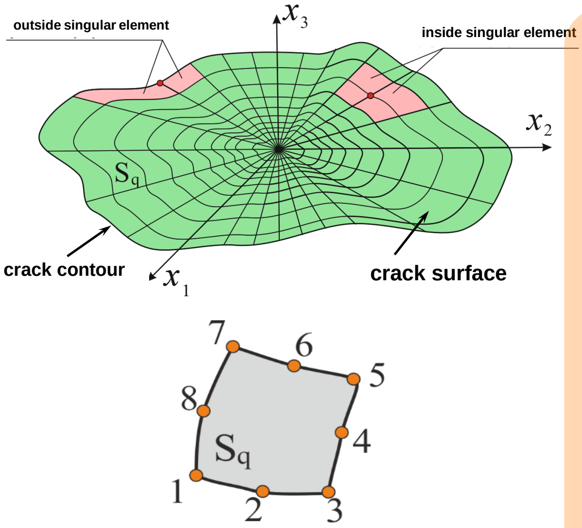
$$\sum_{j=1}^{3}\iint_{S_B} T_{ij}^B(\mathbf{x},\xi)\left[u_j^B(\xi)-u_j^B(\mathbf{x})\right]dS_\xi - \sum_{j=1}^{3}\iint_{S_B} U_{ij}^B(\mathbf{x},\xi)n_j(\xi)t_n(\xi)\,dS_\xi = 0,$$

$$u_i^B(\mathbf{x})+r_i(\mathbf{x})\Delta u_r(\mathbf{x})+\tau_i(\mathbf{x})\Delta u_\tau(\mathbf{x})-\iint_S T_{ij}^M(\mathbf{x},\xi)\left[u_j^B(\xi)-u_j^B(\mathbf{x})\right]dS_\xi+\left[r_j(\mathbf{x})\Delta u_r(\mathbf{x})+\tau_j(\mathbf{x})\Delta u_\tau(\mathbf{x})\right]\times$$
$$\times\iint_S T_{ij}^M(\mathbf{x},\xi)\,dS_\xi-\iint_S T_{ij}^M(\mathbf{x},\xi)\left[r_j(\xi)\Delta u_r(\xi)+\tau_j(\xi)\Delta u_\tau(\xi)\right]dS_\xi+\iint_S U_{ij}^M(\mathbf{x},\xi)n_j(\xi)t_n(\xi)\,dS_\xi-$$
$$-\sum_{s=1}^{3}\sum_{j=1}^{3}\iint_{S_T}\Phi_{sj}(\mathbf{x}^*,\xi)\alpha_j(\xi)q_{si}d_\xi S=u_0(\mathbf{x})$$

$$\sum_{j=1}^{3}\iint_{S_T}\alpha_j(\xi)K_{ij}(\xi,\mathbf{y})\,dS_\xi-\frac{1}{G_M}\sum_{s=1}^{3}\sum_{j=1}^{3}\sum_{m=1}^{3}\iint_{S_B}D_{smj}(\mathbf{y}^*,\xi)n_j(\xi)t_n(\xi)q_{s3}q_{mi}\,dS_\xi+$$
$$+\sum_{s=1}^{3}\sum_{j=1}^{3}\sum_{m=1}^{3}\iint_{S_B}S_{smj}(\mathbf{y}^*,\xi)u_j(\xi)q_{s3}q_{mi}\,dS_\xi=\sum_{j=1}^{3}\sum_{m=1}^{3}\sigma_{jm}^{(0)}(\mathbf{y})q_{j3}q_{mi},\ \mathbf{y},\mathbf{y}^*\in S_T,\ j=\overline{1,3}.$$

crack

Boundary element formulation for crack



$$\alpha_{iq}(\xi)\Big|_{S_q} = \sum_{n=1}^{8} \alpha_{iqn}^{(2)} N^n(\xi_1,\xi_2)$$

$$x_{iq}\Big|_{S_q} = \sum_{n=1}^{8} x_{iqn} N^n(\xi_1,\xi_2)$$

$$N^1(\xi_1,\xi_2) = \frac{1}{4}(1-\xi_1)(1-\xi_2)(-\xi_1-\xi_2-1)$$

$$N^2(\xi_1,\xi_2) = \frac{1}{2}(1-\xi_1^2)(1-\xi_2)$$

$$N^3(\xi_1,\xi_2) = \frac{1}{4}(1+\xi_1)(1-\xi_2)(\xi_1-\xi_2-1)$$

$$N^4(\xi_1,\xi_2) = \frac{1}{2}(1-\xi_2^2)(1+\xi_1)$$

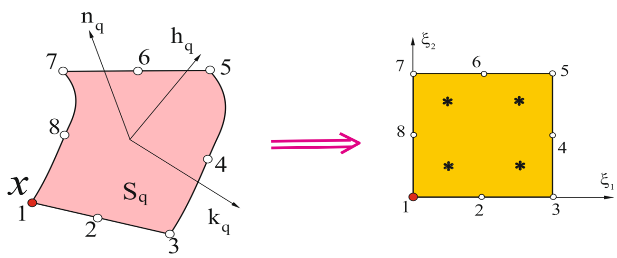
$$N^5(\xi_1,\xi_2) = \frac{1}{4}(1+\xi_1)(1+\xi_2)(\xi_1+\xi_2-1)$$

$$N^6(\xi_1,\xi_2) = \frac{1}{2}(1-\xi_1^2)(1+\xi_2)$$

$$N^7(\xi_1,\xi_2) = \frac{1}{4}(1-\xi_1)(1+\xi_2)(-\xi_1+\xi_2-1)$$

$$N^8(\xi_1,\xi_2) = \frac{1}{2}(1-\xi_2^2)(1-\xi_1)$$

Regularization of a hypersingular integrals: (inside singular element)



$$\iint_{S_q} \frac{\sqrt{L(\eta)}}{|x-\eta|^3}\beta_3(\eta)\,d_\eta S_q = \int_0^1\!\!\int_0^1 \frac{\sqrt{L(\eta(\xi))}}{\left((x_1-\eta_1(\xi))^2+(x_2-\eta_2(\xi))^2\right)^{3/2}}\beta_3(\eta(\xi))J_q(\xi)\,d\xi_1\,d\xi_2 =$$

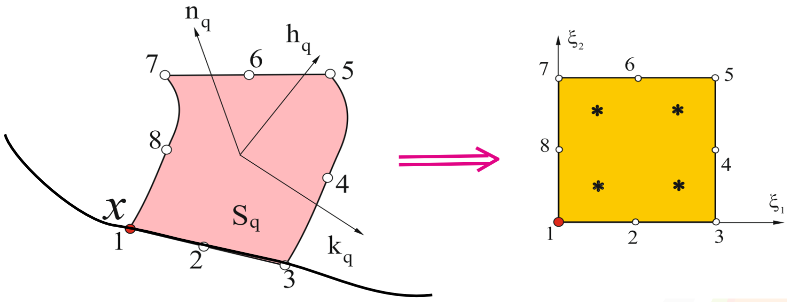
$$= \int_0^1\!\!\int_0^1 \frac{\Psi(x,\xi)}{(\xi_1^2+\xi_2^2)^{3/2}}\beta_3^*(\xi)\,d\xi_1\,d\xi_2 = \left(K_{00}^q(x)+I_{00}^q(x)+L_{00}^q(x)\right)\beta_3^*(0)+\left(K_{10}^q(x)+I_{10}^q(x)\right)\frac{\partial\beta_3^*}{\partial\xi_1}(0)+$$

$$+\left(K_{01}^q(x)+I_{01}^q(x)\right)\frac{\partial\beta_3^*}{\partial\xi_2}(0)+\frac{1}{2}K_{20}^q(x)\frac{\partial^2\beta_3^*}{\partial\xi_1^2}(0)+\frac{1}{2}K_{02}^q(x)\frac{\partial^2\beta_3^*}{\partial\xi_2^2}(0)+K_{11}^q(x)\frac{\partial^2\beta_3^*}{\partial\xi_1\,\partial\xi_2}(0)+\iint_{S_q^\varepsilon}\frac{\Psi(x,\xi)}{(\xi_1^2+\xi_2^2)^{3/2}}\beta_3^*(\xi)\,d\xi_1\,d\xi_2$$

$$\Psi(x,\xi) = \left[\frac{\left(x_{1q1}-\sum_{n=1}^{8}x_{1qn}N^n(\xi_1,\xi_2)\right)^2+\left(x_{2q1}-\sum_{n=1}^{8}x_{2qn}N^n(\xi_1,\xi_2)\right)^2}{\xi_1^2+\xi_2^2}\right]^{-3/2}$$

$$\beta_3^*(\xi) = \beta_3\left(\sum_{n=1}^{8}x_{kqn}N^n(\xi_1,\xi_2)\right)\sqrt{L\left(\sum_{n=1}^{8}x_{kqn}N^n(\xi_1,\xi_2)\right)}J_q(\xi_1,\xi_2)$$

Regularization of a hypersingular integrals: (outside singular element)



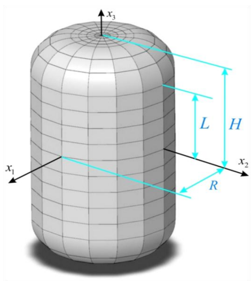
$$\iint_{S_q} \frac{\sqrt{L(\eta)}}{|x-\eta|^3}\beta_3(\eta)\, d_\eta S_q = \int_0^1\int_0^1 \frac{1}{\left(x_1-\eta_1(\xi)\right)^2+\left(x_2-\eta_2(\xi)\right)^2}\beta_3(\eta(\xi))\,J_q(\xi)\,d\xi_1\,d\xi_2 = \int_0^1\int_0^1\frac{\Psi(x,\xi)}{\xi_1^2+\xi_2^2}\beta_3^*(\xi)\,d\xi_1\,d\xi_2$$

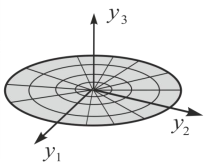
$$=\left(K_{00}^q(x)+I_{00}^q(x)\right)\beta_3^*(0)+K_{10}^q(x)\frac{\partial\beta_3^*}{\partial\xi_1}(0)+K_{01}^q(x)\frac{\partial\beta_3^*}{\partial\xi_2}(0)+\iint_{S_q^\varepsilon}\frac{\Psi(x,\xi)}{\left(\xi_1^2+\xi_2^2\right)^{3/2}}\beta_3^*(\xi)\,d\xi_1\,d\xi_2$$

$$\Psi(x,\xi)=\left[\frac{\left(x_{1q1}-\sum_{n=1}^{8}x_{1qn}N^n(\xi_1,\xi_2)\right)^2+\left(x_{2q1}-\sum_{n=1}^{8}x_{2qn}N^n(\xi_1,\xi_2)\right)^2}{\xi_1^2+\xi_2^2}\right]^{-3/2}$$

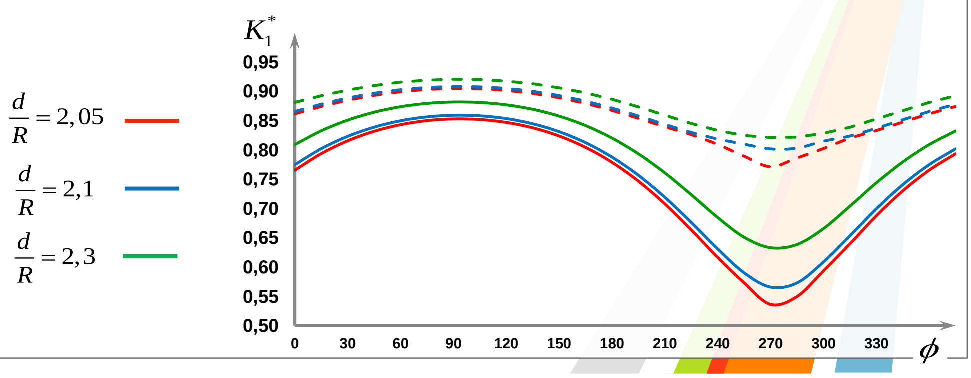
$$\beta_3^*(\xi)=\beta_3\left(\sum_{n=1}^{8}x_{kqn}N^n(\xi_1,\xi_2)\right)\sqrt{L\left(\sum_{n=1}^{8}x_{kqn}N^n(\xi_1,\xi_2)\right)}\,J_q(\xi_1,\xi_2)$$

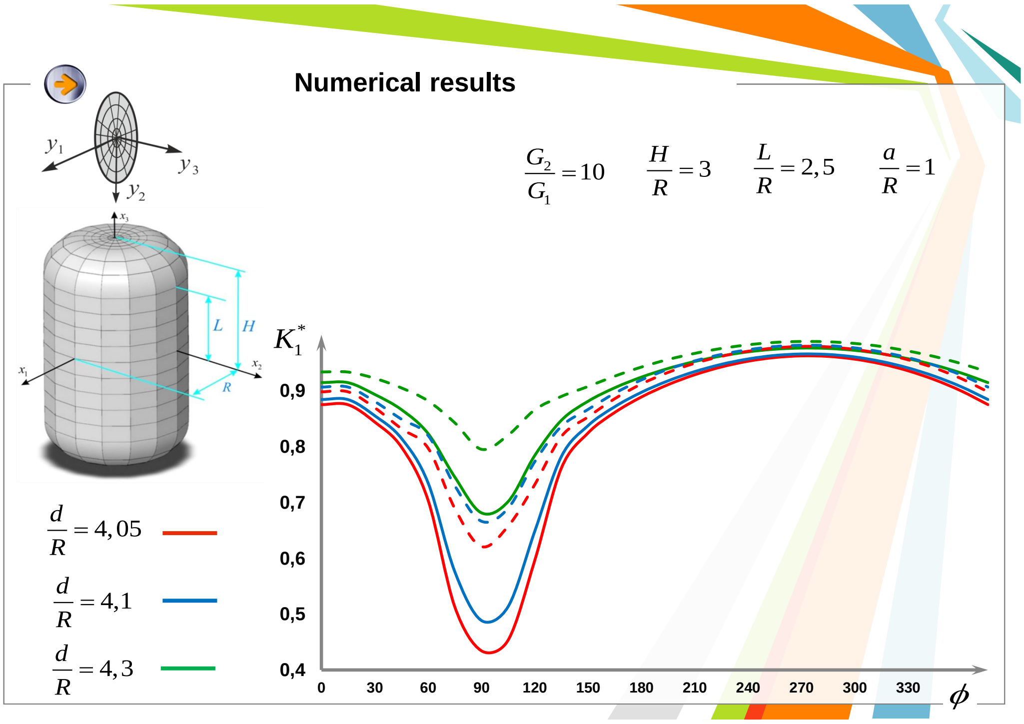
Numerical results

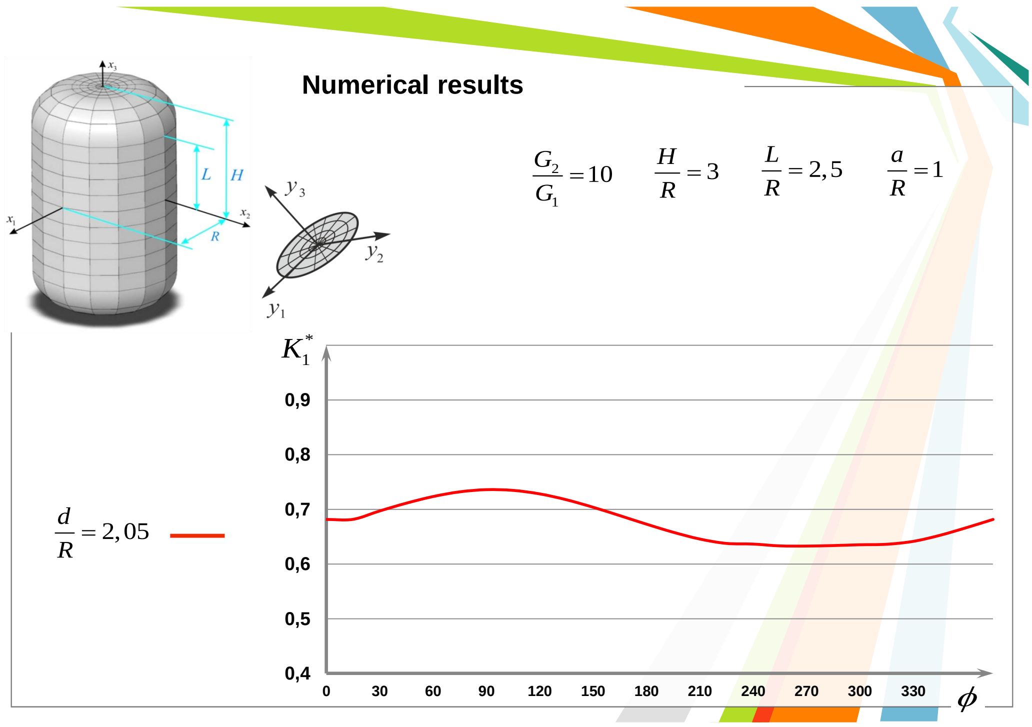




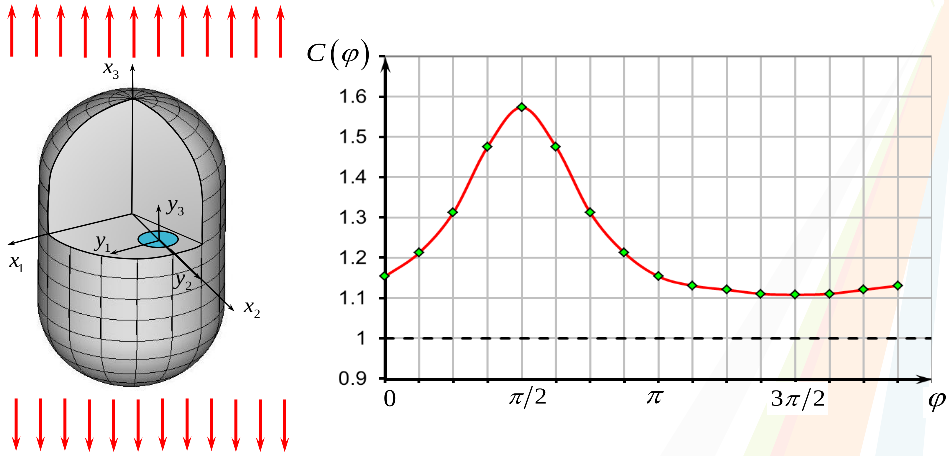
$$\frac{G_2}{G_1}=10 \qquad \frac{H}{R}=3 \qquad \frac{L}{R}=2,5 \qquad \frac{a}{R}=1$$




Numerical results
$\frac{G_2}{G_1} = 10$ $\frac{H}{R} = 3$ $\frac{L}{R} = 2,5$ $\frac{a}{R} = 1$
y_1 y_2 y_3
x_1 x_2 x_3
L H R
$\frac{d}{R} = 4,05$
$\frac{d}{R} = 4,1$
$\frac{d}{R} = 4,3$
K_1^*
0,9
0,8
0,7
0,6
0,5
0,4
0 30 60 90 120 150 180 210 240 270 300 330
ϕ

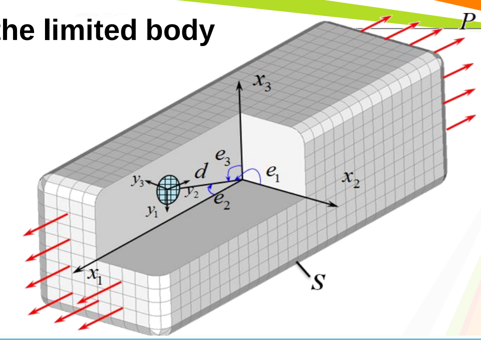

Numerical results
$\frac{G_2}{G_1} = 10$ $\frac{H}{R} = 3$ $\frac{L}{R} = 2,5$ $\frac{a}{R} = 1$
y_1 y_2 y_3
x_1 x_2 x_3
L H R
$\frac{d}{R} = 2,05$
K_1^*
0,9
0,8
0,7
0,6
0,5
0,4
0 30 60 90 120 150 180 210 240 270 300 330
ϕ

Subcase: crack inside of the inclusion



$H/R=2 \qquad G_B/G_M=10 \qquad d=0{,}8R \qquad a=0{,}1R$

Special case: crack in the limited body



body surface

$$\sum_{j=1}^{3}\iint_{S} T_{ij}(\mathbf{x},\xi)\left[u_j(\xi)-u_j(\mathbf{x})\right]d_\xi S-\sum_{j=1}^{3}\iint_{S} U_{ij}(\mathbf{x},\xi)P_j(\xi)\,dS_\xi=\sum_{s=1}^{3}\sum_{j=1}^{3}\iint_{S_T}\Phi_{sj}\left(\mathbf{x}^*,\xi\right)\alpha_j(\xi)l_{si}d_\xi S$$

$$\sum_{i=1}^{3}\iint_{S_T}\alpha_i(\xi)K_{ij}(\xi,\mathbf{y})\,dS_\xi=\frac{1}{G}\sum_{k=1}^{3}\sum_{s=1}^{3}\sum_{m=1}^{3}\iint_{S_P}D_{smk}\left(\mathbf{y}^*,\xi\right)P_k(\xi)l_{s3}l_{mj}\,dS_\xi-$$
$$-\sum_{k=1}^{3}\sum_{s=1}^{3}\sum_{m=1}^{3}\iint_{S_U}S_{smk}\left(\mathbf{y}^*,\xi\right)u_k(\xi)l_{s3}l_{mj}\,dS_\xi,\ \mathbf{y},\mathbf{y}^*\in S_T,\quad j=\overline{1,3}.$$

crack

Numerical results

$l=1м$ $\quad l_0=70см$

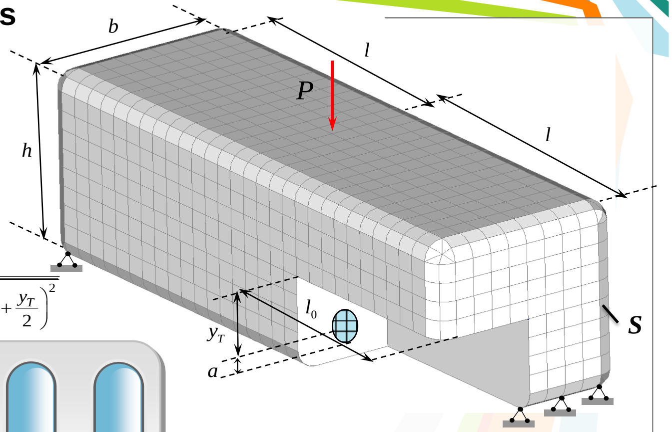
$b=20см$ $\quad h=30см$

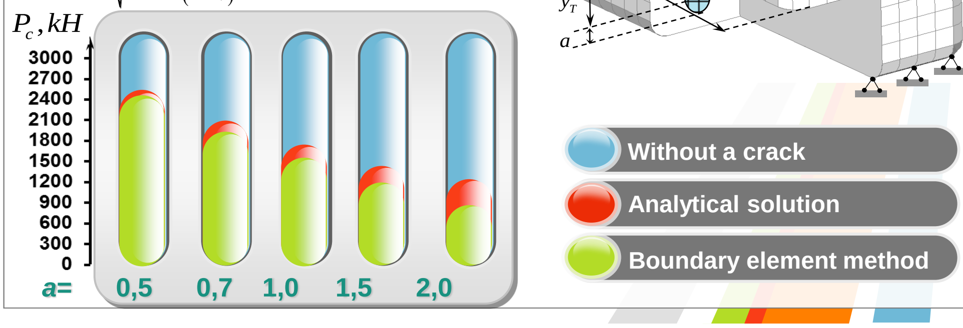
$y_T=10см$ $\quad \nu=0,27$

$K_{IC}=31,0\,МПа\sqrt{м}$

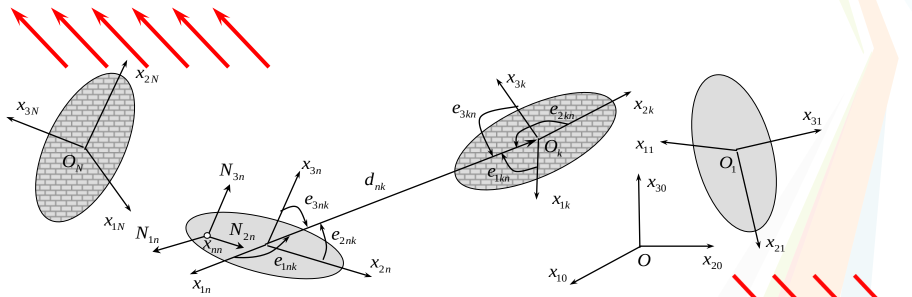
$$(1-\nu)\left(k_1^2+k_2^2\right)+k_3^2\le(1-\nu)K_{IC}^2$$

$$P\le\sqrt{\frac{\pi}{a}}\,\frac{K_{IC}}{24}\,\frac{b\,h^3}{\sqrt{l_0^2\,y_T^2+\frac{1}{(2-\nu)^2}\left(\frac{h}{2}-y_T\right)^2\left(\frac{h}{4}+\frac{y_T}{2}\right)^2}}$$





Thin-walled inclusion- crack interaction



Crack

$$\sigma_{i3n}\left(x_n\right)\Big|_{x_n\in S_n^{\pm}}=0 \qquad j=1,2,3$$

$$\alpha_{jn}\left(x_n\right)=\frac{u_{jn}^{-}\left(x_n\right)-u_{jn}^{+}\left(x_n\right)}{4\pi}\Bigg|_{x_n\in S_n}\neq 0$$

Thin inclusion

$$x_{3k}=\pm h_k\left(x_{1k},x_{2k}\right) \qquad \sigma_{i3k}\left(x_k\right)\Big|_{x_k\in S_k^{+}}=\sigma_{i3k}\left(x_k\right)\Big|_{x_k\in S_k^{-}}$$

$$\sigma_{33k}\left(x_k\right)\Big|_{x_k\in S_k}=4\pi G_k\frac{2(1+\nu_k)}{1-2\nu_k}\frac{\beta_{3k}\left(x_k\right)}{h_k\left(x_k\right)} \qquad \sigma_{j3k}\left(x_k\right)\Big|_{x_k\in S_k}=4\pi G_k\frac{\beta_{jk}\left(x_k\right)}{h_k\left(x_k\right)} \qquad j=1,2$$

Boundary integral equation

Superposition principle

$$\sigma(\mathbf{x})=\sigma^0(\mathbf{x})+\sum_{n=1}^{N}\sigma_n^m(\mathbf{x})+\sum_{k=N+1}^{N+K}\sigma_k^e(\mathbf{x})$$

$$\sum_{n=1}^{N}\sum_{i=1}^{3}\iint_{S_k}\alpha_{in}(\xi)K_{ijnm}(\xi,x_{nm})\,dS_\xi+\sum_{k=N+1}^{N+K}\sum_{i=1}^{3}\iint_{S_k}\beta_{ik}(\xi)K_{ijkm}(\xi,x_{km})\,dS_\xi=\frac{1-\nu}{G}N_{jm}(x_{mm}),\quad j=\overline{1,3}\quad m=\overline{1,N}\quad x_{km}\in S_m$$

$$\sum_{n=1}^{N}\sum_{i=1}^{3}\iint_{S_k}\alpha_{in}(\xi)K_{i3nm}(\xi,x_{nm})\,dS_\xi-4\pi(1-\nu)\frac{G_m}{G}\frac{2(1+\nu_m)}{1-2\nu_m}\frac{\beta_{3m}(x_m)}{h_m(x_m)}+\sum_{k=N+1}^{N+K}\sum_{i=1}^{3}\iint_{S_k}\beta_{ik}(\xi)K_{i3km}(\xi,x_{km})\,dS_\xi=\frac{1-\nu}{G}N_{3m}(x_{mm}),$$

$$m=\overline{N+1,K+N}\qquad x_{km}\in S_m$$

$$\sum_{n=1}^{N}\sum_{i=1}^{3}\iint_{S_k}\alpha_{in}(\xi)K_{ijnm}(\xi,x_{nm})\,dS_\xi-4\pi(1-\nu)\frac{G_m}{G}\frac{\beta_{3m}(x_m)}{h_m(x_m)}+\sum_{k=N+1}^{N+K}\sum_{i=1}^{3}\iint_{S_k}\beta_{ik}(\xi)K_{ijkm}(\xi,x_{km})\,dS_\xi=\frac{1-\nu}{G}N_{jm}(x_{mm}),$$

$$j=1,2\qquad m=\overline{N+1,K+N}\qquad x_{km}\in S_m$$

$$x_{1kn}=e_{1kn}\,d_{kn}+\sum_{s=1}^{2}l_{skn}\,x_{snn}\qquad x_{2kn}=e_{2kn}\,d_{kn}+\sum_{s=1}^{2}m_{skn}\,x_{sn}\qquad x_{3kn}=e_{3kn}\,d_{kn}+\sum_{s=1}^{2}n_{skn}\,x_{sn}$$

Numerical results: thin-walled inclusion- crack interaction

Matrix $G=7\cdot10^4\ MPa$ $\nu=0,3$

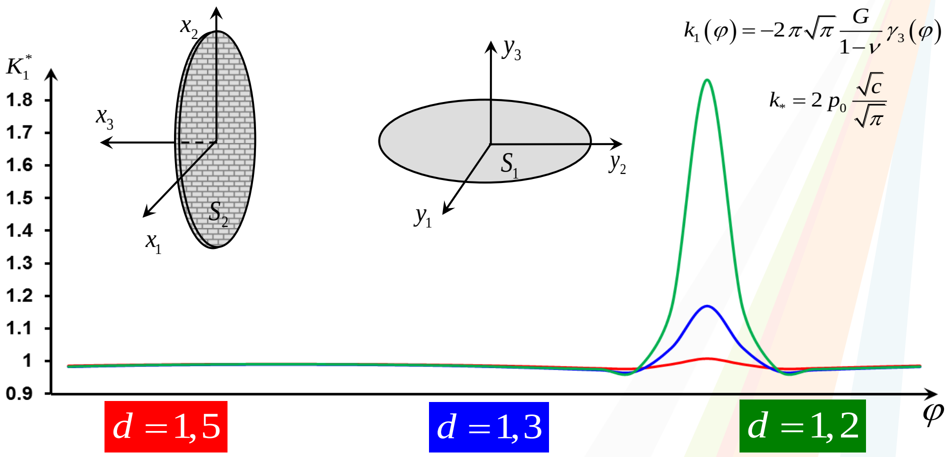
Inclusion $G=7\cdot10^3\ MPa$ $\nu=0,3$ $a=10$ $b=5$ $\varepsilon=0,04$

Crack $c=1$

$$K_1^*=\frac{k_1}{k_*}$$

$$k_1(\varphi)=-2\pi\sqrt{\pi}\,\frac{G}{1-\nu}\gamma_3(\varphi)$$

$$k_*=2\,p_0\frac{\sqrt{c}}{\sqrt{\pi}}$$

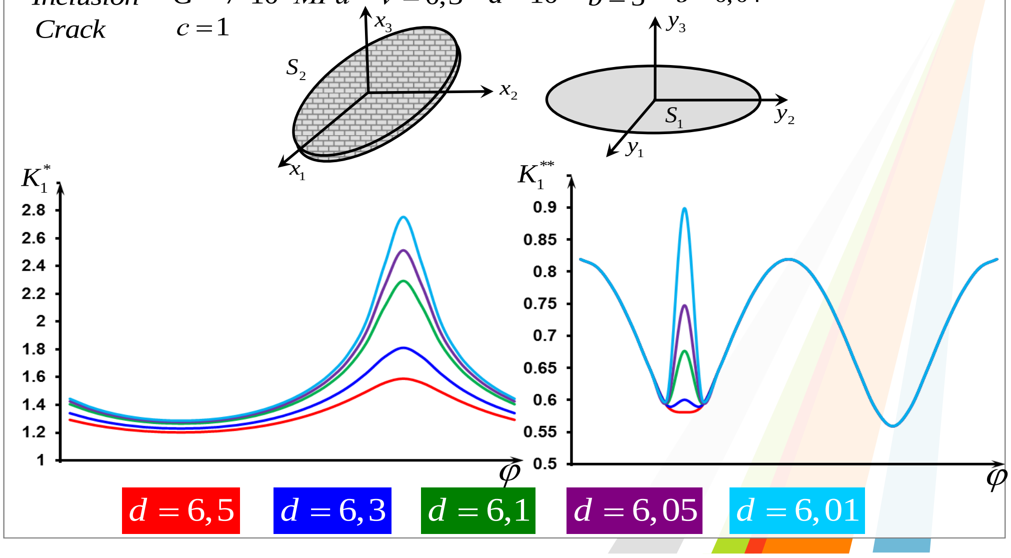


Numerical results: thin-walled inclusion- crack interaction

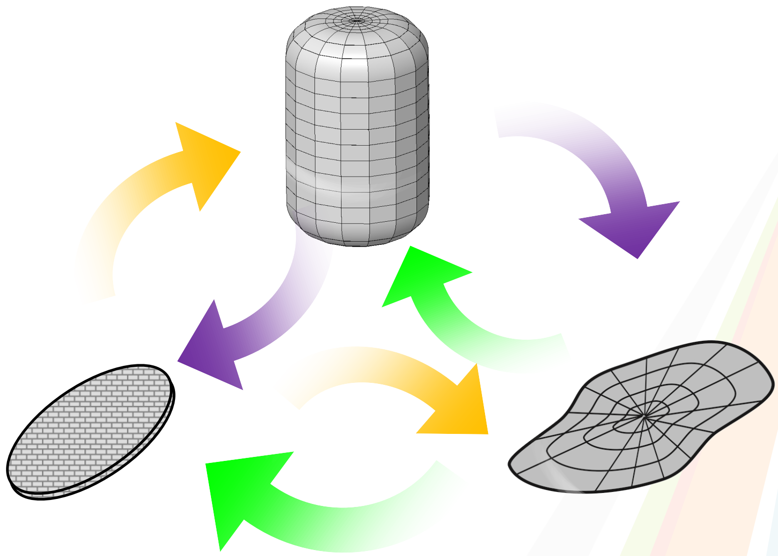
Matrix $G = 7 \cdot 10^4\ MPa$ $\quad \nu = 0,3$

Inclusion $G = 7 \cdot 10^3\ MPa$ $\quad \nu = 0,3$ $\quad a = 10$ $\quad b = 5$ $\quad \varepsilon = 0,04$

Crack $c = 1$



Effective stress field method



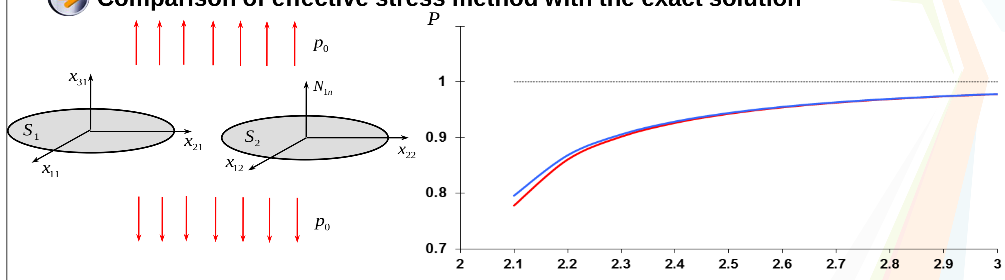
Modification of the boundary integral equation

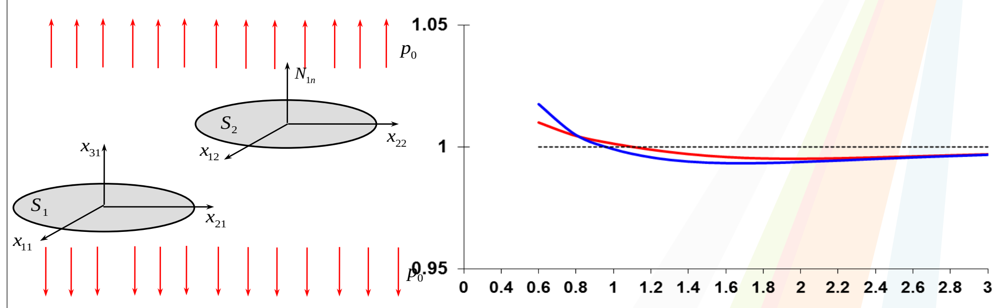
$$\iint_{S_m}\left[\left(\frac{1+\nu}{\left|x_{mm}-\xi\right|^3}-\frac{3\nu\left(x_{2mm}-\xi_2\right)^2}{\left|x_{mm}-\xi\right|^5}\right)\alpha_{1m}(\xi)+\frac{3\nu\left(x_{1mm}-\xi_1\right)\left(x_{2nn}-\xi_2\right)}{\left|x_{mm}-\xi\right|^5}\alpha_{2m}(\xi)\right]d\,S_\xi=\frac{1-\nu}{G}N_{1m}(x_{mm})-$$
$$-\sum_{n'=1}^{N}\sum_{i=1}^{3}\iint_{S_k}\left[\alpha_{in}(\xi)K_{i1nm}(\xi,x_{nm})\right]d_\xi S-\sum_{k=N+1}^{N+K}\sum_{i=1}^{3}\iint_{S_k}\left[\beta_{ik}(\xi)K_{i1km}(\xi,x_{km})\right]d_\xi S-\frac{1}{G}\sum_{j=N+K+1}^{N+K+M}\sum_{i=1}^{3}\iint_{S_{Bj}}D_{13i}^{M}(y,\xi)P_i(\xi)d\,S_{j\xi}+$$
$$+\sum_{j=N+K+1}^{N+K+M}\sum_{i=1}^{3}\iint_{S_{Bj}}S_{13i}^{M}(y,\xi)u_i(\xi)d\,S_{j\xi}$$

$$\iint_{S_m}\left[\frac{3\nu\left(x_{1mm}-\xi_1\right)\left(x_{2mm}-\xi_2\right)}{\left|x_{mm}-\xi\right|^5}\alpha_{1m}(\xi)+\left(\frac{1+\nu}{\left|x_{mm}-\xi\right|^3}-\frac{3\nu\left(x_{1mm}-\xi_1\right)^2}{\left|x_{mm}-\xi\right|^5}\right)\alpha_{2m}(\xi)\right]d\,S_\xi=\frac{1-\nu}{G}N_{2m}(x_{mm})-$$
$$-\sum_{n'=1}^{N}\sum_{i=1}^{3}\iint_{S_k}\left[\alpha_{in}(\xi)K_{i2nm}(\xi,x_{nm})\right]d_\xi S-\sum_{k=N+1}^{N+K}\sum_{i=1}^{3}\iint_{S_k}\left[\beta_{ik}(\xi)K_{i2km}(\xi,x_{km})\right]d_\xi S-\frac{1}{G}\sum_{j=N+K+1}^{N+K+M}\sum_{i=1}^{3}\iint_{S_{Bj}}D_{23i}^{M}(y,\xi)P_i(\xi)d\,S_{j\xi}+$$
$$+\sum_{j=N+K+1}^{N+K+M}\sum_{i=1}^{3}\iint_{S_{Bj}}S_{23i}^{M}(y,\xi)u_i(\xi)d\,S_\xi$$

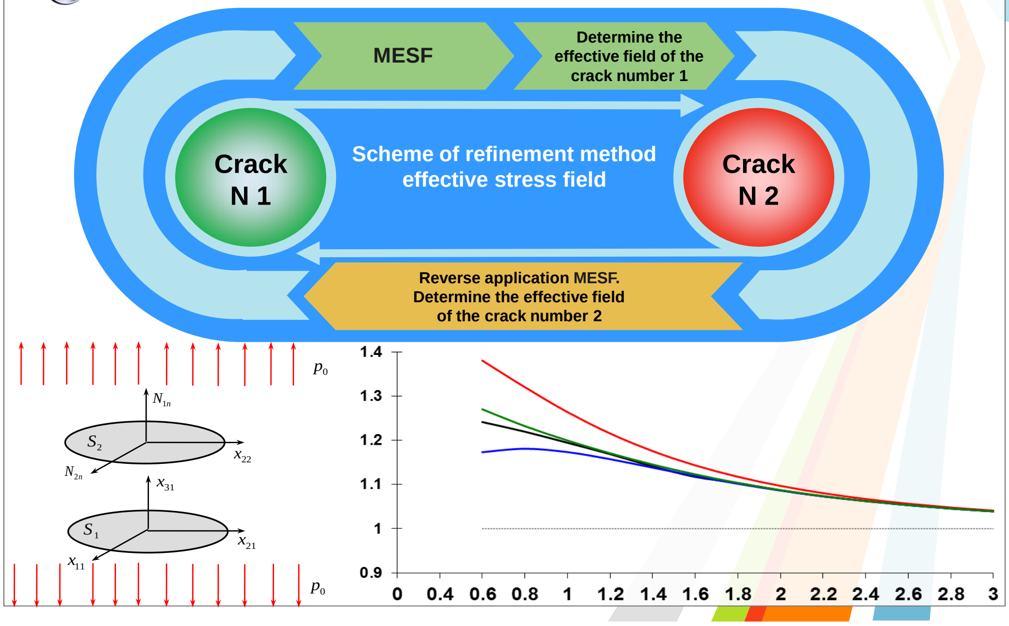
$$\iint_{S_m}\frac{\alpha_{3m}(\xi)}{\left|x_{mm}-\xi\right|^3}d\,S_\xi=\frac{1-\nu}{G}N_{3m}(x_{mm})-\sum_{n'=1}^{N}\sum_{i=1}^{3}\iint_{S_k}\left[\alpha_{in}(\xi)K_{i3nm}(\xi,x_{nm})\right]d_\xi S-\sum_{k=N+1}^{N+K}\sum_{i=1}^{3}\iint_{S_k}\left[\beta_{ik}(\xi)K_{i3km}(\xi,x_{km})\right]d_\xi S-$$
$$-\frac{1}{G}\sum_{j=N+K+1}^{N+K+M}\sum_{i=1}^{3}\iint_{S_{Bj}}D_{33i}^{M}(y,\xi)P_i(\xi)d\,S_{j\xi}+\sum_{j=N+K+1}^{N+K+M}\sum_{i=1}^{3}\iint_{S_{Bj}}S_{33i}^{M}(y,\xi)u_i(\xi)d\,S_\xi$$

Comparison of effective stress method with the exact solution

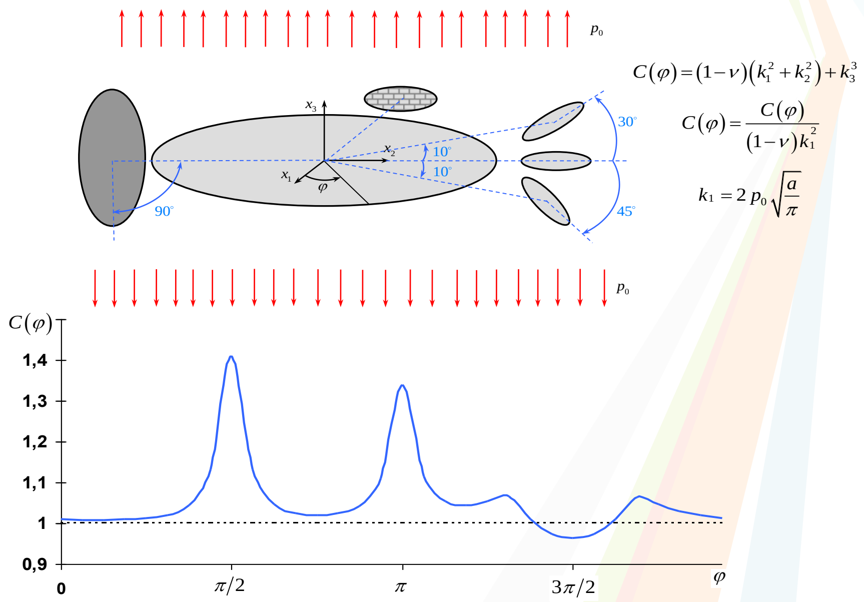




Refinement method effective stress field



Numerical results: method effective stress field




Thank You!