Probabilistic and statistical approaches of integrity and residual lifetime assessment of structural elements

O. Yasniy*

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*Ternopil Ivan Pul'uj National Technical University, Ukraine

Overview

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- Present state of probabilistic methods
- Fracture mechanics concepts
- Failure assessment diagram (FAD)
- Concept of probabilistic assessment
- Types and distributions of input parameters
- Data fit and its verification
- Statistical description of *da*/*dN*-curves
- Statistical description of *C* and *m*
- Ex.1: Probabilistic lifetime assessment of plate with central crack
- Ex.2: Probabilistic assessment of lifetime of railway axle
- Probabilistic assessment of cracked structures limit state
- Ex. 3: Limit state of reactor vessel model
- Conclusions

Present state of probabilistic methods

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- Emergence of probabilistic and reliability analysis in the middle of 20th century
- Probabilistic methods at that time, no certainly unified methods, often very simplified
- Quasi-probabilistic approaches for engineer applications (partial safety factors)
- Probabilistic fracture methods as an add-on for deterministic approaches in different standards: BS7910, R6, SINTAP, FITNET, FM-codes
- FM-software: ProSINTAP, ProSACC, EIFSIM, …

Fracture mechanics concepts

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Common applications and methods of structural elements assessment with cracks under static, cyclic and dynamic loading

- Input values: geometry, loading, material state
- Static loading: FAD concept
- Cyclic loading: crack growth calculations
- Limit state: critical loading and maximal failure size from FAD, accepted crack size

FAD concept

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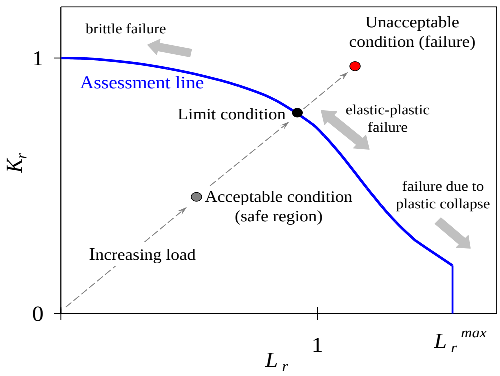

Actual point:

$$K_r = \frac{K_{Ip}}{K_{mat}} + \frac{K_{Is}}{K_{mat}} + \rho$$

$$L_r = \frac{P}{P_L} = \frac{\sigma_{ref}}{\sigma_Y}$$

Limit curve:

$$f(L_r) = \left(1 + \frac{L_r^2}{2}\right)^{-1/2}\left[0.3 + 0.7\exp(-\mu L_r^6)\right], \quad L_r \le 1$$

$$f(L_r) = f(1) L_r^{\frac{N-1}{2N}}, \quad 1 < L_r \le L_r^{\max}$$

$$\mu = \min\left[0.001\left(\frac{E}{\sigma_Y}\right); 0.6\right], N = 0.3\left(1 - \frac{\sigma_Y}{\sigma_U}\right), L_r^{\max} = \frac{1}{2}\frac{\sigma_Y + \sigma_U}{\sigma_Y}$$

Crack growth calculations

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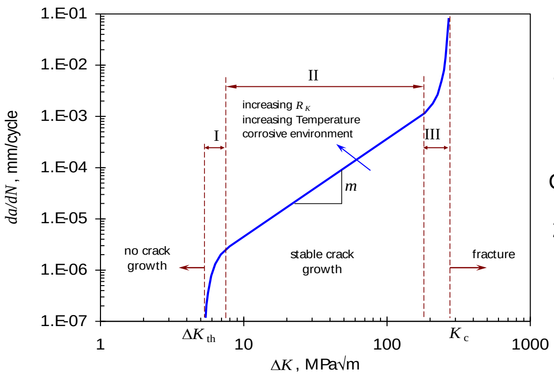

$$\frac{da}{dN} = f(\Delta K, R_K, ...)$$

1. Paris-Erdogan equation:

$$\frac{da}{dN} = C\Delta K^m$$

Constant amplitude and $R_K = K_{min}/K_{max}$

2. NASGRO equation:

$$\frac{da}{dN} = C\left[\left(\frac{1-f_c}{1-R_K}\right)\Delta K\right]^m \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{max}}{K_c}\right)^q}$$

Variable amplitude and R_K

Concept of probabilistic assessment

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- Statistical description of data scatter (geometry, loading, and material state)
- Implementation of probabilistic fracture mechanics calculations with appropriate methods: Monte-Carlo simulations (MCS), MCS-IS, FORM, SORM
- Quantitative description of results: probability of failure, variability of life, initial crack size, etc.

Types and distributions of input parameters

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Distribution type	Parameters of distribution	Probability density function	Eq.
normal	μ - mean σ - variance $-\infty < x < \infty, \sigma > 0$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$	(1)
lognormal	x_0–location parameter m - scale parameter σ - shape parameter $x_0 \leq x < \infty, m > 0, \sigma > 0$	$f(x) = \frac{1}{(x-x_0)\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\left(\ln\frac{x-x_0}{m}\right)^2\right]$	(2)
Weibull	x_0-location parameter β - shape parameter η - scale parameter $x_0 \leq x < \infty, \eta > 0, \beta > 0$	$f(x) = \frac{\beta}{\eta}\left(\frac{x-x_0}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x-x_0}{\eta}\right)^{\beta}\right]$	(3)

Data fit and its verification

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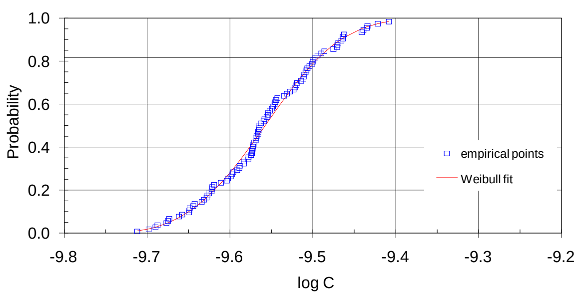
- **Methods:**
  - Regression analysis
  - Method of moments
  - Maximal likelihood estimation

Criteria and their verification:

- Estimation of failure
- Hypothesis, afterwards GOF-Tests (Anderson-Darling, Kolmogorov-Smirnov, χ^2)



RMS error
X: 0.0047523
Y: 0.021266

AD GoF test accepted
Significance level: 0.1
AD statistical: 0.2438
AD critical: 0.637

c.o.v.: 0.32142

median: -9.56156

Statistical description of *da/dN*-curves

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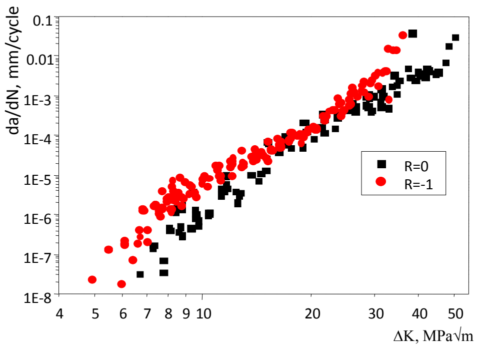
FCG diagrams of steel (0, 45% C) for *R*=0 and *R*=-1



Consideration and description of scatter of

- Paris constants, C and m
- Threshold value, ΔKth
- Fracture toughness, K_c

Statistical description of *C* and *m*

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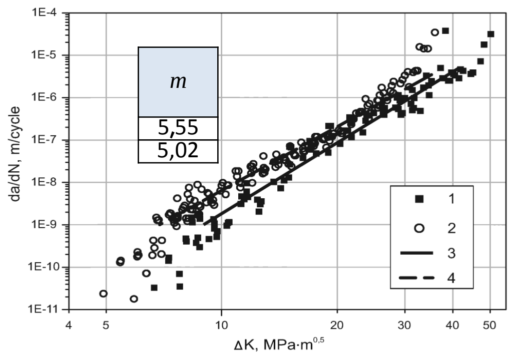
- *C* and *m* are often observed as dependent parameters
- Thus one of this parameters was fixed and another was varied
- *m* was estimated from the fit with least-squares method
- *C* was treated as a random variable obtained from many statistical experiments



Statistical description of *C* and *m*

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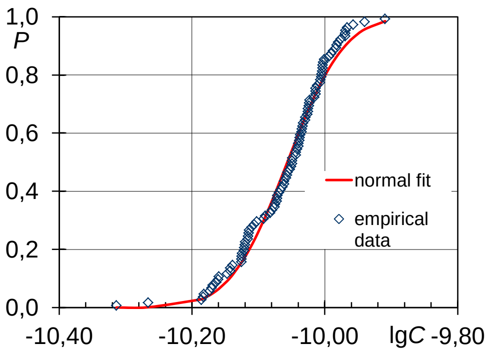
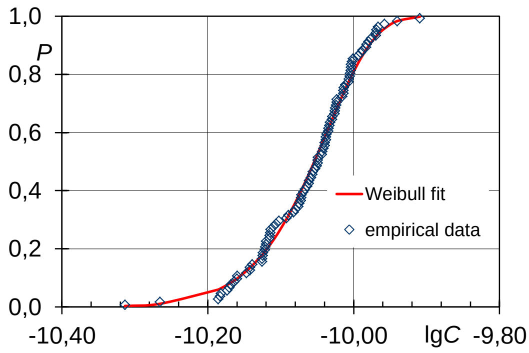


Weibull distribution of lg*C*



$$F(\lg C) = 1 - \exp\left[-\left(\frac{\lg C + 10.81}{0.782}\right)^{12.7}\right]$$

Railway axle with surface defect

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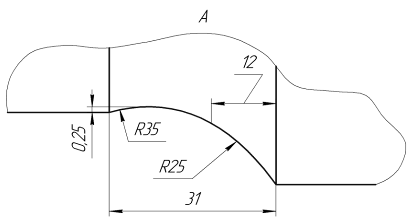

Fig. 1. Dimensions of axle fillet

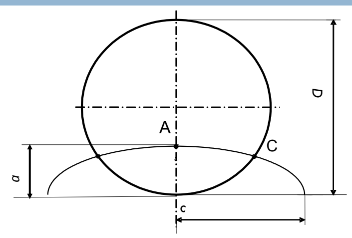

Fig. 2. Cylinder with semi-elliptical surface crack

Semi-elliptical crack with semi-axis ratio of *a/c* = 0.4 was considered (Fig. 2). Crack depth *a* was chosen equal to 0.5 mm, 1.0 mm, 3.0 mm, 8.0 mm, 16.0 mm and 32.0 mm. Axle diameter *D* in the place of crack was 129.5 mm.

The SSS and SIF of railway axle in highest stresses local field were assessed, where the cracks initiate most often - the place of transition from cylindrical part of axle with diameter 130 mm to the fillet with *R* = 25 and 35 mm (Fig.1).

Finite element modelling of railway axle

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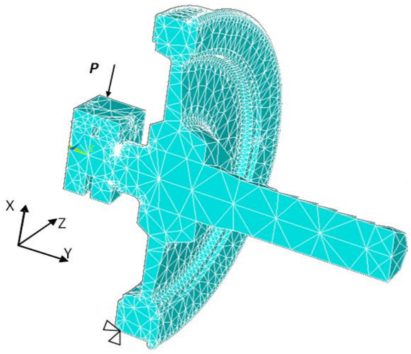

Fig. 1. FEM of axle

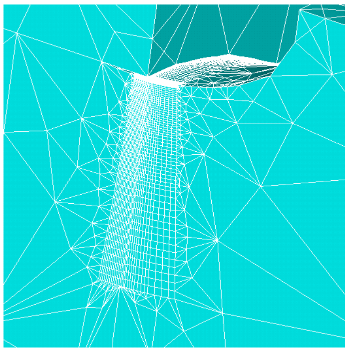

Fig. 2. Meshing fragment

Three-dimensional finite element SOLID95 was chosen for the model. This element contains 20 nodes (including intermediate). The ordered mapped meshing of finite elements was used for modelling and assessment of stress-strain state in the tip of semi-elliptical crack. The loading on axle-box *P* = 260 kN.

Finite element analysis of railway axle

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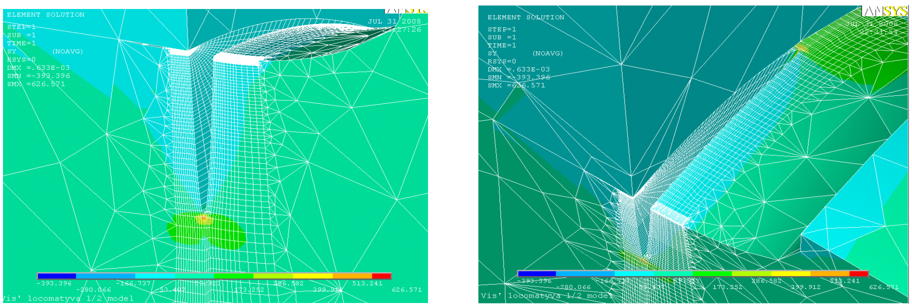

Fig. 1. Stress distribution and detail of the mesh for a crack with
a = 16.0 mm and a/c = 0.4.

Stress intensity factors of railway axle

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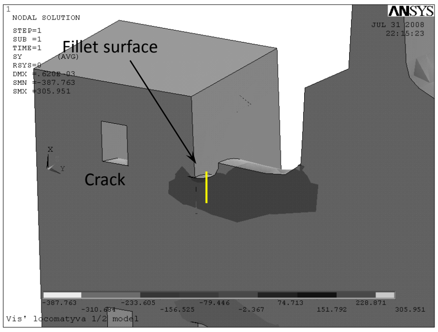

Fig. 1. Stress distribution on the surface of fillet.

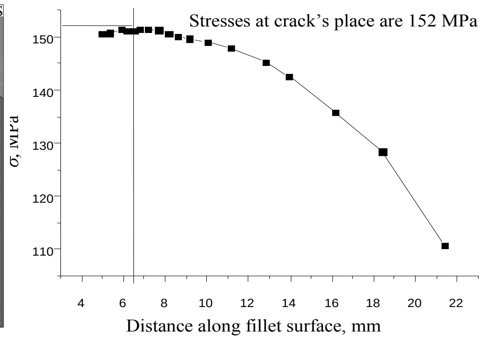

Fig. 2. The distribution of normal stresses along the fillet surface of axle without crack under loading on journal box P = 260 kN

The dimensionless SIF Y was calculated by formula: $Y = K / \left(\sigma_b \sqrt{\pi a} \right)$

where σ_b – normal stress.

Stress intensity factors of railway axle

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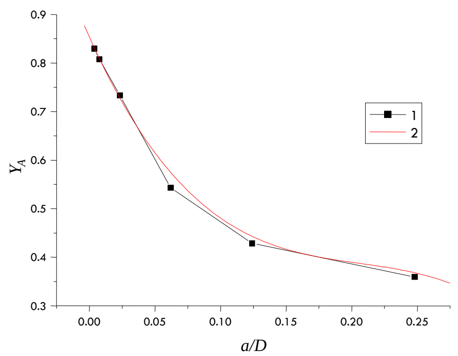

Fig. 1. Dependence of Y_A on a/D: FEM (1); FEM approximation (2)

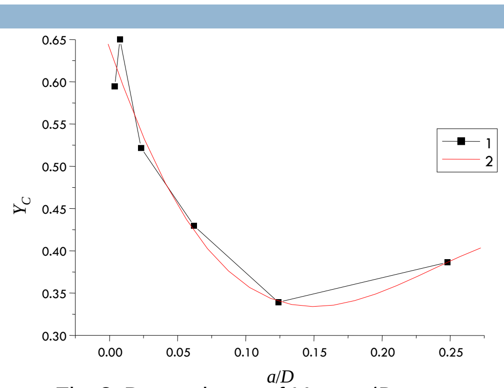

Fig. 2. Dependence of Y_C on a/D: FEM (1); FEM approximation (2)

$$Y_A = 0,854 - 6,027\lambda + 27,839\lambda^2 - 44,290\lambda^3 \quad (1)$$

$$Y_C = 0,642 - 4,865\lambda + 23,757\lambda^2 - 33,466\lambda^3 \quad (2)$$

Example 1: Probabilistic assessment of lifetime of plate with central crack

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Steel 0.45% C. Plate with central crack. t = 5 mm, W=23,5 mm, a_0=5,1 mm. Paris equation, m = 5.06

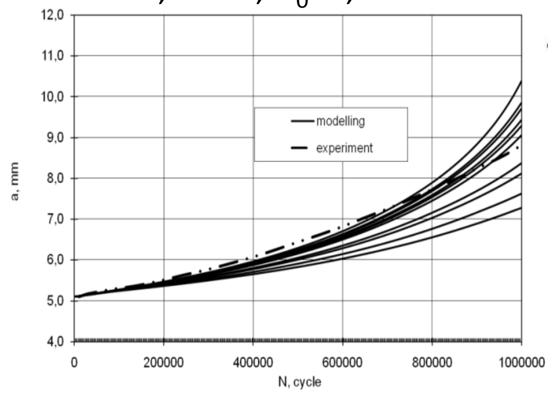

Fig. 1. Crack length a vs. N, a_0=5,1 mm

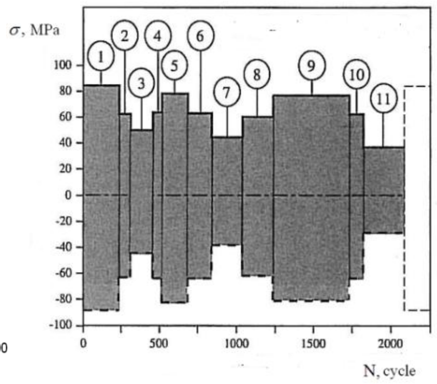

Fig. 2. Block loading [*Zerbst*, 2005]

Probabilistic assessment of lifetime of plate with central crack

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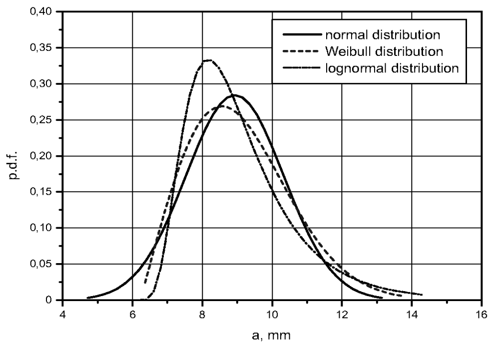

Fig. 1. P. d. f. of final crack length

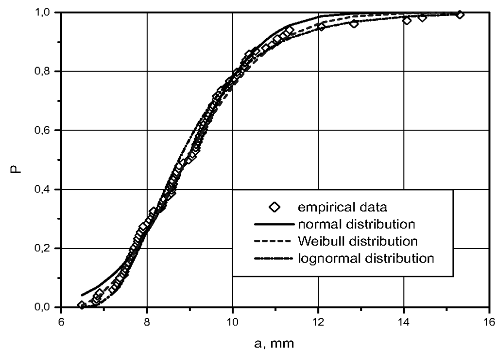

Fig. 2. C. d. f. of final crack length

Example 2: Probabilistic assessment of lifetime of railway axle

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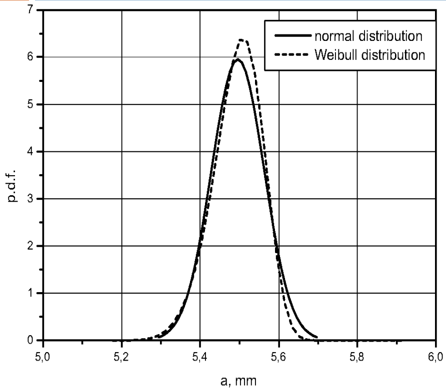

Fig. 1. P. d. f. of final crack depth a_f, a_0=5 mm

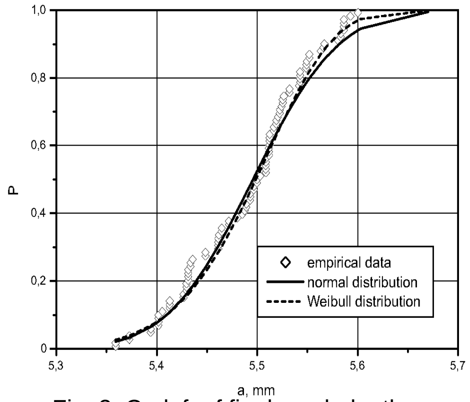

Fig. 2. C. d. f. of final crack depth a_f, a_0=5 mm

a_0, mm	Normal distribution					Weibull distribution					
	μ	σ	α	AD stat	AD critical	x_0	η	β	α	AD stat	AD critical
5,0	5.496	0.067	0.05	0.659	0.754	5.159	0.36029	6.2072	0.05	0.6058	0.757

Probabilistic assessment of lifetime of railway axle

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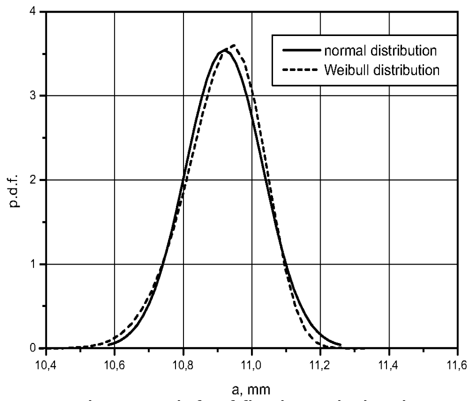

Fig. 1. P. d. f. of final crack depth a_f, a_0=10 mm

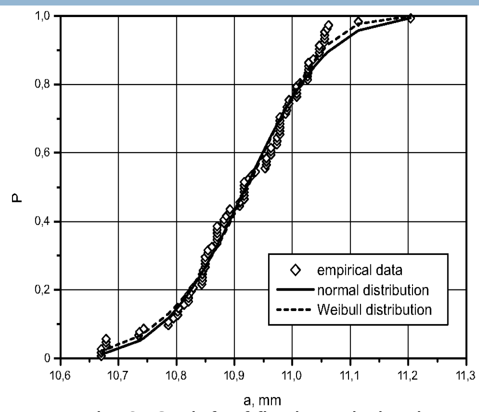

Fig. 2. C. d. f. of final crack depth a_f, a_0=10 mm

a_0, mm	Normal distribution					Weibull distribution					
	μ	σ	α	AD stat	AD critical	x_0	η	β	α	AD stat	AD critical
10,0	10.92	0.1126	0.05	0.8332	0.754	10.351	0.60889	5.8827	0.05	0.5784	0.757

Probabilistic assessment of the limit state of structures with the cracks

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The failure probability is multidimensional definite integral

$$P_f = \int_{g(\mathbf{x})\le 0} f_X(\mathbf{x})dx. \quad (1)$$

Two different limit state functions *g* (*x*) are used

$$g_{FAD}(\mathbf{x}) = g_{FAD}(K_{Ic}, \sigma_{0.2}, a) = f_{FAD} - K_r$$
$$g_{L_r}{}^{\max}(\mathbf{x}) = g_{L_r}{}^{\max}(\sigma_{0.2}, \sigma_U, a) = L_r{}^{\max} - L_r, \quad (2)$$

where $\sigma_U = \sigma_B$ — ultimate tensile strength; *a* — crack size; L_r — ratio of the applied stress to yield stress of the material of the structure with the crack. Limit state functions are based on the standardized procedure SINTAP.

Example 3: Limit state of reactor vessel model

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The probability of failure assessment of reactor vessel model after WPS on the basis of FAD taking into account the statistical distributions

- depth of the crack *a*,
- internal pressure *p*,
- yield strength $\sigma_{0.2}$,
- ultimate tensile strength σ_U
- fracture toughness of the material K_{Ic} for the case of loading-cycle with total unloading of the specimen.

The model of pressure vessel with a crack on the inner wall

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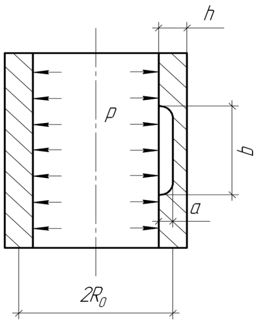

Geometry			
$2R_0$	***h***	***a***	***a/b***
mm			
4130	**140**	**16-30, step 2**	**2/3**

Mechanical properties of 15Cr2MoV (III) steel at 293 K

$\sigma_{0.2}$, MPa	σ_B, MPa	δ, %	ψ, %
1100	1160	16.6	67.2

Stress intensity factor (SIF) and limit load (P_L)

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$$K = \sigma_\Theta \sqrt{\pi a} Y, \text{ where } Y = 1.14 - 0.48\frac{a}{b} + \frac{1}{0.2 + 4.9\left(\frac{a}{b}\right)^{1,2}}\left(\frac{a}{h}\right)^2, \quad \sigma_\Theta = \frac{pR_0}{h}$$

[Helliot J., 1979]

$$P_L = \frac{\sigma_y}{(s+b)} \cdot \left(s \ln\left(\frac{R_2}{R_1}\right) + b\left(\frac{R_1}{R_1 + c}\right) \ln\left(\frac{R_2}{R_1 + c}\right)\right),$$

$$\text{where } s = \frac{bc(1 - c/w)}{MR_1\left(\ln\left(\frac{R_2}{R_1}\right) - \left(\frac{R_1}{R_1+c}\right)\ln\left(\frac{R_2}{R_1+c}\right)\right) - c}$$

[Laham S., 1998]

$$M = \left(1 + \frac{1.61 b^2}{R_1 c}\right)^{1/2}$$

Probability density functions

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Input data	Distribution type	Parameters of distribution
p, MPa	normal	μ_p=32.5; σ_p=3.25; 4.875; 6.5
$\sigma_{0,2}$, MPa	lognormal	x_0=1080, m=20, σ=0.4
σ_B, MPa	lognormal	x_0=1140, m=8, σ=0.6
K_{mat}, MPa√m	Weibull	x_0 = 141, β = 14.39, η = 2.04

FAD of pressure vessel model under alternating pressure for different crack depth (28, 30, 32 mm)

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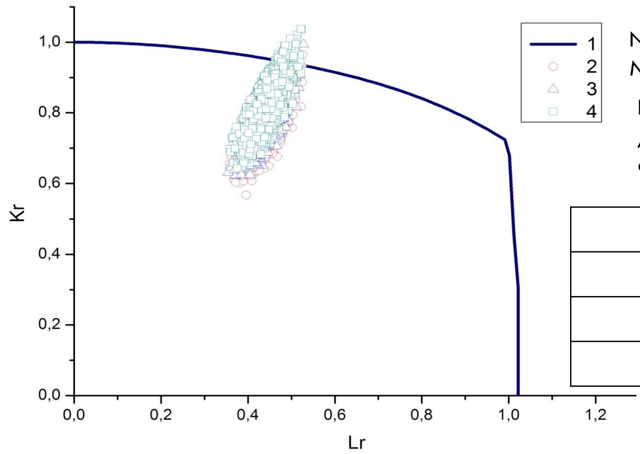

Number of simulations
N = 10^4

Normal distribution of p:
μ_p = 32.5 MPa,
cov_p = 0.05.

a, mm	P_f
28	$2 \cdot 10^{-4}$
30	$3.2 \cdot 10^{-3}$
32	$1.35 \cdot 10^{-2}$

(1) a=28 mm; (2) a=30 mm; (3) a=32 mm

Dependence of failure probability $\boldsymbol{P_f}$ on a crack depth by alternating pressure, calculated by **MCIS**

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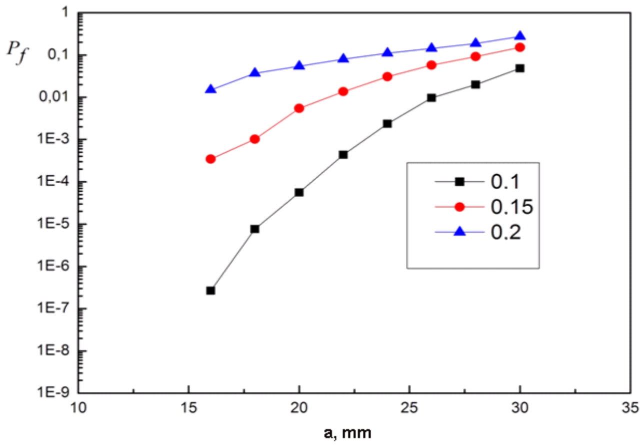

cov_p=0.1 (1); 0.15 (2); 0.2(3)

Dependence of failure probability $\boldsymbol{P_f}$ on a crack depth by alternating pressure, calculated by **FORM**

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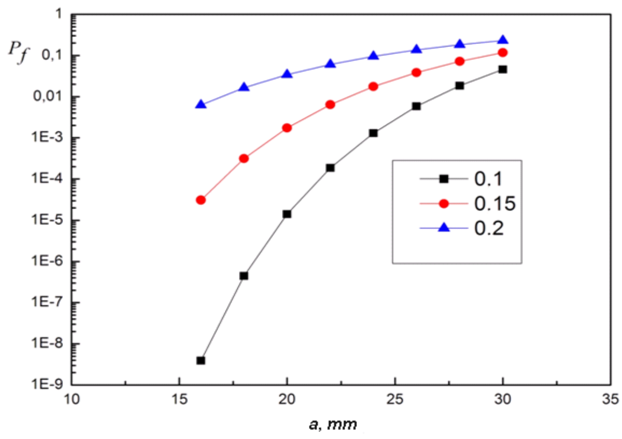

cov_p=0.1 (1); 0.15 (2); 0.2(3)

CONCLUSIONS

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- The c.d.f of cyclic crack resistance characteristics (parameter lg*C* of Paris law) of 0.45% C steel were constructed and tested by Anderson-Darling GOF.
- The probabilistic analysis of lifetime of commuter train axle with the surface semi-elliptical crack was performed.
- The distribution functions for final crack depth in axle after 10^6 cycles of block loading were obtained depending on initial defect size.
- The dependencies of reactor model failure probability on the crack depth *a* were obtained by method of MCIS and FORM.
- The FAD with Monte Carlo method for different crack depth were constructed, considering the pressure as normally distributed random variable.